Gaussian-mode Analysis of "Thin" Mirrors

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Abstract: We have developed a technique for calculating the Gaussian beam-mode scattering matrices of a shaped, off-axis mirrors. In its simplest form, the technique gives the coupling efficiency between the incoming and outgoing lowest-order Gaussian modes. In its more complete form, the technique gives the full scattering matrix between higher-order modes, which in turn characterises the aberrative behaviour of the mirror. In this paper, we model a mirror as a thin phase-transforming surface located at the tangent plane to the centre of the mirror. The scattering matrix is determined in the usual manner by evaluating the overlap integrals across some surface in the system for which the amplitude and phase of the field are known. Because, however, the modes are not orthonormal over the surface of the mirror a system of linear equations has to be solved. The shape of the mirror is simply described by the height of the surface above the tangent plane, and therefore, the technique is particularly flexible and can be used for a variety of applications. Eventually, we hope to use the technique for synthesising systems of shaped mirrors.

1 Introduction

Perhaps one of the biggest deficiencies in Gaussian beam-mode optics at the present time is the inability to describe the behaviour of shaped, off-axis mirrors. It is true that some mirrors can be described as ideal quadratic phase-transforming surfaces, but in many cases this approximation is inadequate and a more detailed analysis is required. For example, when ellipsoidal mirrors are used at frequencies away from the nominal design frequency, phase errors occur which can limit the efficiency of a system. Moreover, in array receivers most of the beams pass through the optics off-axis, and one has to worry about the distortion of individual beams in addition to the large-scale aberrations of the image. At the present time, we are particularly interested in applying phase-retrieval methods to submillimetre-wave optics, and in this case, one has to be certain that the mirror used to defocus the beam does not scatter power between modes. Another interesting application is to use the technique, together with phase-retrieval methods, for designing systems of shaped mirrors that modify beams in particular ways.

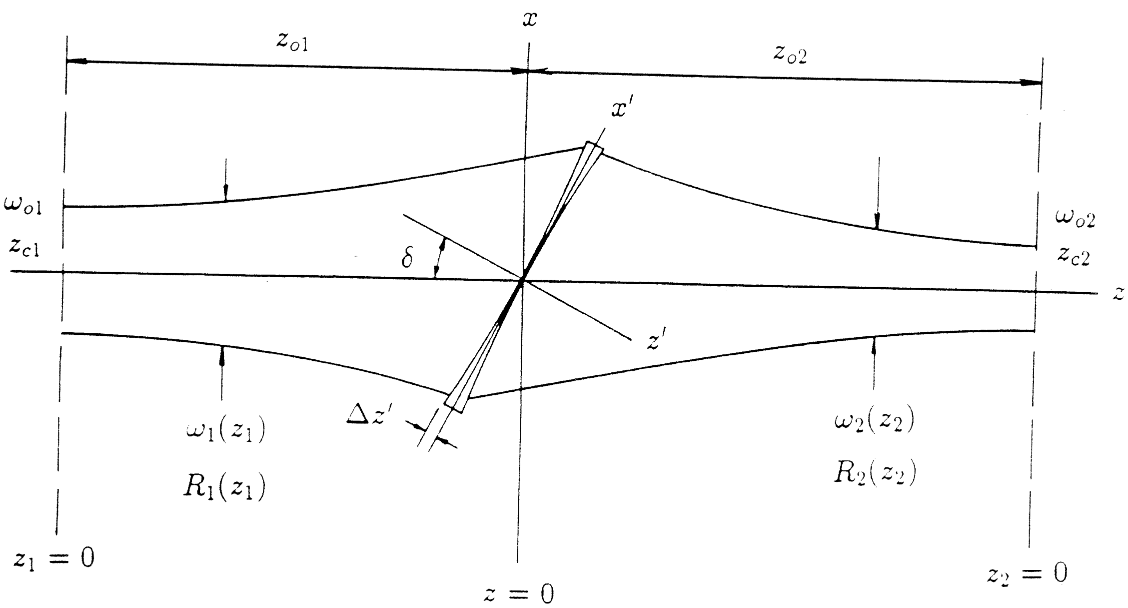

Figure 1: The coordinate system used in the analysis. The mirror is shown as though in transmission, and the tangent plane simply separates the input and output coordinate frames.

We have developed a technique for studying the behaviour of shaped, off-axis mirrors. The technique is aimed at calculating the scattering matrix relating the mode coefficients of the input beam to the mode coefficients of the output beam. To this end, we introduce the concept of a *thin* mirror. By analogy with a thin lens, we represent an off-axis mirror as a thin phase-transforming surface located at the tangent plane to the centre of the mirror. The phase transformation across the plane can be any arbitrary function, allowing ellipsoidal, paraboloidal, spherical, or other forms to be analysed. Once the reflected field is known, the scattering matrix can be determined by evaluating the overlap integrals over the surface of the mirror and inverting the system of linear equations that results from the modes not being orthogonal.

