CONVERSION LOSS OF A RESISTIVE STATE SUPERCONDUCTING MIXER

by

Orest Vendik, Electrotechnical University, St. Petersburg, Russia

Erik Kollberg, Chalmers University of Technology, Göteborg, Sweden

Abstract

We have investigated theoretically superconducting hot-electron type mixers operating in a frequency regime, where the signal and LO is below the bandgap frequency of the superconducting device. Then the signal and LO may see a device impedance that is modulated by the IF current, which leads to creation of an image current. Three cases are investigated: open and short-circuited image, and equal impedance at the signal and image frequency. We find that for open circuited image it is theoretically possible to obtain a conversion gain G = 0 dB, while for short-circuited image $G_{opt} \approx -7.7$ dB and for equal impedance at the signal and image frequency $G_{opt} \approx -4.7$ dB. These results can be compared to the optimum conversion gain of -6 dB for the classical bolometer mixer.

1. Introduction

Hot-electron bolometer (HEB) mixers utilising superconducting films in the resistive state [1-3], have a potential to offer low conversion loss and noise temperature from frequencies of a few GHz to several THz. Below about 700 GHz, the frequency approximately corresponding to the superconductor bandgap of niobium (Nb), Nb trilayer S-I-S mixers show unchallenged performance [4- 6]. Particular for frequencies above about one THz HEB mixers are predicted to have superior performance compared to any other type of mixer such as SIS and Schottky mixers.

However, also at lower microwave frequencies the superconducting resistive state mixer may be very useful. E. g. in a microwave system, that needs cooling e. g. in order to use superconducting narrow band low loss filters, it may be practical to have the mixer integrated in the same circuit. Besides low noise, a large dynamic range is often required. Both requirements can be fulfilled by choosing the device volume large enough, of requiring large enough LO power. Experiments at 20 GHz using HEB-devices made from thin and narrow strips of niobium, show excellent mixer performance, with a conversion loss < 6 dB and noise temperature of only a few hundred K [3].

The common theory of HEB mixers is essentially the same as the one presented in the early paper by Arams [7], originally developed for InSb type submillimeter wave bolometric mixers. The basic assumptions used in this theory may are not necessarily true for a superconducting HEB mixer, particularly not for "low frequency" operation. The theoretical work reported below has the Arams' results as a special case.

2. Approach to the mixer analysis

A superconducting HEB device (Fig. 1) normally consists of one superconducting strip with micrometer dimensions or smaller, which is deposited on a substrate of e. g. silicon, single crystalline quartz or sapphire. The strip is cooled to the superconducting state and then under the influence of DC and microwave power undergoes the transition to the resistive state, where the superconductor will gradually become normal.

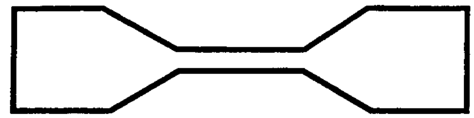

Fig.1 Layout of a typical bolometric superconducting thin film device.

The resistance of the device in the resistive state may be explained by several possible physical phenomena, such as formation of normal domains, phase slip centers, and moving magnetic vortices[1]. The DC and microwave currents transfer the electron subsystem of the device into a nonequilibrium state. It is common to describe these nonequilibrium electrons as hot electrons. In general, however, it is not possible to define a precise thermodynamic temperature of the electron subsystem, although we may consider an "effective electron temperature".

In designing a theory for mixing, we will discuss two cases, Case **A** when the signal frequency (f_s) and the local oscillator (LO) frequency is (f_{LO}) *larger* than the frequency corresponding to the quasiparticle bandgap ($2\Delta/h$) and Case **B** when the signal and the LO frequency is *lower* than $2\Delta/h$. These two cases are illustrated in Fig. 2. For both cases the response time of the device is related to the time constant τ, which determines decay rate of the excess energy absorbed by the heated electrons. The available maximum IF is determined by this relaxation time of the electrons, i. e. $f_{IF} < 1/(2\pi\tau)$ [3].

Case A:

In Arams' theory, which is applicable to Case A and commonly used to explain HEB-mixer conversion loss [7], it is assumed that when the LO and signal currents are added in the bolometer, the device is fast enough to respond to the difference frequency (IF) ($|f_{LO}-f_s|=f_{IF}$) power variation, but not to frequencies of the order the signal and the LO frequency. It is also assumed that the signal and LO device impedance is constant, i. e. not modulated by the IF signal. Since the device resistance at f_{LO} or f_s,is constant, there will of course be no generation of any high frequency harmonics $n\times f_{LO}$ or $n\times f_s$. The modulation of the dissipated power at the IF will cause a modulation of the electron temperature, causing a related resistance modulation (R(t)) noticeable at DC and at the IF. The DC bias current applied to the device will consequently cause a "DC" voltage modulation $I_{DC}\times R(t)$ at the IF and an IF signal can be extracted in a separate IF load resistance (see Fig. 3 below).

