Gaussian Beam Mode Analysis of Imaging Arrays

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Abstract

In this paper we consider the Gaussian beam mode analysis of imaging arrays on submillimetre-wave radio telescopes. We show how it is possible to describe any of the off-axis beams generated by the array in terms of the same on-axis beam mode set. The mapping process can be easily analysed using a modal approach.

1. Introduction

Focal plane arrays for submillimetre-wave radio telescopes are now being developed at a number of observatories. The imaging process in such systems is nearly always described in terms of Fourier optics [1]. It is more physically meaningful, however, in a telescope coupled to an array of mixers by a quasi-optical beam guide to develop the corresponding Gaussian beam mode model. In general, multi-mode Gaussian Beam Mode Analysis (GBMA) of beam guides is particularly powerful in terms of its conceptual accessibility, offering an intuitive understanding of the way in which submillimetre-wave optical systems operate [2].

This is a new application of Gaussian Beam Mode Analysis (GBMA), as the approach is normally used in modelling a single on-axis beam produced by some horn antenna in a single pixel detection process [3][4][5][6]. Nevertheless, since GBMA should be applicable to any field which is well defined on some input plane, it should be possible, at least in theory, to model an *off-axis* feed horn of an array system in terms of *on-axis* beam modes. However, if the on-axis beam mode set appropriate for a single horn is used, it is obvious that very high order modal contributions are necessary to describe the off-axis beams. The feasibility of the GBMA depends on finding a more appropriate mode set which can describe any off-axis beam of an array with a modest number of modes, and thus be efficient computationally. In section 2 we describe how the best choice mode set (from a computational point of view) can be selected in a practical way for an array of beams propagating through an optical system of finite throughput. The mode set is parameterized in terms of the most appropriate choice for the beam waist radius, W_o. The mode set, for which the optics can transmit the maximum number of modes intact, is the "best" choice mode set, in the sense that it is the one which minimises the number of modes required in a modal expansion description of the any of the off-axis beams on the output plane (the sky). The number of modes in the best choice set turns out to have a natural relationship to the throughput of the optics and, consequently, the number of independent channels by which information can be transmitted by the optics. An example case of a telescope fed by a 4×4 array of horns is presented to show the power of the approach.

The natural modes of an imaging system of *finite* throughput are the eigenfunctions of the diffraction integral which maps the input aperture (taken to be the field of view of the telescope) onto the sky [2]. Although these modes are not the same as Gaussian beam modes, since finite throughput implies truncation, nevertheless the best choice mode set is a good approximation to the true modes of the system. This is discussed in section 3. In section 4 mapping and image reconstruction is discussed in terms of GBMA. Describing image reconstruction turns out to be very natural from this viewpoint, with the Nyquist rate being related to the total number of modes required in a modal expansion of the image, again emphasising the fundamental physical significance of the best choice mode set. In section 5 we briefly discuss how aberrations can be included in the description.

2. GBMA applied to an array of beams.

2.1. Choosing optimum mode set

Gaussian beam modes are solutions to the wave equation parameterized in terms of an arbitrary beam waist radius W_0 and beam waist position. In practice, for computational efficiency these are chosen so the beam can be described to high accuracy with as few modes as possible [7]. If we are interested in describing the output beam from an optical system fed by an *off-axis* horn, it is not necessary to describe the very high spatial frequency content of the fields of the horn, because truncation and aberrations will cause spatial filtering. Thus, the image produced at the output plane by the optical system will be band limited. Since the spatial frequency content of a mode depends both on the mode number n and the beam width parameter, W_o, we expect that both the best choice for W_o and the number of modes required to describe the beam at the output plane will be affected by the spatial filtering characteristics of the optical system as well as on the input field distribution. The optimum mode set is the one with the largest number of modes not significantly affected by truncation and vignetting in the process of propagation through the optics from the input plane to the output plane; this is the mode set which requires the minimum number of modes to describe the fields adequately at the output plane.

Two stops are, in fact, required to define the optimum mode set uniquely (the same is true for the definition of throughput). Consider the simple, but instructive, example of an arbitrary field of finite extent at some plane with a finite sized lens in the far field (see figure 1). We assume the field has zero curvature on the input plane, so that the beam waist is best chosen to be there. We define the input and output stop widths in terms of the extent of the input field (field stop) and the radius of the lens (aperture stop). At both the input and lens stop the truncation level depends on the local value of r/W, where r is the radius of the stop. The greater r/W at an aperture is, the greater the number of unscattered modes that will squeeze through. However, the fact that the lens is in the far field of the input plane implies an inverse relationship between the respective beam widths since $W_1W_2 = \lambda z/\pi$. Therefore, if we decrease W_1, so as to reduce truncation effects on the modes at the entrance stop, and thus allow more modes through without scattering, we unfortunately increase W_2 at the lens causing a greater level of truncation at the exit stop. In this situation we get the greatest number of unscattered modes to squeeze through both apertures, if we arrange that the level of truncation, defined by $t = r/W$, to be the same at both, so that

$$t_1 = \frac{r_1}{W_1} = t_2 = \frac{r_2}{W_2}$$

which since $W_1W_2 = \lambda z/\pi$, implies

$$W_1^2 = \frac{\lambda z r_1}{\pi r_2}$$

In order to determine the number of modes that can squeeze through a stop, we choose to define a mode as significantly truncated if the coupling of overall power to the transmitted *propagating* mode, $\psi_{n,trans}$, from the incident *propagating mode* is less than 50% (see figure 2). That is, $|\int_A(\psi^*_{n,inc}\psi_{n,trans})dA|^2 = (\int_A |\psi_n^2|dA)^2 < 0.5$, where the subscript A denotes surface integration over the area of the stop. For one-dimensional Hermite modes, it is found that the mode number of the highest order mode which just squeezes through with 50% coupling loss at the stop of width r is given by $n \approx (1.2\, r/W)^2$. The number of modes which "squeeze through" is therefore given approximately by

$$N \approx \frac{4r_1r_2}{\lambda z}.$$

The throughput of a stop is defined, in terms of classical optics, to be the product of the area of the stop and the viewing solid angle, $A\Omega$. In this case we can take A to be the