2 Scattering-matrix theory

The coordinate system to be used in the analysis is shown in Fig. 1, and a detailed view of the mirror is shown in Fig. 2. According to this scheme, the mirror is shown as though in transmission, and the tangent plane simply separates the input and output coordinate frames. If a generic point in the (x, y, z) frame is denoted by s, and a generic point in the, inclined, (x', y', z') frame is denoted by s', then the two frames are connected on the

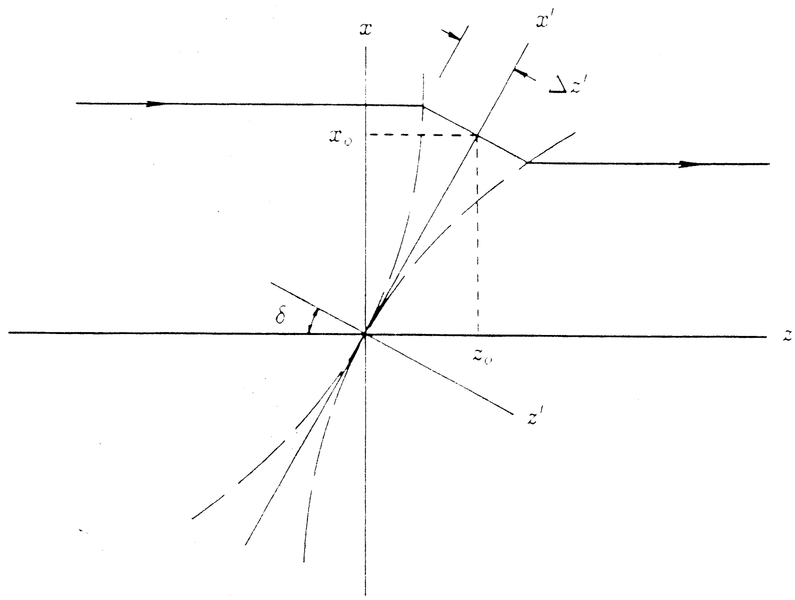

Figure 2: A detailed view of the mirror.

input surface by

$$
\begin{aligned}
x &= x' \cos\delta + \Delta z' \sin\delta = x_o + \Delta x \\
y &= y' = y_o \\
z &= x' \sin\delta - \Delta z' \cos\delta = z_o - \Delta z
\end{aligned}
\tag{1}
$$

and on the output surface by

$$
\begin{aligned}
x &= x' \cos\delta - \Delta z' \sin\delta = x_o - \Delta x \\
y &= y' = y_o \\
z &= x' \sin\delta + \Delta z' \cos\delta = z_o + \Delta z
\end{aligned}
\tag{2}
$$

where the form of the mirror is described by $\Delta z' = f(x', y')$. The incident and reflected fields can be written in the inclined frame to give

$$
E_i(s') \exp\left[-jk\left(z_o - \Delta z' \cos\delta\right)\right] \quad \text{and} \quad E_o(s') \exp\left[-jk\left(z_o + \Delta z' \cos\delta\right)\right] \; . \tag{3}
$$

respectively, and because these points represent the same physical position, we have

$$
E_o(s') = E_i(s') \exp\left[jk2\Delta z' \cos\delta\right] = E_i(s') \exp\left[j\phi(x', y')\right] \; . \tag{4}
$$

The mirror is thus being described as an inclined phase-transforming surface, and the obliquity factor enters naturally as expected.

For long-focal-length optics, the input and output beams can be decomposed into sums of propagating free-space modes:

$$E_i(s') = \sum_{m,n} A_{mn}\psi^{i}_{mn}(s') \quad \text{and} \quad E_o(s') = \sum_{r,s} B_{rs}\psi^{o}_{rs}(s') \ . \tag{5}$$

To determine the mode coefficients, we evaluate the overlap integrals over the surface of the mirror in the output coordinate frames. Unlike usual analyses, however, the modes are not orthonormal over the plane of interest; nevertheless

$$\int_M E_o(s')\,\psi^{o*}_{ij}(s')\,dM = \int_T E_o(s')\,\psi^{o*}_{ij}(s')\left[1+\left(\frac{\partial f}{\partial x'}\right)^2+\left(\frac{\partial f}{\partial y'}\right)^2\right]^{\frac{1}{2}} dx\,dy' \ , \tag{6}$$

where we have projected the surface of the mirror M onto the tangent plane T. For thin mirrors

$$\left(\frac{\partial f}{\partial x'}\right)^2 \ll 1 \text{ and } \left(\frac{\partial f}{\partial y'}\right)^2 \ll 1 \ , \tag{7}$$

and the surface integral is easy to evaluate numerically. We can now project this integral onto the $z = 0$ plane by making the substitutions $x' = x_o/\cos\delta$ and $y' = y_o$ to give the s_o frame (x_o, y_o, z_o). Finally, the overlap integral becomes