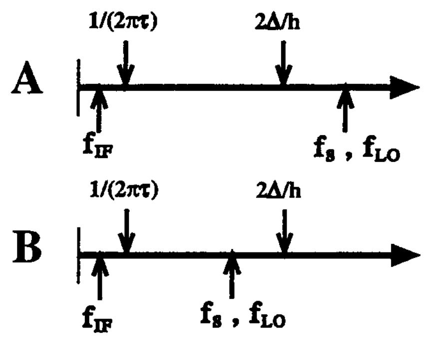

Fig. 2 The signal, local oscillator, and intermediate frequencies compared to the characteristic frequencies of the superconducting device. $1/(2\pi\tau)$ is the relaxation frequency of the electron subsystem of the device, $2\Delta/h$ is the bandgap frequency of the superconductor. For Case **A** the signal and LO resistance of the strip is equal to R_N, while in Case **B** the signal and LO resistance of the strip is modulated by f_{IF}.

---

[1]Note that the validity of the circuit-based model which we will present in the next section does not depend on the details of the microscopic model.

*Case **B***:

However, for f_{LO}, $f_S < 2\Delta/h$ some parts of the strip will be superconducting also for the high frequency components(Fig. 2, Case **B**). In this case the superconducting bridge should be described as a resistor with a mean value of the resistance determined by the DC and LO power but complemented with a *modulation term at the IF*. The modulated resistor is interacting with all current components present, i.e. with the IF, LO, input and image signals. This is the case in several recent experimental mixers (Nb [3], NbN [2]). Particularly mixers based on HTS [8-11] with very high band gap frequencies case **B** of Fig. 2 should be dominating. Indeed for f_{LO}, $f_S<2\Delta/h$ the *Arams theory is not valid.*

It is known that development of the thermal domains in the superconducting film bridge can result in a negative dynamic resistance region of the *I-V* curve [3]. If negative resistance is available, it can be used for increasing the conversion gain, and even make it larger than one. This phenomenon will not be discussed here.

3. Modelling the resistance of the superconducting bridge

Assuming ohmic heating, proportional to the current squared ($I(t)^2$), to be a dominating physical reason for the resistance variations, we suggest the following simplistic equation for describing the current dependent resistance, i. e.

$$R(I(t)) = -R_0 + A_0\langle I(t)^2\rangle \tag{1}$$

The operator $\langle\ \rangle$ indicates an averaging of the total current squared over a time interval determined by the time constant τ of the bolometric device. The signal and LO frequency terms in the spectrum of $R(I(t))$ are suppressed, while the IF term is saved. The model we adopt for the current dependent resistance is further illustrated in Fig. 3. The solid line shows the DC and IF resistance vs. the sum of the high frequency and low frequency currents.

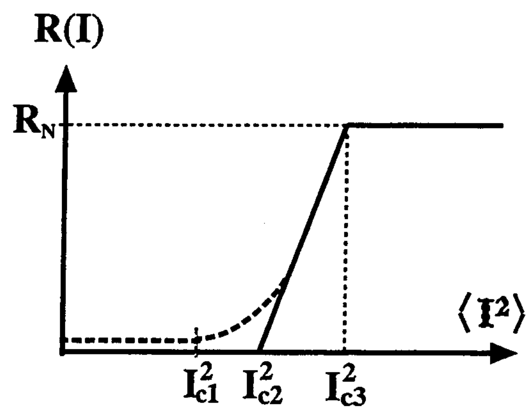

Fig. 3. The model assumes that the resistance of a the superconducting thin film bridge on the averaged depends on the square of the current. The solid line is the DC resistance. The dashed line shows the resistance experienced by the microwave current.

Thus, for DC *and* IF current one has:

$$R(I) = \begin{cases} 0 & \text{for } \langle I^2\rangle \le I^2_{c,2} \\ R_N\dfrac{\langle I^2\rangle - I^2_{c,2}}{I^2_{c,3}-I^2_{c,2}} & \text{for } I^2_{c,2} \le \langle I^2\rangle \le I^2_{c,3} \\ R_N & \text{for } I^2_{c,3} \le \langle I^2\rangle \end{cases} \tag{2}$$

where R_N is the resistance of the bridge in the normal state.

We shall consider the case when the sum of DC and LO current squared is in the region $I_{c,2}^2 \le \langle I^2 \rangle \le I_{c,3}^2$. For this particular case $R(I)$ can be written as:

$$R(I) = -R_N \frac{I_{c,2}^2}{I_{c,3}^2 - I_{c,2}^2} + R_N \frac{1}{I_{c,3}^2 - I_{c,2}^2} \cdot \langle I^2 \rangle \tag{3}$$

Comparing (3) with the equation (1) gives:

$$R_0 = R_N \frac{1}{k^2 - 1}, \quad A_0 = \frac{R_N}{I_{c,2}^2} \cdot \frac{1}{k^2 - 1} = \frac{R_0}{I_{c,2}^2} \tag{4}$$

where

$$k = \frac{I_{c,3}}{I_{c,2}} \tag{5}$$

The coefficient k ($k > 1$) characterizes the slope of the resistive state characteristic. The dashed line in Fig. 3 indicates the high frequency resistance of the superconducting bridge. The region between $I_{c,1}$ and $I_{c,2}$ has been investigated for NbN films at liquid helium temperature and was termed the potentialless (no DC resistance) resistive state [12]. To obtain mixing in the potentialless resistive state we must have a situation as shown in Fig. 2B, i. e. $1/(2\pi\tau) < f_s, f_{LO} < \Delta/h$.