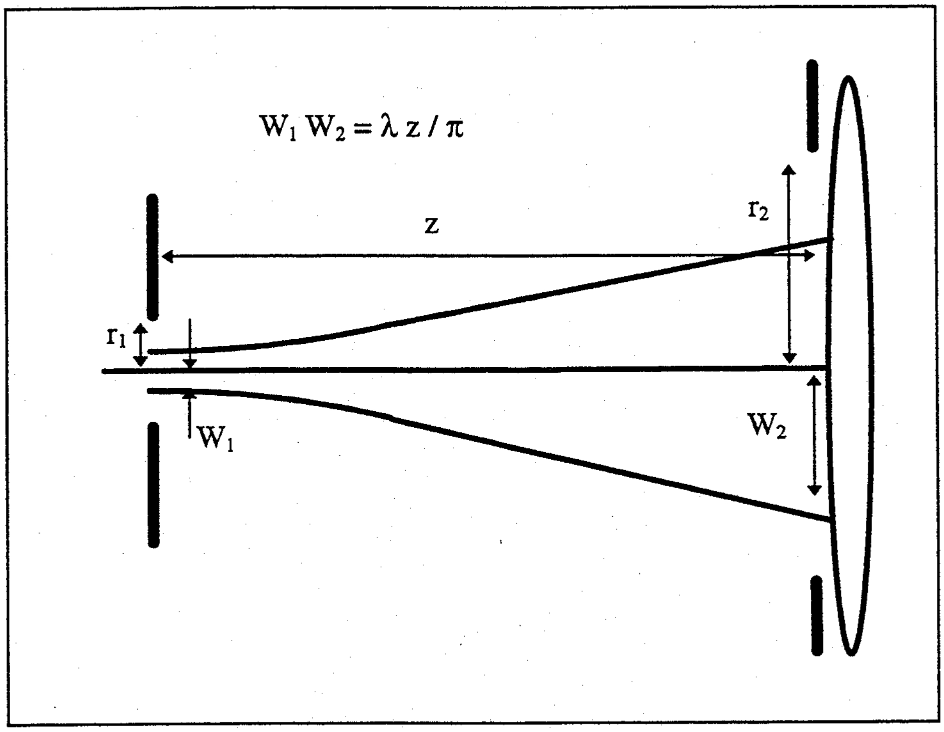

Figure 1. Simple single lens optical system

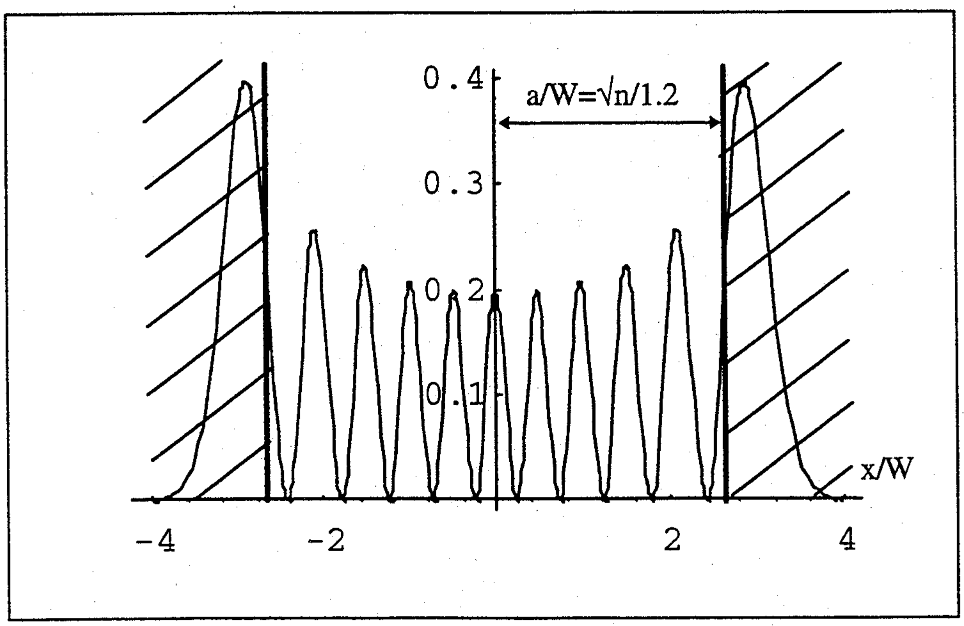

Figure 2. 50% power coupling loss due to beam truncation for h_{10} mode.

area of the entrance stop at the input plane,which defines the physical extent of the field, and Ω to be the solid angle subtended by the lens at the input plane. Thus, since for square apertures, $\Omega = (2r_2/z)^2$ and $A = (2r_1)^2$, we can write $A\Omega = (4r_1r_2/z)^2 = N^2\lambda^2$. Thus, the number of modes transmitted turns out to have a natural relationship to the throughput of the optics.