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_o(s_o)\,\psi^{o*}_{ij}(s_o)\,J dx_o dy_o \ , \tag{8}$$

where the Jacobian is defined in the usual way through

$$dx'dy' = \begin{vmatrix} \frac{\partial x'}{\partial x_o} & \frac{\partial x'}{\partial y_o} \\ \frac{\partial y'}{\partial x_o} & \frac{\partial y'}{\partial y_o} \end{vmatrix} dx_o dy_o \ . \tag{9}$$

Because the mirror slopes in the x direction, $dx'dy' = \sec\delta dx_o dy_o$ where δ is the angle of incidence. Also we have $x' = x_o\sqrt{1+\tan^2\delta}$ and $y' = y_o$. At this stage, we also note that the input modes have their waists at $z_1 = 0$, and the output modes have their waists at $z_2 = 0$; consequently, the relationship between the two coordinate frames is simply $z_1 = z_2 + z_{o1} + z_{o2}$. Thus, the tangent plane is described in the frame of the input modes by $z_1 = x_o\tan\delta + z_{o1}$ and in the frame of the output modes by $z_2 = x_o\tan\delta - z_{o2}$.

Substituting the modal expansion into the overlap integrals we get

$$\int_M E_o(s')\psi^{o*}_{ij}(s')dM = \sum_{r,s} B_{rs}\beta_{ijrs}\sec\delta \tag{10}$$

where

$$\beta_{ijrs} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \psi^{o}_{rs}(s_o)\psi^{o*}_{ij}(s_o)dx_o dy_o \ . \tag{11}$$

Through (4), we can express the reflected field in terms of the incident field and write

$$\int_M E_o(s')\psi^{o*}_{ij}(s')dM = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_i(s_o)\exp\left[\phi(x',y')\right]\psi^{o*}_{ij}(s_o)\,J dx_o dy_o \ . \tag{12}$$

Substituting the modal expansion then leads to

$$\int_M E_o(s')\,\psi_{ij}^{o*}(s')\,dM = \sum_{m,n} A_{m,n}\alpha_{ijmn}\sec\delta \tag{13}$$

where

$$\alpha_{ijmn} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \psi_{mn}^{i}(s_o)\exp\left[j\Delta\phi(x_o/\cos\delta, y_o)\right]\psi_{ij}^{o*}(s_o)dx_o dy_o\ . \tag{14}$$

Finally, combining these two results we find

$$\sum_{r,s} B_{rs}\beta_{ijrs} = \sum_{m,n} A_{mn}\alpha_{ijmn} \text{ for all } ij\ . \tag{15}$$

Clearly, this set of equations can be expressed in matrix form:

$$\boldsymbol{\alpha A} = \boldsymbol{\beta B}\ . \tag{16}$$

In this equation, $\boldsymbol{A}$ and $\boldsymbol{B}$ are vectors of mode coefficients, $\boldsymbol{\alpha}$ is a matrix describing the scattering of power between modes, and $\boldsymbol{\beta}$ is a matrix representing the nonorthogonality of the modes across the surface of the mirror: notice that $\boldsymbol{\beta}$ is Hermitian. We can now define a scattering matrix $\boldsymbol{S}$, relating the mode coefficients at the two waists, through

$$\boldsymbol{B} = \boldsymbol{\beta}^{-1}\boldsymbol{\alpha A} = \boldsymbol{SA}\ . \tag{17}$$

Let us choose propagating Hermite polynomials as a basis set. A suitable expansion function is

$$\begin{aligned}\psi_{ij} \quad = \quad & \frac{\sqrt{2}}{w(z)}h_i\left(\frac{\sqrt{2}x}{w(z)}\right)h_j\left(\frac{\sqrt{2}y}{w(z)}\right)\exp\left[-j\frac{\pi}{\lambda R(z)}\left(x^2+y^2\right)\right] \\ & \exp\left[j(i+j+1)\tan^{-1}\frac{z}{z_c}\right]\exp\left[-jkz\right]\end{aligned} \tag{18}$$

where

$$h_m(u) = \frac{H_m(u)\exp\left[-\frac{u^2}{2}\right]}{(\sqrt{\pi}2^m m!)^{\frac{1}{2}}}\ . \tag{19}$$