4. Modelling the mixer.

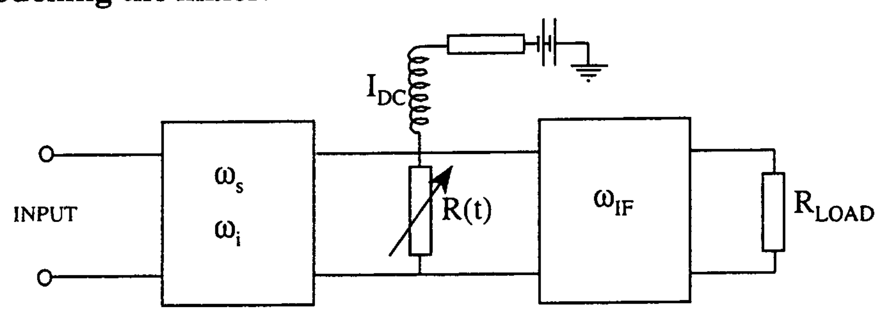

Fig. 4 Block diagram of the mixer.

Fig. 4 presents a block diagram of the mixer. Notice that the embedding impedances of the mixer may be different for the signal and the image frequency respectively.

The total current in the device is

$$\underset{A}{I(t) = I_{dc}} + \underset{B}{I_{LO}\cos\omega_{LO}t} + \underset{C}{I_s\cos\omega_s t} + \underset{D}{I_i\cos\omega_i t} + \underset{E}{I_{IF}\cos\omega_{IF}t} \tag{6}$$

where I_{dc}, I_{LO}, I_s, and I_{IF} are the DC, LO, signal, image and the intermediate frequency currents respectively. The capital letters A - E in equation (6) denotes the different terms and will be used below. The following relations between frequencies are assumed

$$\begin{aligned} \omega_{LO} + \omega_{IF} &= \omega_i \\ \omega_{LO} - \omega_{IF} &= \omega_s \end{aligned} \tag{7}$$

and we will use

$$\begin{aligned} I_{dc} \cong I_{LO} &>> I_s \cong I_i \cong I_{IF} \\ \omega_{LO} \cong \omega_s \cong \omega_i &>> \omega_{IF} \end{aligned} \tag{8}$$

For simplicity, let us suppose that the impedance with respect to all frequencies of the mixer current are real and consequently no phase shifts between current and voltage components take place. Substituting (6) into (1) and neglecting the high frequency terms (ω_{LO}, ω_i, ω_s) and the quadratic terms of the small signal current amplitudes (I_s^2, I_i^2, I_{IF}^2 etc.), one obtains:

$$R(t) = -R_0 + \underbrace{A_0(I_{dc}^2 + 0.5 I_{LO}^2)}_{a} + \underbrace{A_0 I_{LO} I_s \cos\omega_{IF} t}_{b} + \underbrace{A_0 I_{LO} I_i \cos\omega_{IF} t}_{c} + \underbrace{2A_0 I_{dc} I_{IF} \cos\omega_{IF} t}_{d} \qquad (9)$$

The small letters a - d defining the different terms in eq. (9) will be used for the discussion below. The voltage drop over the resistive bridge can now be calculated as $U(t)=R(t)\cdot I(t)$. The resulting frequency components are listed in Table 1.

TABLE 1

	U(t) = R(t) I(t) =	ω_s	ω_i	ω_{IF}
$a \times A$	$= R_B \cdot I_{dc} +$			
$a \times B$	$+ R_B \cdot I_{LO} \cos\omega_{LO} t +$			
$a \times C$	$+ R_B \cdot I_s \cos\omega_s t +$	+		
$a \times D$	$+ R_B \cdot I_i \cos\omega_i t +$		+	
$a \times E$	$+ R_B \cdot I_{IF} \cos\omega_{IF} t +$			+
$b \times A$	$+A_0 \cdot I_{LO} I_s I_{dc} \cos\omega_{IF} t$			+
$b \times B$	$+A_0 \cdot I_{LO} I_s I_{LO} 0.5(\cos\omega_s t + \cos\omega_i t) +$	+	+	
$b \times C$	0			
$b \times D$	0			
$b \times E$	0			
$c \times A$	$+A_0 \cdot I_{LO} I_i I_{dc} \cos\omega_{IF} t$			+
$c \times B$	$+A_0 \cdot I_{LO} I_i I_{LO} 0.5(\cos\omega_s t + \cos\omega_i t) +$	+	+	
$c \times C$	0			
$c \times D$	0			
$c \times E$	0			
$d \times A$	$+2A_0 I_{dc} I_{IF} I_{dc} \cos\omega_{IF} t +$			+
$d \times B$	$+A_0 I_{dc} I_{IF} I_{LO}(\cos\omega_s t + \cos\omega_i t)$	+	+	
$d \times C$	0			
$d \times D$	0			
$d \times E$	0			