For a more complicated quasi-optical system the best value of W_o, is effectively defined by the two most truncating stops in the optical train (see figure 3). These can be found by estimating the effect of changing W_o on the resulting truncation levels $t_i = r_i/W_i$ at *each* truncating stop. The various t_i for all of the truncating stops can be plotted as a function of W_o, as illustrated in figure 3. What we seek is the value of $W_o = W_{o,opt}$ for which the value of t_i for the two most truncating stops is as large as possible. As will be seen from observation of figure 3, this occurs when the two relevant curves $t_i(W_o)$ cross. $W_{o,opt}$ then parameterizes the mode set for which the greatest number of modes is transmitted through the system.

2.2. Reconstructing the beam pattern of an array

In practice if we wish to reconstruct the output field to high accuracy we can use a larger number of modes than that defined by the highest order mode that suffers 50% power loss; $2N$ modes is usually more than sufficient. Although we intimated above that any mode that suffered less than 50% power truncation at any stop effectively squeezes through the optics, in practice, of course, the higher order modes do suffer some power attenuation and scattering to other modes [8]. A full scattering analysis needs to be made of the system and the scattering matrix $\mathbf{S}$ evaluated, but just including $(2N)^2$ modes [9]. Since the optimum mode set is used in determining $\mathbf{S}$, it will tend, in fact, to be almost diagonal. The significance of this is discussed further in the next section.

As an example we take an 4×4 square array of *long* square scalar horns of sidelength a, feeding a telescope with, for simplicity, a square aperture of sidelength b. The telescope reflector we will assume to be in the far field of the array, at distance of $z = f$, the focal length of the telescope. We can take the aperture field of the horn to be given by $E_h = E_o \cos(\pi x/a)\cos(\pi y/a)$. The best fit Gaussian beam to E_h has a beam width parameter at the horn apertures of $W_h = 0.350a$; thus, to a good approximation the field at a horn mouth can be taken to be $E_h = E_o \exp(-(x^2+y^2)/W_h^2)$. If we assume a 10dB edge taper at the telescope aperture (of width D_t), then for the Gaussian beam approximation to E_h at the telescope, $W_{ht} = 0.466D_t$. This implies that $W_h = \lambda f/\pi W_{ht} = 0.684\lambda F$, fixing $a = 1.95\lambda F$, where F is the F-ratio of the telescope beam given by $F = f/D_t$. For a single on-axis horn beam it would be adequate to use a mode set parameterized by a beam width parameter of W_h, to describe the beam propagation. The best choice of W_f for the array on the focal plane, however, depends also on the extent of the array, which

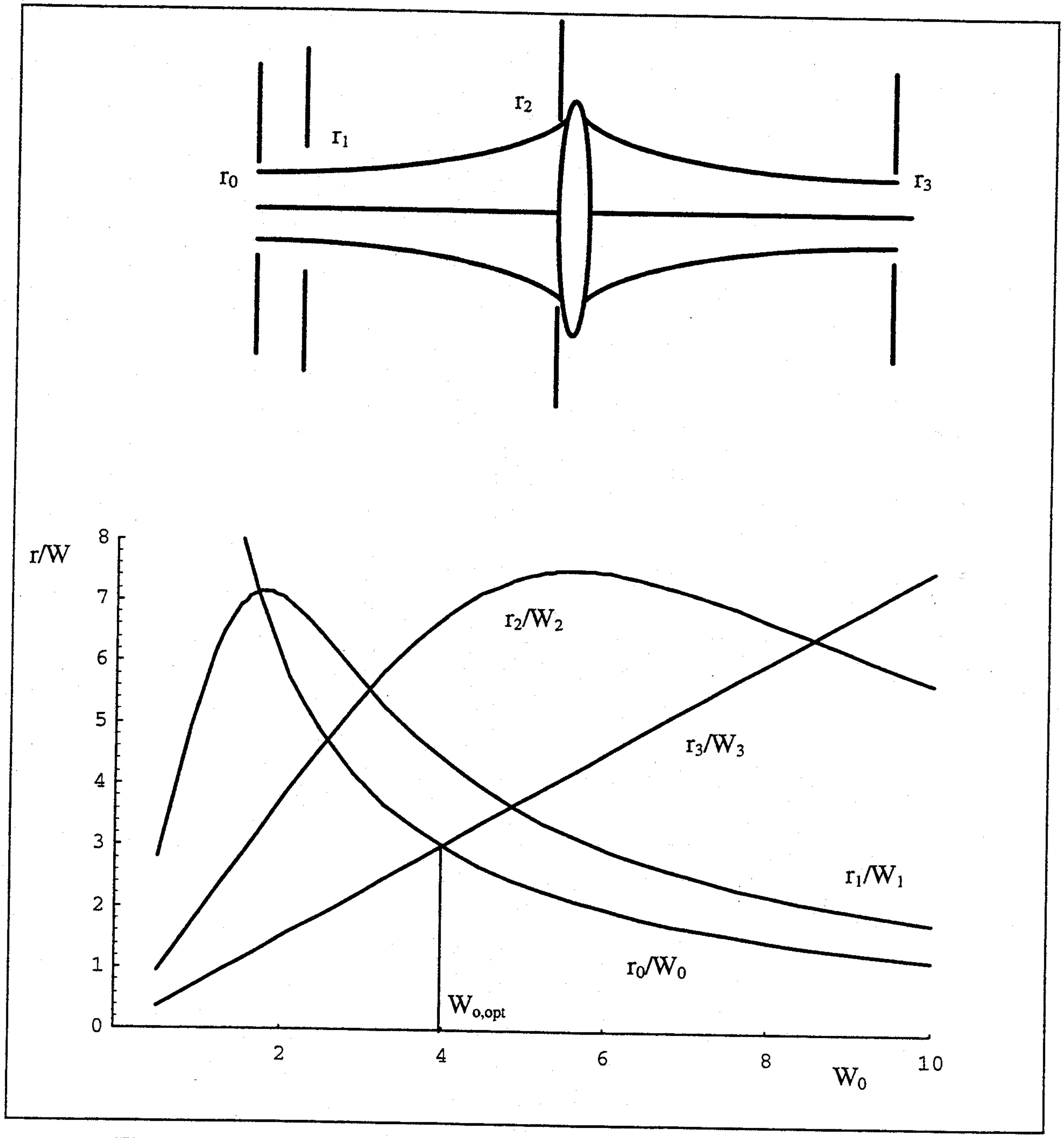

Figure 3: Truncation levels as a function of W_0 for stops in optical system.