This set is normalised in the sense that the total power is unity. As shown in Fig. 1, the input beam is described in terms of coordinate z_1 and the output beam is described in terms of coordinate z_2. The waist and confocal distance of the input modes are denoted by w_{o1} and z_{c1} respectively, and the waist and confocal distance of the output modes are denoted by w_{o2} and z_{c2}. The Gaussian radii of the input and output beams are given by

$$w_2(z_2) = w_{o2}\left[1+\left(\frac{z_2}{z_{c2}}\right)^2\right]^{\frac{1}{2}} \quad \text{and} \quad w_1(z_1) = w_{o1}\left[1+\left(\frac{z_1}{z_{c1}}\right)^2\right]^{\frac{1}{2}}\ . \tag{20}$$

and the radii are curvature are given by

$$R_1(z_1) = z_1\left[1+\left(\frac{z_{c1}}{z_1}\right)^2\right] \quad \text{and} \quad R_2(z_2) = z_2\left[1+\left(\frac{z_{c2}}{z_2}\right)^2\right] \tag{21}$$

respectively.

The parameters of the modes associated with the output beam are chosen so that the waists at the centre of the mirror are equal:

$$w_1(z_{o1}) = w_2(z_{o2}) \,, \tag{22}$$

and after taking into account the focusing effect of the mirror, the radii of curvature match:

$$\frac{1}{R_1(z_{o1})} + \frac{1}{R_2(z_{o2})} = \frac{1}{f} \,, \tag{23}$$

where f is the focal length of the mirror. In some cases, it may be difficult to decide what the value of f should be. For example, an off-axis spherical mirror is astigmatic, and the sagittal and tangential focal lengths are different.

The position and size of the output waist can be found by satisfying the above two requirements [1], and this leads to

$$\frac{z_{o2}}{f} = 1 + \frac{(z_{o1}/f) - 1}{[(z_{o1}/f) - 1]^2 + (\pi w_{o1}^2/\lambda f)^2} \tag{24}$$

and

$$\left(\frac{w_{o2}}{w_{o1}}\right)^2 = \frac{1}{[(z_{o1}/f) - 1]^2 + (\pi w_{o1}^2/\lambda f)^2} \,. \tag{25}$$

Substituting the mode set into (11) gives

$$\begin{aligned} \beta_{ijrs} = & \int_{-\infty}^{+\infty} \frac{2}{w_2^2(z_2+\Delta z)} h_r\left(\frac{\sqrt{2}(x_o - \Delta x)}{w_2(z_2+\Delta z)}\right) h_s\left(\frac{\sqrt{2}(y_o)}{w_2(z_2+\Delta z)}\right) h_i\left(\frac{\sqrt{2}(x_o - \Delta x)}{w_2(z_2+\Delta z)}\right) \\ & h_j\left(\frac{\sqrt{2}(y_o)}{w_2(z_2+\Delta z)}\right) \exp\left[j\,(r+s-i-j)\tan^{-1}\frac{(z_2+\Delta z)}{z_{c2}}\right] dx_o dy_o \end{aligned} \tag{26}$$

At this stage, the integral can be evaluated as it stands, or various levels of approximation can be introduced. For the purposes of this paper, we will assume that all of the offsets can be ignored when calculating the β matrix. This approximation is equivalent to following the tangent plane rather than the surface of the mirror when calculating field amplitudes. Clearly, this approximation is reasonable if the mirror is thin. After making this assumption, we can write

$$\beta_{ijrs} = \int_{-\infty}^{+\infty} \frac{\sqrt{2}}{w_2(z_2)} h_r\left(\frac{\sqrt{2}x_o}{w_2(z_2)}\right) h_i\left(\frac{\sqrt{2}x_o}{w_2(z_2)}\right) \exp\left[j\,(r-i)\tan^{-1}\frac{z_2}{z_{c2}}\right] dx_o \delta_{js} \,, \tag{27}$$

where we have taken advantage of the fact that, because the mirror is inclined in the x direction, the modes in the y direction remain orthogonal. As a consequence β becomes a block diagonal matrix; a feature which can be exploited when the matrix is inverted. Also notice that β only depends on the angle of incidence: it does not depend on the shape of the mirror. It may also be seen that, to a large extent, the modes remain orthogonal in a long-focal-length system because the waist changes very little over the region occupied by the mirror. When coupling does occur, one finds that a mode is preferentially coupled to modes having similar orders, and this implies that some form of perturbation analysis would be appropriate.