In the table the terms from the expressions (6) and (9), A, B, C, D, E and a, b, c, d are used. R_B is a constant resistance which is seen by the DC, IF, signal and LO currents and is equal to

$$R_B = -R_0 + A_0(I_{dc}^2 + 0.5I_{LO}^2) \tag{10}$$

Equating the terms of the frequency components $(\omega_s, \omega_i, \omega_{LO})$ to the corresponding voltage drops which appear over the resistance bridge, one obtains the following system of equations:

$$\begin{aligned} &\text{for } \omega_s && Z_{11}I_s + Z_{12}I_i + Z_{13}I_{IF} = U_s \\ &\text{for } \omega_i && Z_{21}I_s + Z_{22}I_i + Z_{23}I_{IF} = U_i \\ &\text{for } \omega_{IF} && Z_{31}I_s + Z_{32}I_i + Z_{33}I_{IF} = U_{IF} \end{aligned} \tag{11}$$

where

$$\begin{aligned} &Z_{11} = R_B + 0.5A_0I_{LO}^2 && Z_{12} = 0.5A_0I_{LO}^2 && Z_{13} = A_0I_{dc}I_{LO} \\ &Z_{21} = 0.5A_0I_{LO}^2 && Z_{22} = R_B + 0.5A_0I_{LO}^2 && Z_{23} = A_0I_{dc}I_{LO} \\ &Z_{31} = A_0I_{dc}I_{LO} && Z_{32} = A_0I_{dc}I_{LO} && Z_{33} = R_B + 2A_0I_{dc}^2 \end{aligned} \tag{12}$$

These equations reflect the fact that both the LO and signal see a resistance that is *modulated* by the IF current, causing new current and voltage components at $f_{LO} \pm f_{IF} = f_s$ or f_i, where $f_i = 2f_{LO} - f_s$ is the image frequency.

For additional information about the meaning of the resistance R_B see Appendix 2

5. Mixer performance for different image frequency load impedances

Let us suppose that the embedding impedance at the output (intermediate) frequency is R_L and at the image frequency is R_i. Then

$$U_{IF} = -R_L \cdot I_{IF}, \quad U_i = -R_i \cdot I_i \tag{13}$$

For convenience we introduce a factor b which we use to define R_i accordingly

$$R_i = b \times \frac{U_s}{I_s} \tag{14}$$

Three cases will be analysed:

b=∞	for opencircuited image
b=1	for equal impedance at the image and the short circuit
b=0	for shortcircuited image

Equation (11) can now rewritten in the form

$$\begin{aligned} Z_{11}I_s + Z_{12}I_i + Z_{13}I_{IF} &= U_s \\ Z_{21}I_s + \left(Z_{22} + b \times \frac{U_s}{I_s}\right)I_i + Z_{23}I_{IF} &= 0 \\ Z_{31}I_s + Z_{32}I_i + (Z_{33} + R_L)I_{IF} &= 0 \end{aligned} \tag{15}$$

where the elements Z_{ij} are defined above. It is possible to derive an expression for the input impedance U_s/I_s (see Appendix 1), allowing expressions for the signal, image and IF currents. Then it is possible to determine the input power P_s and the IF output power P_{IF} as

$$P_s = 0.5U_sI_s, \qquad P_{IF} = 0.5I_{IF}^2R_L. \tag{16}$$

The conversion gain is finally defined as

$$G = \frac{P_{IF}}{P_s} \tag{17}$$

5.1. Infinite impedance at the image frequency. ($Z_i = \infty$)

For the special case when the image is terminated by an infinite impedance (b=∞, I_i=0), it is possible to find an analytical solution. Equation (11) is simplified to

$$\begin{aligned} Z_{11}I_s + Z_{13}I_{IF} &= U_s \\ Z_{31}I_s + (Z_{33}+R_L)I_{IF} &= 0 \end{aligned} \tag{18}$$

For the signal and IF current we get

$$I_s = U_s\left(Z_{11} - \frac{Z_{31}Z_{13}}{Z_{33}+R_L}\right)^{-1} \qquad I_{IF} = -I_s\frac{Z_{31}}{Z_{33}+R_L} \tag{19}$$

which together with (16) and (17) yields

$$G = \frac{Z_{31}^2 R_L}{(Z_{33}+R_L)\left(Z_{11}(Z_{33}+R_L) - Z_{13}Z_{31}\right)} \tag{20}$$

A maximum G is obtained for

$$R_L^{opt} = Z_{33}\cdot\sqrt{1-M} \tag{21}$$

where

$$M = \frac{Z_{31}^2}{Z_{22}Z_{33}}. \tag{22}$$

After some transformations one obtains the optimized conversion gain:

$$G^{opt} = \frac{1-\sqrt{1-M}}{1+\sqrt{1-M}} \tag{23}$$

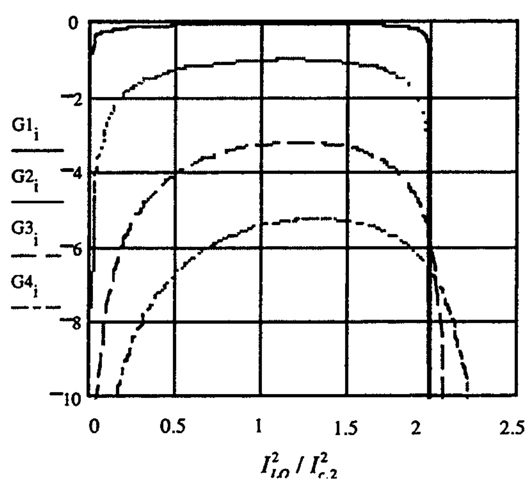

Fig. 5. Conversion gain (dB) for R_i=∞ vs. $I_{LO}^2 / I_{c,2}^2$ for α=1.00005, 1.005, 1.05 and 1.15 respectively.