defines the input pupil as well as the exit pupil. W_f is given by

$$W_f = \sqrt{\frac{\lambda f.4a}{\pi D_t}} \approx 1.58\lambda F = 0.808a.$$

We expect to be able to adequately describe the 4×4 array of beams on the sky with $(2N \times 2N)$ modes, where

$$N \approx (1.2 \times 2a/W_f)^2 \approx 9.$$

Figure 4 shows a 1-dimensional cut through the array of beam patterns reconstructed on the sky.

3. Integral Analysis - interpreting the best choice mode set.

The natural modes of an imaging system of finite throughput are the eigenfunctions of the diffraction integral equation which maps the input plane onto the output plane[2]. Assume that we have some optical system, and that we wish to find the field at the output plane, E_2, corresponding to a given field at the input plane, E_1. By linear superposition, the field at the output plane is expressible in the form [2]

$$E_2(\mathbf{r}_2) = \int_{P_1} E_1(\mathbf{r}_1) K(\mathbf{r}_2|\mathbf{r}_1) dS$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are coordinates in the input and output plane respectively, and $K(\mathbf{r}_2|\mathbf{r}_1)$ is the appropriate kernel, also known as the point spread function or transfer function. In the case of any array the kernel is the field pattern on the sky of a point radiator (or detector) at the plane defined by the imaging array. This function includes the influences of all the optical components.

From the modal (or eigenfunction) point of view we can look for field distributions, $\phi(\mathbf{r})$ that remain unchanged after passing through the system:

$$\lambda\phi(\mathbf{r}_2) = \int_{P_1} \phi(\mathbf{r}_1) K(\mathbf{r}_2|\mathbf{r}_1) dS$$

The fact that in general $|\lambda| \leq 1$, implies that mode attenuation in the system is possible. This happens through a process of truncation and diffraction which nevertheless reproduces the input field distribution at the output plane. These eigenfunctions can be thought of as modes of the system rather than free space modes. In fact, if the optical system is perfect, in the sense of producing no truncation or aberrational effects, the $\phi(\mathbf{r})$ clearly have the same functional form as the Gaussian beam modes, at least in the limit of the Fresnel approximation [10], [11]. We can expand any field in terms of these normal modes. Thus, $E_1(\mathbf{r}_1) = \sum A_n \phi_n(\mathbf{r}_1)$, while $E_2(\mathbf{r}_2) = \sum A_n \lambda_n \phi_n(\mathbf{r}_2)$.

