

## Embedded Control Systems, Winter 2009, Problem Set 3<sup>1</sup>

Issued: Wednesday February 4, 2009

due: Wednesday February 11, 2009

In this problem set you will use Matlab to obtain several plots. My major concern is that you understand these plots, so please include explanation with them when you turn them in!

1. Consider a low pass filter with frequency response

$$H(j\omega) = \frac{1}{j\omega\tau + 1}.$$

- (a) Using the Matlab file prob1\_ps3.m, plot the frequency response of this filter for  $\tau = 1$ . Note that in Matlab,  $j\omega$  is replaced with the Laplace operator,  $s$ . At the top of prob1\_ps3.m you will have to insert the value for the time constant,  $\tau$ , and specify the transfer function numerator and denominator coefficients of  $s$  in descending order.
- (b) Using the frequency response plot, determine  $y_{ss}(t)$ , the sinusoidal steady state response of the filter output, to an input signal  $u(t) = \sin(10t)$  (in other words, what is the magnitude and phase of the output sinusoid?).
- (c) Consider two sinusoids with the same frequency  $\omega_0$ :  $u_1(t) = A_1 \sin(\omega_0 t)$  and  $u_2(t) = A_2 \sin(\omega_0 t + \phi)$ . At what times do each of these sinusoids cross zero? Let  $t_1$  and  $t_2$  denote the times of corresponding zero crossings for the two sinusoids. Compute  $\Delta t = t_2 - t_1$  in terms of  $\omega_0$  and  $\phi$ .
- (d) The m-script file prob1\_ps3.m also plots the time response of the filtered output in response to the input  $u(t) = \sin(10t)$ . Does the time response agree with the magnitude and phase predicted using the frequency response plot? Check both the magnitude and phase difference; as shown in part (1c) the latter may be computed from the time between zero crossings.
- (e) How do the answers above change for different values of  $\tau$ , for example,  $\tau = 0.1$  and  $\tau = 10$ ? NOTE: You may need to change the time interval for the simulation so that the transient response has time to die out.

HAND IN: At least one set of frequency and time response plots, and be very careful to explain what happens with the other plots. If you just hand in plots, do explain why they look the way they do and why you see the trends you see. This problem is fundamental to understanding linear filtering and the relation between bandwidth and frequency response.

NOTE: The Matlab command “ginput” is useful in picking numerical values from Matlab figures. (Type “[x,y]=ginput” and hit return – you can then pick points off the current figure with the crosshairs until you hit return again.)

2. Consider a DC motor, as shown in Figure 1. The DC motor in the EECS 461 lab has parameter values:  $K_M = 0.0302$  N-m/A,  $K_V = 0.0302$  V/(rad/sec),  $R = 0.317$  ohms,  $L = 0.0000823$  H,  $J = 1.380 \times 10^{-5}$  N-m/(rad/sec)<sup>2</sup>, and  $B = 0.00131$  N-m/(rad/sec).

- (a) Suppose that this motor is driven by a constant voltage source  $V$  and undergoes a constant load  $T_L$ . Then, as we showed in Lecture 5, the steady state torque and speed are related by

$$T_M + \left(\frac{K_M K_V}{R}\right) \Omega = \left(\frac{K_M}{R}\right) V$$

Using the Matlab file prob2a\_ps3\_newbox.m, sketch the associated torque speed curves for several values of the input voltage  $V$ . Why do these curves not depend on the inductance  $L$  or the inertia  $J$ ? Hand in the plot of torque-speed curves with the different load torques identified. Note: prob2a\_ps3.m is incomplete; you will need to incorporate the equation for motor torque in the “FOR” loop on line 26.

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<sup>1</sup>Revised February 2, 2009.

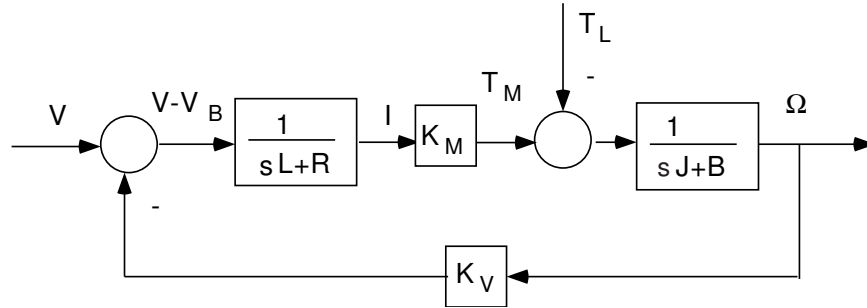


Figure 1: DC Motor

- (b) For a given input voltage  $V$ , the specific values of steady state torque and speed will depend on the value of the friction coefficient  $B$  and the constant load torque  $T_L$ , according to the relations

$$\Omega = \frac{K_M V - R T_L}{K_M K_V + R B}, \quad T_M = \frac{K_M (B V + K_V T_L)}{K_M K_V + R B}.$$