For convenience we introduce the parameter α

$$\alpha = \left(I_{DC}^2 + 0.5 I_{LO}^2\right) / I_{c,2}^2 \tag{24}$$

Notice that α is proportional to the total power absorbed and that it indicates the bias point in the R(I) vs. ⟨I²⟩ diagram, Fig. 4. In Fig. 5 is depicted G^{opt} vs. $I_{LO}^2 / I_{c,2}^2$ for different values of α. Notice that $I_{LO}^2 / I_{c,2}^2$ is proportional to the LO power.

The maximum conversion gain is obtained for $\alpha \to 1$, yielding the optimum value for $0.5 \cdot I_{LO}^2 / I_{c,2}^2$ of $2-\sqrt{2}=0.5858$. This means that optimum performance is expected when the absorbed LO power is approximately equal to the DC power $\left(0.5\, I_{LO}^2 \cong I_{dc}^2\right)$.

The impedance levels are as well of interest. In Table 2 below the impedances are normalized with respect to $R_0=R_N/(k^2-1)$ (see equation (3)), viz.

$$R_{L,opt}=\frac{U_{IF}}{I_{IF}}=r_L R_0 \qquad R_{s,opt}=\frac{U_s}{I_s}=r_s R_0 \qquad R_B=\frac{U_{DC}}{I_{DC}}=r_B R_0 \tag{26}$$

Table 2

r_B	$I_{LO}^2 / I_{c,2}^2$	G^{Opt} dB	r_s	r_L
0.00005	1.17	-0.105	0.00706	0.01002
0.005	1.19	-1.045	0.0711	0.1002
0.05	1.23	-3.216	0.233	0.330
0.15	1.34	-5.267	0.608	0.441

Notice that $r_B=\alpha-1$. Maximum conversion gain is obtained when the parameter $r_B \to 0$ and $\alpha \to 1$ while the signal and output impedances approaches zero ohms . This means that the mixer is operating with a very low power dissipation and practically without heating. The nature of the nonlinearity in this regime could be due to kinetic effects (depairing of Cooper pairs when the current slightly exceeds the critical value). This is an interesting difference as compared to the bolometric regime where heating is assumed.

5.2. Equal impedance at signal and image frequency ($R_i = R_s$)

For this case b =1. We have calculated the conversion gain vs. $I_{LO}^2 / I_{c,2}^2$ and for

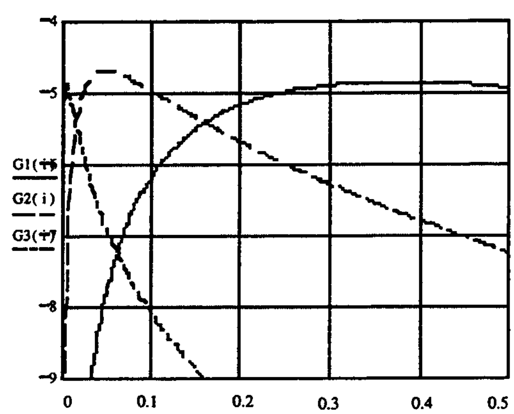

Fig. 6 Conversion gain (dB) vs. $I_{LO}^2 / I_{c,2}^2$ for α=1.00005 (r_B=0.00005), r_L=0.1, 0.01, 0.001 for G1(i), G2(i) and G3(i) respectively in the case of the equal signal and image frequency impedance. Maximum conversion gain is obtained approximately for r_L=0.01 (G2-curve) and $I_{LO}^2 / I_{c,2}^2$=0.05.

different load resistances r_L (see Fig 6). Note that the maximum conversion gain ≈ -4.7 dB is not strongly dependent on r_L. However, decreasing the IF load resistance r_L increases the conversion gain and requires lower LO power.

5.3. Zero impedance at image frequency ($R_i = 0$)

For shortcircuited image, the maximum gain is lower than for the opencircuited image, or for equal signal and image impedances. A maximum conversion gain of -7.7 dB is obtained when α->1. The dependence on r_L is not so strong, and for r_B=0.00005 (α=1.00005) a value of 0.01 is close to optimum yielding a result for the conversion that is only moderately dependent on the LO power (Fig. 7).

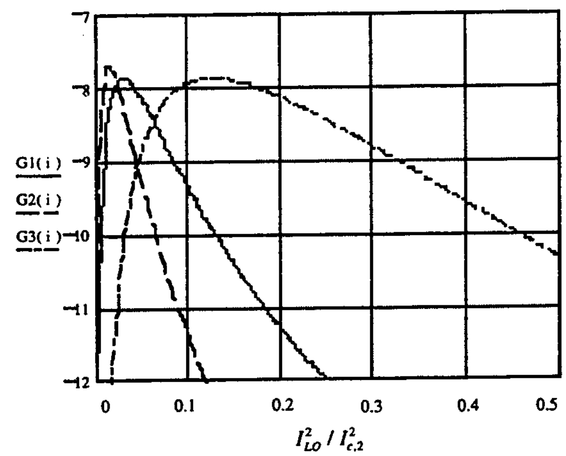

Fig. 7. Conversion gain (dB) vs. $I^2_{LO} / I^2_{c,2}$ for α=1.00005 (r_B=0.00005), r_L= 0.1, 0.01, 0.001. The case of zero image frequency impedance. For G1(i), the $I^2_{LO} / I^2_{c,2}$ scale is expanded by a factor of 20. Maximum conversion gain is obtained approximately for $I^2_{LO} / I^2_{c,2}$=0.01 (G2-curve).