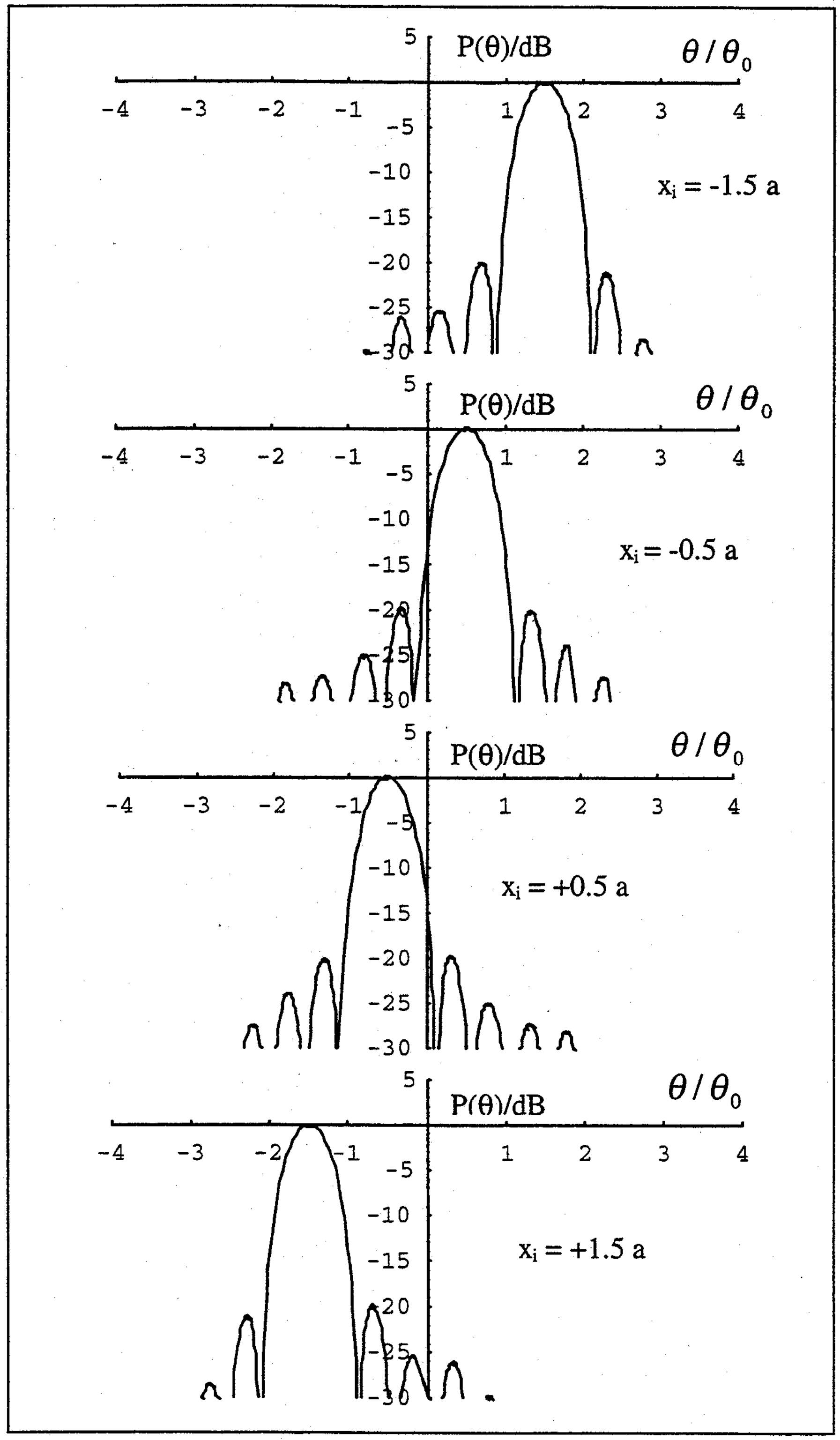

Figure 4: Beam patterns on sky due to an array of horn antennas (produced with an 18×18 mode set). Note: $\theta_0 = a / f$

In any real quasi-optical system clearly the optical components (lenses, mirrors, stops, pupils etc.) making up the beam guide will be finite in size, and may introduce aberrations into the beam. If we analyse propagation through such a system using propagating Gaussian beam modes, rather than applying the diffraction integral, we have to take into account the fact that the beam is no longer in free space as it propagates in the vicinity of an optical component. Boundary conditions are introduced by such components, and only certain linear combinations of the free space modal solutions are possible at the aperture (analogous to the modes in a waveguide). This situation can be handled using scattering matrix analysis [9], where an incident free space mode is regarded as having its power attenuated both because of truncation, and because the truncated mode that results is not a pure propagating mode, so that there is scattering of some of the transmitted power to other modes as well. This scattering process may happen many times in a real quasi-optical system and an overall scattering matrix for the system $\mathbf{S}$ can be computed.

For the best choice of W the scattering effect is minimised on the maximum number of modes, resulting in the scattering matrix S for the optical system being approximately diagonal. These beam modes, ψ_n, are therefore good approximations to the true eigenmodes of the optical system for those modes for which the magnitude of the eigenvalue is effectively unity, $|\lambda| = 1$. We can in fact derive the eigenvalues and Gaussian beam mode expansions for the eigenfunctions of the integral equation by computing the eigenvalues and eigenfunctions of $\mathbf{S}$, defined by $\mathbf{S}\boldsymbol{\chi}^n = \lambda_n \boldsymbol{\chi}^n$. Here $\boldsymbol{\chi}_i$ is a vector of mode coefficient $\{\chi_i^n\}$, in terms of which ϕ_n can be expanded $\phi_n = \sum_i \chi_i^n \psi_i$. (Note that even though it may not be always possible to derive ϕ_n analytically by solving the eigenvalue integral equation, it often still possible to obtain a mode expansion of the solution.)

4. GBMA of Mapping

Consider a source field with a brightness distribution $B(\theta, \phi)$, which is to be mapped by a radio telescope fed by an array of horns. When the source is observed with an antenna of power pattern $P(\theta, \phi)$, a flux density S_o is observed given by [12]

$$S_o = \int_{source} B(\theta, \phi) P(\theta, \phi) d\theta d\phi$$

In the imaging system $P(\theta, \phi)$ represents one of the array of beam patterns produced by the telescope. Consider off-axis horn i, with a field across its aperture expressed as an expansion in terms of the optimum mode set, and given by

$$E_i(x, y) = \sum_{m,n} A_{mn}^i h_m(x; W) h_n(y; W).$$

Here $h_m(x;W)$, etc., represents a Hermite Gaussian mode of order m, parameterized by the optimum beam width parameter, W, and defined by

$$h_m(x;W) = \frac{H_m(\sqrt{2}x/W)}{\sqrt{2^m m! \sqrt{\pi/2}\, W}} \exp\left(-\frac{x^2}{W^2} - j\frac{\pi x^2}{\lambda R}\right).$$

The corresponding telescope beam pattern is given by $P_i(\theta,\phi) = |E_{i,s}(\theta,\phi)|^2$, where $E_{i,s}(\theta,\phi)$, the field pattern on the sky, is given by $E_{i,s}(\theta,\phi) = \sum_{m,n} C^i_{mn} h_m(\theta;\omega) h_n(\phi;\omega)$, with the C^i_{mn} related to the A^i_{mn} via the usual scattering relationship

$$C^i_{mn} = \sum_{m,\mu,n,\nu} S_{m\mu,n\nu}\, A^i_{\mu\nu},$$

(in matrix notation $\mathbf{C} = \mathbf{SA}$.) Note, that here ω is the corresponding best choice beam width (in angle) on the sky, assuming small angles. Since we have chosen to analyse the problem in terms of the optimum mode set, we can define the scattering matrix to be finite in size so that $m_{\max} = \mu_{\max} = n_{\max} = \nu_{\max} = N$, where N is mode number of the highest order mode that just suffers less than 50% power coupling loss at each of the two most truncating stops. Thus, C^i_{NN} is the highest order mode coefficient, and the highest order mode in the expansion at the output plane for $E_{i,s}(\theta,\phi)$ is $h_N(\theta;\omega)h_N(\phi;\omega)$.