We can also substitute the mode set into (14) to give

$$\begin{aligned}\alpha_{ijmn} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\sqrt{2}}{w_1(z_1-\Delta z)}\frac{\sqrt{2}}{w_2(z_2+\Delta z)}h_m\left(\frac{\sqrt{2}(x_o+\Delta x)}{w_1(z_1-\Delta z)}\right)h_i\left(\frac{\sqrt{2}(x_o-\Delta x)}{w_2(z_2+\Delta z)}\right)\\ &\quad \exp\left[j\,(m+n+1)\tan^{-1}\frac{(z_1-\Delta z)}{z_{c1}}\right]\exp\left[-j\,(i+j+1)\tan^{-1}\frac{(z_2+\Delta z)}{z_{c2}}\right]\\ &\quad h_n\left(\frac{\sqrt{2}y_o}{w_1(z_1-\Delta z)}\right)h_j\left(\frac{\sqrt{2}y_o}{w_2(z_2+\Delta z)}\right)\exp\left[-j\frac{\pi}{\lambda R_1(z_1-\Delta z)}\left((x_o+\Delta x)^2+y_o^2\right)\right]\\ &\quad \exp\left[+j\frac{\pi}{\lambda R_2(z_2+\Delta z)}\left((x_o-\Delta x)^2+y_o^2\right)\right]\exp\left[j\Delta\phi(x',y')\right]dy_o dx_o\,. \qquad (28)\end{aligned}$$

This equation is rather involved, and it is not possible to simplify for the general case. Nevertheless, it is straightforward to evaluate numerically. Again we can follow the tangent plane when calculating the amplitudes. It is of course important to follow the surface of the mirror when calculating the phase. This approximation leads to

$$\begin{aligned}\alpha_{ijmn} &= \int_{-\infty}^{+\infty}\frac{2}{w_1(z_1)w_2(z_2)}h_m\left(\frac{\sqrt{2}(x_o)}{w_1(z_1)}\right)h_i\left(\frac{\sqrt{2}(x_o)}{w_2(z_2)}\right)\exp\left[j\,(m+n+1)\tan^{-1}\frac{z_1}{z_{c1}}\right]\\ &\quad \exp\left[-j\,(i+j+1)\tan^{-1}\frac{z_2}{z_{c2}}\right]\int_{-\infty}^{+\infty}h_n\left(\frac{\sqrt{2}y_o}{w_1(z_1)}\right)h_j\left(\frac{\sqrt{2}y_o}{w_2(z_2)}\right)\\ &\quad \exp\left[-j\frac{\pi}{\lambda R_1(z_1-\Delta z)}\left((x_o+\Delta x)^2+y_o^2\right)\right]\\ &\quad \exp\left[+j\frac{\pi}{\lambda R_2(z_2+\Delta z)}\left((x_o-\Delta x)^2+y_o^2\right)\right]\exp\left[j\Delta\phi(x',y')\right]dy_o dx_o\,. \qquad (29)\end{aligned}$$

To complete the analysis we need to make a modification to the last equation. The problem is that the last three terms, which constitute the phase error across the surface of the mirror, are based on the paraxial approximation. As a consequence, the equation correctly describes the long-wavelength behaviour, but does not adequately describe the short-wavelength behaviour. We can correct this deficiency, albeit in a heuristic manner, by replacing the parabolic phase front of the Gaussian mode with a spherical phase front. The appropriate phase factor is given by

$$\begin{aligned}&\exp\left[-j\left(\frac{2\pi}{\lambda}\right)\frac{(x_o+\Delta x)^2+y_o^2}{R_1(z_1-\Delta z)+\sqrt{R_1^2(z_1-\Delta z)+(x_o+\Delta x)^2+y_o^2}}\right]\\ &\exp\left[+j\left(\frac{2\pi}{\lambda}\right)\frac{(x_o-\Delta x)^2+y_o^2}{R_2(z_2+\Delta z)-\sqrt{R_2^2(z_2+\Delta z)+(x_o-\Delta x)^2+y_o^2}}\right]\,. \qquad (30)\end{aligned}$$

Once the α and β matrices are known, the scattering matrix can be evaluated through (17).