5.4. Discussion

In table 3 is shown the conversion gain for different values of r_l and optimized $I^2_{LO} / I^2_{c,2}$.

Table 3: Optimum mixer performance for r_B=0.00005

	r_L=0.1				r_L=0.01				r_L=0.001			
	G_{max} dB	optim. $\frac{I^2_{LO}}{I^2_{c,2}}$	$\left(\frac{P_{LO}}{P_{DC}}\right)_{opt}$	$\frac{r_s}{r_B}$	G_{max} dB	optim. $\frac{I^2_{LO}}{I^2_{c,2}}$	$\left(\frac{P_{LO}}{P_{DC}}\right)_{opt}$	$\frac{r_s}{r_B}$	G_{max} dB	optim. $\frac{I^2_{LO}}{I^2_{c,2}}$	$\left(\frac{P_{LO}}{P_{DC}}\right)_{opt}$	$\frac{r_s}{r_B}$
$R_i=\infty$	-0.276	0.26	0.149	142	-0.105	1.17	1.41	141	-0.276	1.86	13.2	141
$R_i=Z_S$	-4.85	0.005	0.0025	2.40	-4.69	0.05	0.026	2.48	-4.86	0.375	0.231	2.40
R_i=0	-7.87	0.0015	0.00075	1.42	-7.70	0.014	0.0070	1.41	-7.87	0.125	0.067	1.41

The reason why the maximum possible conversion loss is as high as 0 dB for opencircuited image, and as low as 7.7 dB for shortcircuited image is related to power loss µ $0.5R(t)\cdot I(t)^2$ in the bolometric resistance. The current $I(t)$ is larger for the short-circuited case than for the open-circuited case when I_i=0. Small r_B as compared to r_L and r_s indicate low losses in the device itself.

The formula for the conversion gain of the classical mixer as originally derived by Arams [7] is obtained assuming I_i=0 and R_S=R_i=R_B (see Appendix C). The classical theory for the bolometric mixer yields an the optimum load impedance R_L is equal to $(dV/dI)_{DC}$ in the operating point, i. e. (much) larger than the R_B, and the optimum LO is small compared to the DC power (see reference [3], Fig. 4). According to Table 3, this is also a result of the present analysis. The analysis also shows that the optimum source impedance R_s is larger than the DC resistance R_B,

6. Taking Into Account the Intrinsic Microwave Loss of the Device

The largest conversion gain is obtained, if the effective current formed by the sum of DC and LO current squared is just larger than $I_{c,2}^2$. The intrinsic microwave resistance of the bridge is presented by the dashed line in Fig. 3. We now assume that this additional resistance of the bridge is the microwave resistance of the bridge when it is in the superconducting equilibrium state. Hence, we suggest the following approximation for this intrinsic microwave resistance R_{MW} and the normal resistance of the bridge R_N,

$$R_{MW} = R_{MW,SUR} \cdot \frac{l}{w} \qquad R_N = R_{N,SUR} \cdot \frac{l}{w} \tag{27}$$

where $R_{MW,SUR}$ is the microwave surface resistance of the film in the superconducting state, and $R_{N,SURF}$ is the surface resistance of the film in normal state. l and w are the length and the width of the bridge respectively. The loss related to R_{MW} is due to a voltage redistribution over a series connected resistance. Thus a corrected conversion gain can by written as follows:

$$G^{cor} = G^{opt} \cdot \frac{R_{s,opt}}{R_{s,opt} + R_{MW}} = G^{opt} \cdot \frac{1}{1 + \frac{\delta}{r_s}} \tag{28}$$

where

$$\delta = \frac{R_{MW}}{R_N} \cdot (k^2 - 1) \tag{29}$$

δ is a phenomenological parameter responsible for the contribution of the intrinsic microwave resistance of the bridge. For f =10 GHz we may estimate the microwave surface resistance of a superconducting film to be $R_{sur} = 10^{-3}$ Ohm/□ and the normal state film resistance to R_N =1 Ohm/□ . For $l/w = 5$ and $k = 1.4$ we then have $\delta = 10^{-3}$. In Table 4, we have calculated the corrected mixer gain for R_L/R_0=0.01, R_B/R_0=0.0005. and $\delta = 10^{-3}$.

Table 4: Correction of the conversion gain for r_L=0.01, r_B=0.0005. and $\delta = 10^{-3}$

	r_S	G^{cor}/G^{opt}	$(G_{max})^{cor}$ dB
$R_i = \infty$	0.07	0.98	-0.2
$R_i = R_S$	0.0012	0.55	-7.3
$R_i = 0$	0.0007	0.42	-11.5

Notice that in practice a conversion gain below -3 dB should be possible to achieve. However, the impedance levels may be quite low. For R_0=5 Ohm we have R_s=0.35 Ohm and R_L=0.05 Ohm.

7. Conclusion

A microwave mixer based on a superconducting bridge partly in a resistive state has been investigated assuming that the signal and the LO frequency is below the quasiparticle bandgap frequency. In our model, the resistance of the superconducting film is dependent on the current squared and is inertial with respect to the signal and LO frequencies. The mixer circuit is assumed matched at the signal input and intermediate frequency output.