$P_i(\theta,\phi)$ can be written as the bandlimited *bi-modal* sum

$$P_i(\theta,\phi) = \sum_{m,m',n,n'}^{N} (C^i_{mn})^*(C^i_{m'n'})\,[h_m(\theta;\omega)^* h_{m'}(\theta;\omega)][\,h_n(\phi;\omega)^* h_{n'}(\phi;\omega)].$$

It is possible, however, to write this in a more convenient form. The product $[h_m(\theta;\omega)^* h_{m'}(\theta;\omega)]$ is a polynomial p of order $m+m'$ in θ/ω, multiplied by a Gaussian term $\exp(-(\theta/\omega)^2)$, to be squared

$$\begin{aligned}[h_m(\theta;\omega)^* h_{m'}(\theta;\omega)] &\propto H_m(\sqrt{2}\theta/\omega)\exp(-(\theta/\omega)^2)\, H_{m'}(\sqrt{2}\theta/\omega)\exp(-(\theta/\omega)^2).\\ &= p_{m+m'}(\theta/\omega)\exp(-2(\theta/\omega)^2)\end{aligned}$$

But *any* polynomial of order $m+m'$ can be written as a *finite* orthogonal series of Hermite polynomials,

$$p_{m+m'}(\theta/\omega) = \sum_{0}^{m+m'} \eta_k H_k(c(\theta/\omega)),$$

where c is any constant. Therefore, if we define $\omega_P = \omega/\sqrt{2}$, the bi-modal product $[h_m(\theta;\omega)^* h_{m'}(\theta;\omega)]$ can clearly be re-expressed precisely in terms of a finite sum of Hermite Gaussian modes (with the Gaussian beam width, ω_P),

$$[h_m(\theta;\omega)^* h_{m'}(\theta;\omega)] = \sum_{k}^{m+m'} \chi_k h_k(\theta;\omega_P)$$

Thus, $P_i(\theta,\phi)$ becomes a standard beam mode expansion of the form

$$P_i(\theta,\phi) = \sum_{k,l}^{2N} P^i_{kl}\; h_k(\theta;\omega_P)\; h_l(\phi;\omega_P),$$

where the mode coefficients P^i_{kl} are

$$P^i_{kl} = \sum_{m,m',n,n'=0}^{N} \left(C^*_{mn} C_{m'n'} \xi_{mm'k} \xi_{nn'l}\right),$$

and

$$\xi_{mm'k} = \int h_k(\theta;\omega_P)\; [h_m(\theta;\omega)^* h_{m'}(\theta;\omega)]\; d\theta$$

Note that $h_k(\theta)$ is real (i.e. $R=\infty$), so $h_k(\theta) = h_k(\theta)^*$. Clearly, if the mode expansion for the horns A^i_{mn} and the scattering matrix for the system $S_{m\mu,n\nu}$ are known, then the P^i_{kl} are readily calculable. ω_P is best choice beam width parameter for the mode set for describing the beam pattern on the sky, $P_i(\theta,\phi)$, as any other value for ω would require a larger number of modes than $2N \times 2N$ in the modal expansion.

V_i represents a measurement of the flux coupled from the celestial source to the ith horn, [12], [13].

$$V_i = \int P_i(\theta,\phi) B(\theta,\phi)\; d\theta d\phi = \int \left|E_{i,s}(\theta,\phi)\right|^2 B(\theta,\phi)\; d\theta d\phi.$$

Using the natural mode expansion for $P_i(\theta,\phi)$, the relationship for V_i becomes

$$V_i = \int \sum_{k,l}^{2N} P^i_{kl}\; h_k(\theta;\omega_P)\; h_l(\phi;\omega_P) B(\theta,\phi)\; d\theta d\phi,$$

which can clearly be re-expressed as

$$V_i = \sum_{k,l}^{2N} P^i_{kl} B_{kl},$$

where

$$B_{kl} = \int h_k(\theta;\omega_P) h_l(\phi;\omega_P) B(\theta,\phi)\; d\theta d\phi.$$

We therefore have generated a set of simultaneous equations for the B_{kl}. Assuming the equations are linearly independent, we can therefore, by measuring V_i at $2N \times 2N$ positions of the horn, solve for the B_{kl}. The equation for B_{kl} indicates that it is possible to express $B(\theta,\phi)$ as a mode expansion

$$B(\theta,\phi) = \sum_{k,l}^{2N} B_{kl}\; h_m(\theta;\omega_P)\; h_n(\phi;\omega_P).$$

However, the sum is finite, since B_{kl} can only be determined for k and $l \leq 2N$, indicating in terms of mapping that only a bandlimited image of $B(\theta, \phi)$ (in terms of spatial frequencies) can be reconstructed from a set of measurements V_i.