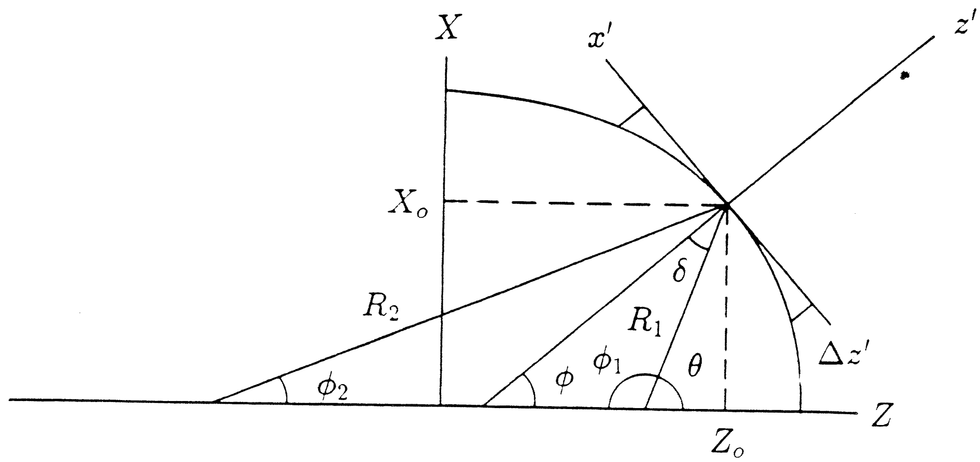

Figure 3: The geometry of an ellipsoidal mirror.

3 Ellipsoidal mirrors

To complete the analysis, we need to find the phase factors $\phi(x', y')$, for the mirrors of interest. Clearly, we need to express the height of the surface of the mirror above the tangent plane in the (x, z) coordinate frame. The geometry of an ellipsoidal surface is shown in Fig. 3.

We choose $R1$ and $R2$ to match the radii of curvature of the input and output beams respectively. Once the angle of incidence has been fixed, we can calculate the lengths of the major and minor axes through

$$a = \frac{(R1 + R2)}{2} \quad \text{and} \quad b = \sqrt{R1R2}\cos\delta \ . \tag{31}$$

We now need to determine the angle of the normal to the tangent plane with respect to the axis of rotation of the ellipsoid. Consideration of the geometry shows

$$\tan\phi 1 = \frac{\sin(2\delta)}{\left[\frac{R1}{R2} - \cos(2\delta)\right]} \tag{32}$$

and also $\phi = \pi - \phi 1 - \delta$ and $\theta = \pi - \phi 1$. The vertex of the mirror is at

$$X_o = R1\sin\theta \text{ and } Z_o = R1\cos\theta + ae \ , \tag{33}$$

where e is the eccentricity. By shifting and rotating the (X, Z) coordinate frame through ϕ, it is straightforward to show that the thickness of the mirror can be found by solving a quadratic equation; more specifically,

$$\Delta z' = \left|\frac{-B - \sqrt{B^2 - 4AC}}{2A}\right| \ . \tag{34}$$

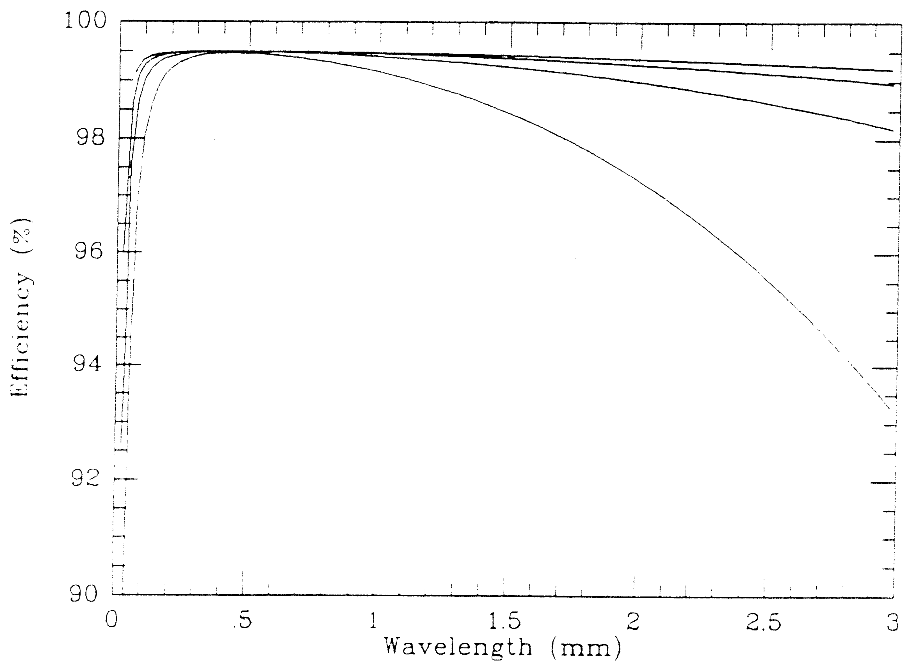

Figure 4: The fundamental-mode coupling efficiencies, as a function of wavelength, of ellipsoidal mirrors having focal lengths of 50mm, 100mm, 150mm, and 200mm. The focal ratio of the illuminating beam is 5, the design wavelength is 0.63mm, the angle of incidence is 45°, and the system is confocal.