We found that the conversion gain can be as low as -4.7 dB for equal embedding resistance at the signal and image frequency. The conversion gain improves and approaches 0 dB if $R_i \rightarrow \infty$. However, if the image is shortcircuited, the conversion gain goes down and is only about -7.7 dB as best. These differences are due to the losses in the device itself. Maximum gain is obtained operating the mixer as close to the critical current $I_{c,2}$ as possible, which also means that the input and output impedances are very low. Adding the microwave losses to the model suggests that the bias current should be increased. The ordinary Arams' case is a special case of the theory presented above.

The case **B**, corresponding to when the signal and the LO frequency is below the quasi particle bandgap frequency, is in practice valid only for comparatively "low" frequencies. What this means in practice is not known in detail. At "ordinary" microwave frequencies this case must be seriously considered, and for mixers using high temperature superconductors the upper limiting frequency may well be at several hundred GHz.

At "ordinary" microwave frequencies it should be possible to design a circuit preventing any image frequency current. A couple of interesting engineering problems can be identified: **i.** how to develop a device (thin film bridge) with an as large as possible nonlinearity and with low intrinsic microwave loss,es and **ii.** how to design impedance transforming circuits with very low intrinsic loss. The first problem is probably the more difficult, while the second problem for reasonably low frequencies can be solved using a superconducting matching circuit.

8. Acknowledgements

The authors would like to acknowledge Prof. Sigfrid Yngvesson for his useful remarks. We as well would like to thank for financial support from the Swedish Royal Academy of Sciences and the Swedish National Board for Industrial Research.

References

[1] E.M.Gershenzon et al, "Millimeter and Submillimeter Range Mixer Based on Electronic Heating of Superconducting Films in the Resistive State", *Sov.Phys.Superconductivity*, 3, pp.1582-1597, 1990.

[2] B.S. Karasik, G.N. Gol'tsman, B.M. Voronv, S.T. Svechnikov, E.M.Gershenzon, H. Ekström, S. Jacobsson, E. Kollberg, and K.S. Yngvesson,"Hot Electron Quasioptical NbN Superconducting Mixer", *IEEE Trans. Appl. Supercond.*, vol. 5, pp. 2232-2235, 1995

[3] H. Ekström, B. Karasik, E. Kollberg, and K.S. Yngvesson, "Conversion Gain and Noise of Niobium Hot - Electron Mixer", IEEE Trans. on MTT, vol. MTT-43, N0. 4, pp. 938 - 947, 1995

[4] R. Blundell, C.E. Tong, "Submillimeter Receivers for Radio Astronomy," *Proc. IEEE*, vol. 80, pp.1702-1720, 1992.

[5] J.Mees, et al, "An Airborne SIS-Receiver for Atmospheric Measurements at 630 and 720 GHz", *Proceedings of the Fifth International Symposium on Space Terahertz Technology*, 1994

[6] Zimmermann R&R&P, "All solid state radiometers for environmental studies to 700 GHz," *Proceedings of the Third International Symposium on Space Terahertz Technology*, pp. 706-723, 1992.

[7] F.Arams, et al., "Millimeter Mixing and Detection in Bulk InSb", *Proc. IEEE*, vol. 54, pp.308-318, 1966.

[8] H. Chaloupka, M. Jeck, S.G. Kolesov, O.G. Vendik, " Frequency Conversion in High-Tc Superconductor Microwave Circuits", *Proc. of 22nd European Microwave Conference*, v.1, pp. 189-194, 1992.

[9] T.B. Samoilova, "Nonlinear Microwave Effects in Thin Superconducting Films", *Supercond. Sci. Technol.*, vol. 8, pp. 259-277, 1995.

[10] J.Wang, D.P. Butler, Z. Celik-Butler, " Modeling of High Tc Superconductor Parametric Amplifiers and Mixers", *Physica C*, vol. 231, pp. 271-276, 1994.

[11] M. Jeck et al. Investigation of Electrical Nonlinearity of HTS Thin fFlms as Applied to Realization of a Microwave IC Mixer", *Journal of Superconductivity*, vol. 8, No. 6, pp. 705-714, 1995

[12] A.B. Kozyrev et al. "Destruction of the Superconductive State in Thin Film by Microwave Pulses", *Solid State Commun.*, vol. 77, No. 6, pp. 441-445, 1991.

Appendix 1: Calculation of the signal, image and if currents.

$$I_s = \frac{U_s}{\Delta}\left[\left(Z_{22} + b\frac{U_s}{I_s}\right)(Z_{33} + R_L) - Z_{23}Z_{32}\right] \tag{A1.1}$$

$$I_i = \frac{U_s}{\Delta}\left[Z_{21}(Z_{33} + R_L) - Z_{23}Z_{31}\right] \tag{A1.2}$$

$$I_{IF} = \frac{U_s}{\Delta}\left[Z_{21}Z_{32} - \left(Z_{22} + b\frac{U_s}{I_s}\right)Z_{31}\right] \tag{A1.3}$$

$$\begin{aligned}\Delta = Z_{11}\left[\left(Z_{22} + b\frac{U_s}{I_s}\right)(Z_{33} + R_L) - Z_{23}Z_{32}\right] - Z_{12}\left[Z_{21}(Z_{33} + R_L) - Z_{23}Z_{31}\right] + \\ + Z_{13}\left[Z_{21}Z_{32} - \left(Z_{22} + b\frac{U_s}{I_s}\right)Z_{31}\right] = a_1 + b\frac{U_s}{I_s}a_2\end{aligned} \tag{A1.4}$$

From (A1.1) and (A1.4) we get a second degree equation:

$$a_1 + b\frac{U_s}{I_s}a_2 - \frac{U_s}{I_s}(Z_{22}Z_{33} + Z_{22}R_L - Z_{23}Z_{32}) - b\left(\frac{U_s}{I_s}\right)^2(Z_{33} + R_L) = 0 \tag{A1.5}$$

From (A1.5) we obtain the input impedance U_S/I_S. Hence, by inserting U_S/I_S into (A1.1)-A(1.4) the currents I_S, I_i, and I_{IF} can now be calculated.