The relationship with Nysquist sampling can be derived in a straightforward manner. Consider the case where the beam width parameter at the telescope focal plane is W_f, and the corresponding beam width at the telescope is W_t. If the telescope is in the far field of the focal plane then, $W_f W_t = \lambda f / \pi$. For an array with a field of view of d_v, the number of propagating modes is given by both $N = (1.2 d_v / 2W_f)^2$, and $N = (1.2 D_t / 2W_t)^2$, so that we obtain

$$N = 1.44 \frac{\pi d_v D_t}{4 \lambda f} \approx \frac{d_v}{F\lambda}.$$

In order to obtain the reconstruction of the source brightness to the highest resolution of the telescope we need to take approximately $2N$ measurements with the horn array. But, this implies a sampling interval of approximately $2d_v/F\lambda$, exactly the same as the Nyquist sampling rate! The measurements are best made at equal intervals, though this is clearly not necessary for the recovery process. In fact beam distortion (due to aberrations in the optical system feeding the telescope) in the case of arrays often results in slight differences in the inter-beam intervals on the sky.

Since the Gaussian beam modes are not the true eigenmodes for the system, but rather approximations to the true eigenmodes, choosing the value for N to be given by the order of the mode that suffers just less than 50% power coupling loss is to some extent arbitrary. However, if N is set much larger, the highest order C^i_{mn} will have very small values, and the inversion process will become unstable. A similar concern arises in a Fourier optics approach, as the highest spatial frequency contributions to the image can only be measured with vanishingly small accuracy, which in the presence of noise may not be the reliably extracted.

We illustrate the mapping process with a one-dimensional example. Again we consider an array of 4 horns. Figure 5 illustrates the inversion process. The input source intensity pattern is a uniform extended source with sharp edges. The image on the focal plane of the telescope has elongated wings. The reconstructed (deconvolved) field using the beam mode approach shows sharp edges, with some ringing effects because of the finite number of modes used (inevitable with a top-hat distribution).

5. Aberrations

We can deal with aberrations in a natural way within the beam mode description of mapping, since the effect is to change the mode coefficients C^i_{mn} for the output field expansion [14],[15]. Aberrations cause mode distortion at an optical component, introducing extra