where

$$
\begin{aligned}
A &= \frac{\sin^2\phi}{b^2} + \frac{\cos^2\phi}{a^2} \\
B &= \frac{2x_o\cos\phi\sin\phi}{\cos\delta}\left(\frac{1}{b^2}-\frac{1}{a^2}\right) + \frac{2X_o\sin\phi}{b^2} + \frac{2Z_o\cos\phi}{a^2} \\
C &= \frac{y_o^2}{b^2} + \left(\frac{x_o}{\cos\delta}\right)^2\left(\frac{\cos^2\phi}{b^2}+\frac{\sin^2\phi}{a^2}\right) + 2\left(\frac{x_o}{\cos\delta}\right)\left(\frac{\cos\phi X_o}{b^2} - \frac{\sin\phi Z_o}{a^2}\right)
\end{aligned}
\quad . \quad (35)
$$

Again, this equation is easy to evaluate numerically for any design of mirror. For the purposes of finding the position of the output waist, we define the focal length of the mirror to be $f = R1R2/(R1 - R2)$.

4 Results

We have used the above technique to calculate the bandwidths of a number of ellipsoidal, off-axis mirrors. In Fig. 4 we show the fundamental-mode coupling efficiencies, as a function of wavelength, of ellipsoidal mirrors having focal lengths of 50mm, 100mm,

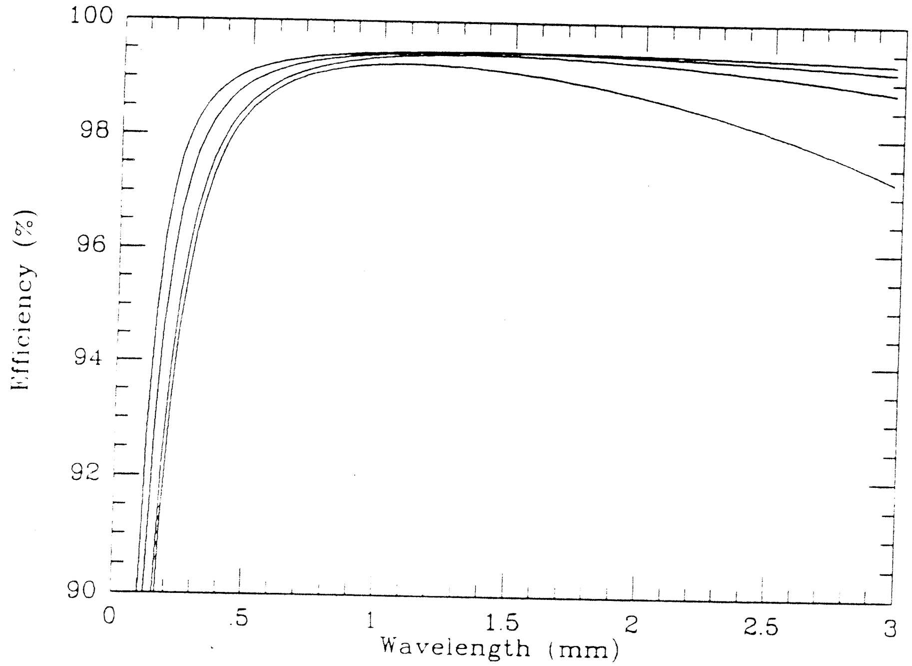

Figure 5: The fundamental-mode coupling efficiencies, as a function of wavelength, of ellipsoidal mirrors having focal lengths of 50mm, 100mm, 150mm, and 200mm. The focal ratio of the illuminating beam is 5, the design wavelength is 2.0mm, the angle of incidence is 45°, and the system is confocal.

150mm, and 200mm. At every wavelength the mirror is illuminated with a Gaussian beam having a waist focal ratio of 5. That is to say, we scale the waist with frequency so that the opening angle of the incoming beam is constant. This is the situation, for example, when a horn illuminates a reflecting antenna, in a frequency independent way, through a Gaussian-beam telescope. In each case, we have designed the mirror to match the input and output radii of curvature at a frequency of 475GHz. It can be seen that in each case the efficiency peaks at the design frequency and falls to the short and long wavelength sides due to the radii of of curvature of the incoming and outgoing beams not being matched. It is clear that high efficiencies can be achieved over a wide range of parameters as expected.