Appendix 2. Deriving the I-V curve from the model of the current dependent resistor.

Using (10) the expression for DC I - V characteristics of the nonlinear resistor can be written as:

$$U_{dc} = R_B \cdot I_{dc} = -R_0 I_{dc} + A_0 \cdot I_{dc}^3 + 0.5A_0 \cdot I_{LO}^2 I_{dc} \qquad \text{(A2.1)}$$

We get

$$\left(\frac{dU}{dI}\right)_{dc} = -R_0 + 3A_0 \cdot I_{dc}^2 + 0.5A_0 \cdot I_{LO}^2 . \qquad \text{(A2.2)}$$

Substituting (4) into (A2.1) one obtains

$$U = \frac{R_N I_{c,2}}{k^2 - 1}\left(0.5\frac{I_{LO}^2}{I_{c,2}^2} + \frac{I_{dc}^2}{I_{c,2}^2} - 1\right) \cdot \frac{I_{dc}}{I_{c,2}} . \qquad \text{(A2.3)}$$

The same in normalized form:

$$z = (x^2 + 0,5y^2 - 1) \cdot x \qquad \text{(A2.4)}$$

where

$$x = \frac{I_{dc}}{I_{c,2}}, \qquad y = \frac{I_{LO}}{I_{c,2}}, \qquad z = \frac{U}{R_N I_{c,2} / (k^2 - 1)} . \qquad \text{(A2.5)}$$

Fig. A2.1 shows the normalized I - V curves for different values of the LO power. Qualitatively the curve obtained are in agreement with the known form of pumped I - V curves (see e. g. [9]).

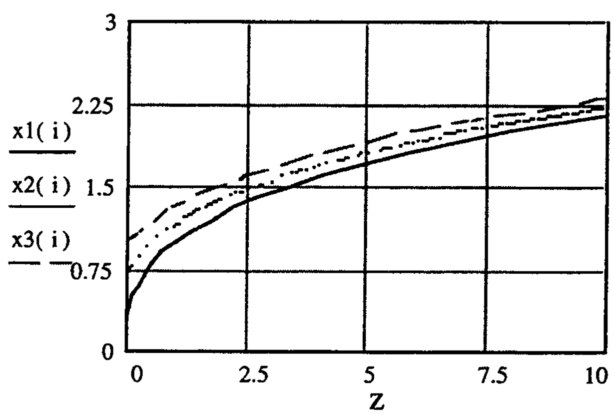

Fig. A2.1 IV curves (x1, x2 and x3) calculated using equation A2.4 for $0.5y^2$=1, 0.5 and 0 respectively..

Appendix 3. Comparison with Arams' theory for bolometric mixers.

In order to find the equivalence of the Arams' theory, we must arrange the model so that the signal and LO power absorbed have the same effect on the nonlinear resistance as the IF power. This leads to the conclusion that we should use R_B as the device impedance for LO, signal and DC currents. If we do so, we can transform (10) - (12) into the following simplified system.

$$
\begin{aligned}
R_B \cdot I_s &= U_s \\
A_0 \cdot I_{dc} I_{LO} I_s + (R_B + 2A_0 \cdot I_{dc}^2) I_{IF} &= U_{IF}
\end{aligned}
\qquad \text{(A3.1)}
$$

Introducing the load resistance ($U_{IF} = -R_L \cdot I_{IF}$) we obtain :

$$
I_{IF} = I_s \cdot \frac{A_0 \cdot I_{dc} I_{LO}}{R_B + 2A_0 \cdot I_{dc}^2 + R_L} \qquad \text{(A3.2)}
$$

which together with (A2.2) leads to

$$
G = (A_0 \cdot I_{dc} I_{LO})^2 \cdot \frac{R_L}{R_B} \left(\left(\frac{dU}{dI} \right)_{DC} + R_L \right)^{-2} \qquad \text{(A3.3)}
$$

The gain of the ordinary bolometric mixer can be expressed as (see Eq. (5) of reference [3]):

$$
G = \frac{1}{2} \frac{P_{LO}}{P_{DC}} \frac{R_L}{R_B} \left(1 - \frac{R_B}{(dU/dI)_{DC}} \right)^2 \left(1 + \frac{R_L}{(dU/dI)_{DC}} \right)^2 \qquad \text{(A3.4)}
$$

Assuming that

$$
P_{DC} = R_B \cdot I_{DC}^2 \qquad\qquad P_{LO} = \frac{1}{2} R_B \cdot I_{LO}^2 \qquad \text{(A3.5)}
$$

and using (A2.2) and (10) to develop the terms within the parenthesises of (A3.4), equation (A3.4) can be transformed into the same form as Eq. (A3.3).