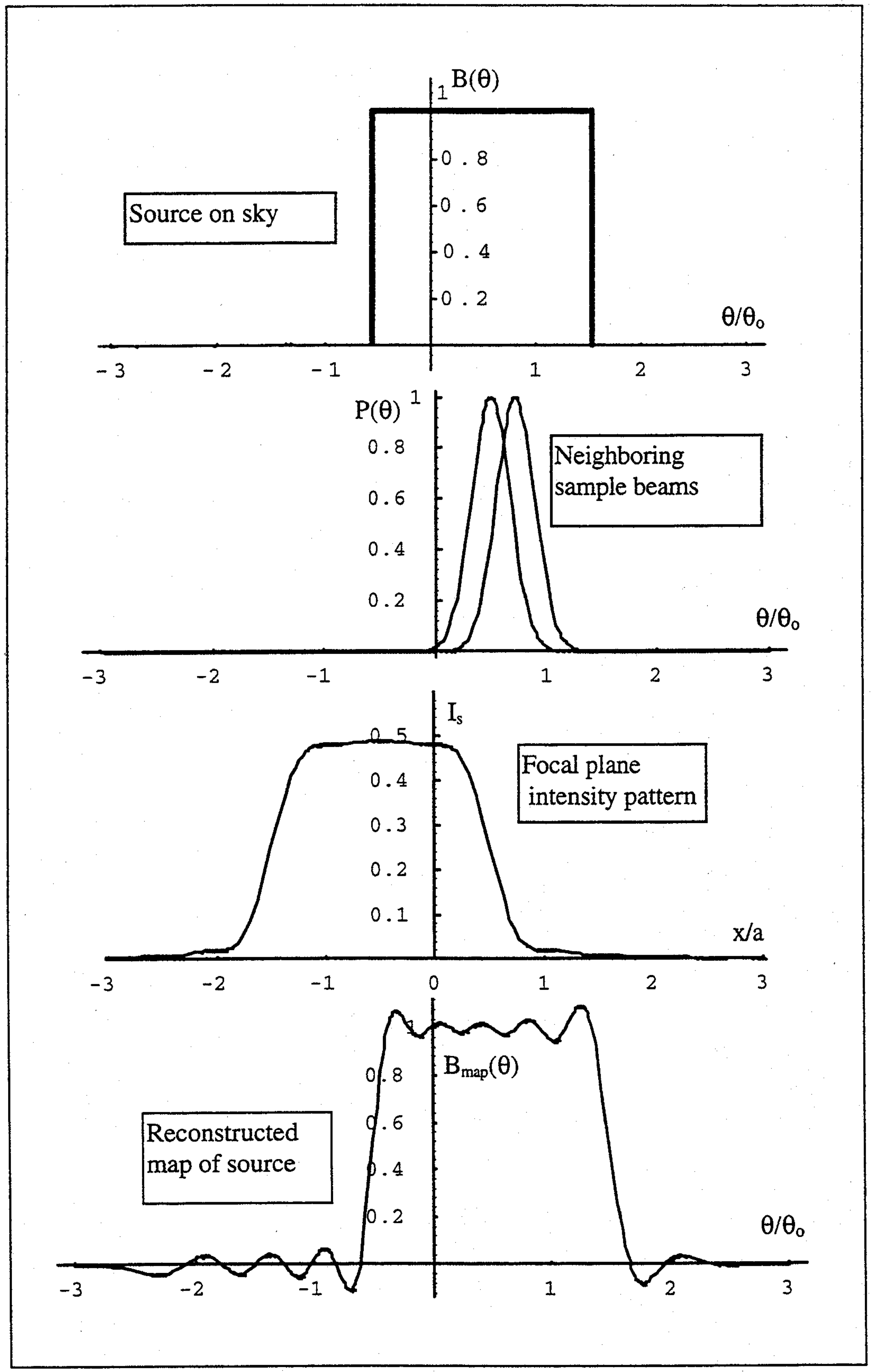

Figure 5. Mapping of extended sources

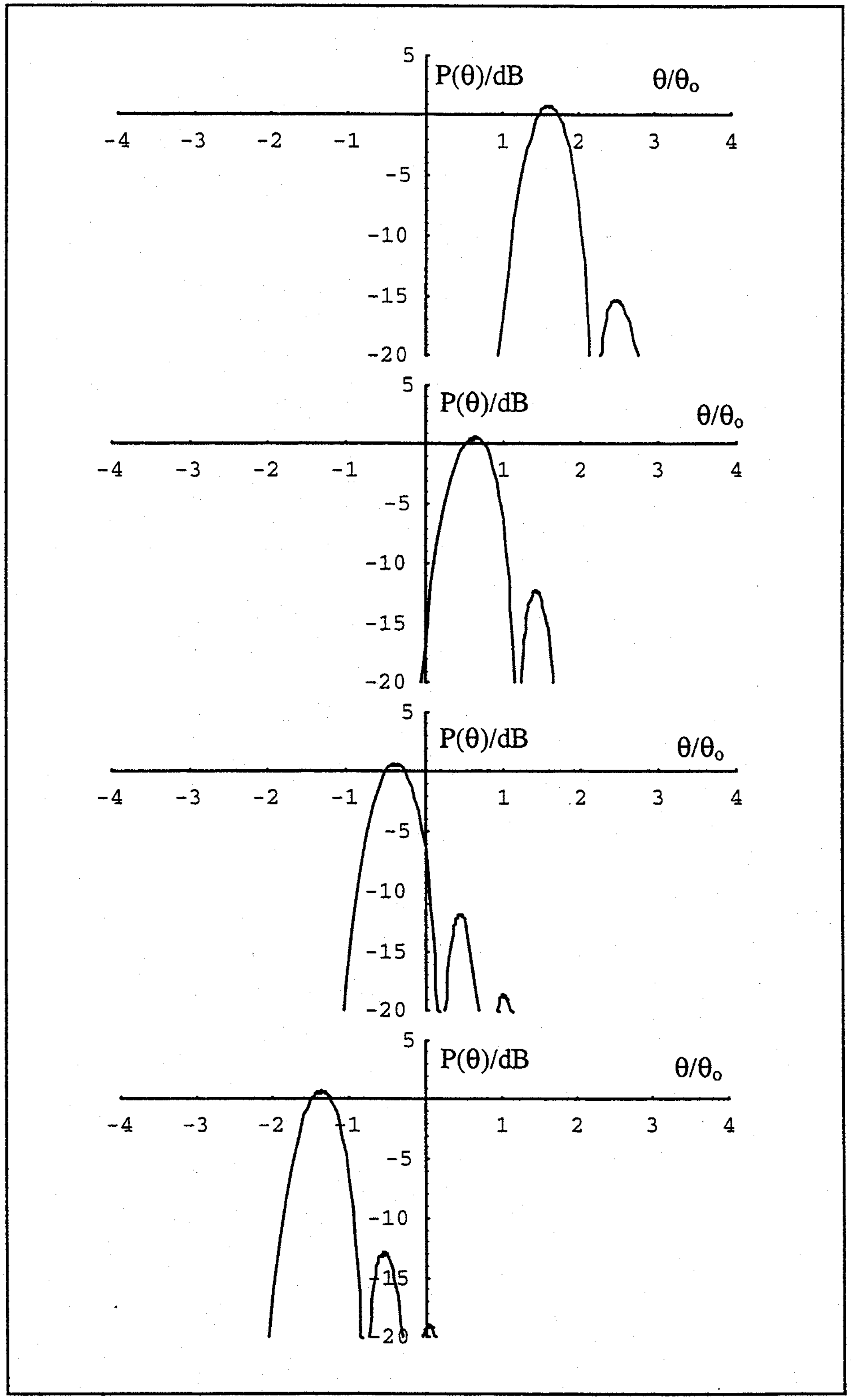

Figure 6. Effect on output beam patterns of a one dimesional aberration function of form $\Phi(x) = \exp(-j\alpha x^2)$ at aperture stop of telescope. Note: $\theta_0 = a / f$

inter-modal scatterings of the propagating modes (see figure 6). In severe cases such an effect may result in more severe attenuation at subsequent stops in the system than would otherwise be the case, with a consequent reduction in the overall total number of modes that can propagate from the input to the output plane. Also, for beams at the edge of the array we might expect more severe aberrational effects, so that the output beams on the sky are not ideal replicas to the on-axis beam. In the theory of Fourier optics this variation in the beam pattern across the output plane causes problems for the analysis, whereas in Gaussian beam mode theory only the mode coefficients for the beam are affected, and thus no difficulty arises. In fact, different types of horns could be used for the array, generating a non-identical set of output beams on the sky, without causing any problems for the reconstruction process (provided the mode coefficients for the horns are known, as well as the scattering matrix of the system.) Figure 6 illustrates the distorted set of beam patterns calculated using GBMA, that result in a one-dimensional array with a non-negligible phase error across the aperture stop of the imaging array.

6. Conclusions

We have considered in detail how to choose the best mode set, in terms of computational efficiency, with which to describe beam propagation in quasi-optical multi-beam systems. For the best choice only a finite and well defined number of modes are propagated by the system. This number is related to the throughput of the optical system giving physical significance to that mode set choice.

We have also considered how the mapping of incoherent sources can be expressed in terms of modal analysis. The reconstruction process becomes one of solving a set of straightforward linear equations. Aberrational effects can also be included.

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