In Fig. 5 we show a similar set of plots, but in this case the mirrors are designed for a centre frequency of 150GHz. As expected, the high-frequency fall-off occurs at a lower frequency than in the previous case. Also, it can be seen that the efficiency only peaks at the design wavelength when the focal length of the mirror is reasonably large. This effect occurs because the efficiency contains two terms: one describes the degree to which the phase fronts are matched across the surface of the mirror, and the other describes the way in which the amplitudes match across the inclined plane. The amplitude projection effect is reasonably independent of wavelength, but it does depend to some extent on

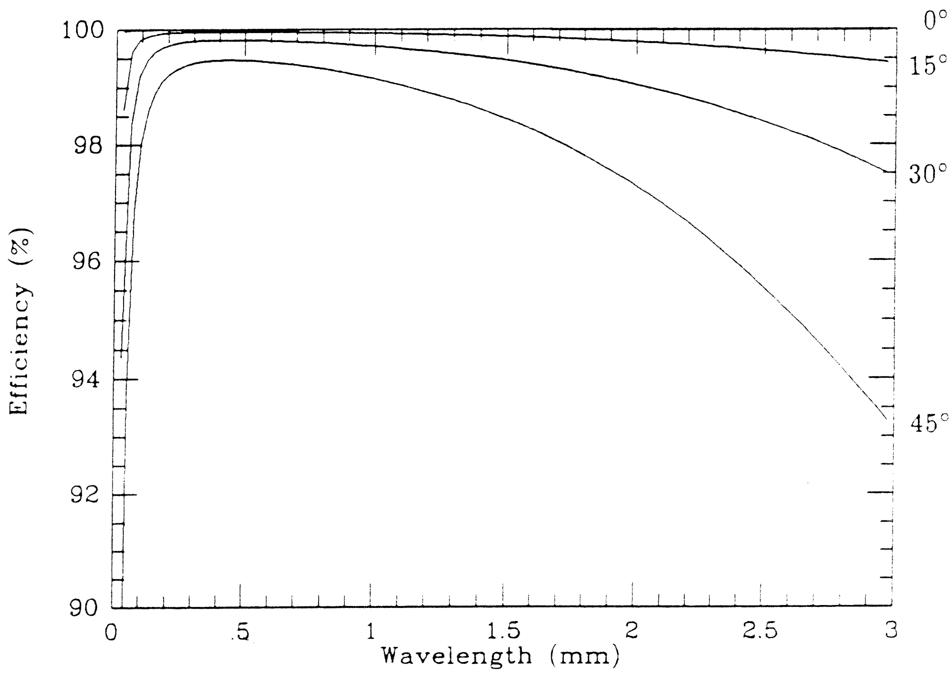

Figure 6: The fundamental-mode coupling efficiencies, as a function of wavelength, of an ellipsoidal mirror having a focal length of 50mm at various angles of incidence. The focal ratio of the illuminating beam is 5, the design wavelength is 0.63mm, and the system is confocal.

the beams involved, and in the present case peaks at the short wavelength end. The net effect is to move the maximum coupling beyond the wavelength at which the phase error is essentially zero.

Clearly, a broadband system can be made by designing the mirrors for the shortest wavelength of operation and keeping the focal lengths of the mirrors greater than about 200mm. This is essentially equivalent to designing the mirrors for far-field operation. This explains the reason for using mirrors having focal lengths of greater than 250mm in the broadband receiver we are currently building [2]. It should be noted that when a mirror is designed for a short-wavelength confocal system, the mirror becomes parabolic, and the formulation for the ellipsoidal surface given in this paper may suffer from round-off errors. There is a formulation that describes an ellipsoidal mirror in the parabolic limit in a more careful manner, but for certain designs of mirror this can introduce problems with the signs of angles, and so we have kept to the formulation described here. We have found this formulation to be perfectly adequate for most purposes.

Finally, in Fig. 6 we show the performance of a mirror, having a focal length of 50mm, which is designed to operate with a beam having a focal ratio of 5 at a wavelength of 0.63mm. The plots show how the coupling efficiency changes as the mirror is tilted. It is clear that there is a maximum efficiency for any mirror which is solely determined by the beam and the angle of incidence. This efficiency tends to unity as the mirror is turned upright.

5 Conclusions

We have developed a technique for calculating the Gaussian beam-mode scattering matrices of shaped off-axis mirrors. The technique can be applied to any shaped mirror and can therefore be used for a variety of applications. We have demonstrated the technique by investigating the bandwidths of off-axis ellipsoidal mirrors. It is clear that very broad band submillimetre-wave optical systems can be manufactured by keeping the focal lengths of the mirrors greater than about 200mm.

Although we have not shown all of the elements of the scattering matrices, it is clear that good-quality mirrors can be used for phase-retrieval experiments. This is possible because long-focal-length mirrors do not scatter power between modes at any significant level. It may be possible to use the proposed technique, together with phase-retrieval methods, for synthesizing systems of shaped mirrors.

References

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