Using the Matlab file prob2b\_ps3\_newbox.m, compute the steady state values of speed and torque for several values of constant load torque, and plot them on the torque-speed curves. You'll note that this m-script is also incomplete: you will have to insert the necessary equations for torque and speed.

- (c) Complete the Matlab file prob2c\_ps3\_newbox.m, and use it to plot the frequency responses of speed (in Hz) and torque (in N-m) to a voltage input (in V). What are the DC gains of each response? Do these agree with the torque-speed equation in part (2a)? Why or why not? Explain. Hand in the frequency response plot.
- (d) Develop a SIMULINK model of this motor, as shown in Figure 2. Use a “step” block to generate the constant voltage input. Use your model, in conjunction with the Matlab file prob2d\_ps3\_newbox.m

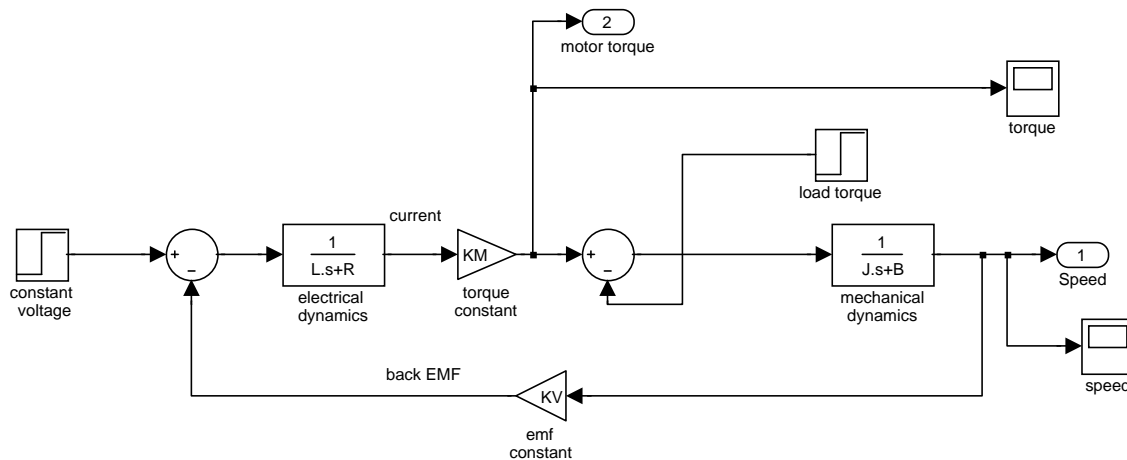


Figure 2: DC Motor with Load Torque Disturbance

to verify that the torque speed curve you constructed agrees with your model (to do so, run your simulation with several different values of the input voltage and the constant load torque). Hand in one of these time response plots.

- (e) Now modify your Simulink model to use a PWM signal to drive the motor; to do so, use a “pulse generator” block to create a PWM signal with a constant duty cycle that alternates between 0 and 5 V. You need not supply a torque disturbance in this part of the problem, so you can eliminate this block, or set the torque disturbance to zero. The frequency spectrum of a PWM signal alternating

between 0 and 5 will have a nonzero DC component. The gain that this frequency component will see is obtained from the frequency response at  $\omega = 0$ . The gain seen by the first harmonic is obtained from the frequency response plots evaluated at the switching frequency  $f = 1/T$  Hz. By inspecting your plots from part (2c), you can determine the relative contribution to the steady state speed and torque signals from the two frequency components of the PWM signal. Use your Simulink model together with the Matlab file `prob2e_ps3_newbox.m` to plot the responses of the motor speed and torque to a 25kHz PWM signal with a 50% duty cycle. Hand in this plot. Are these plots consistent with the frequency response plot from part (2c)? Explain. Why does the torque exhibit a greater response to the PWM switching frequency than does the speed? Explain.

- (f) Examine the speed and torque responses to the PWM input, and determine the steady state value in each case (strictly speaking neither response reaches a steady state due to the high frequency switching noise, so use approximate values). Show how these values may be computed by using the amplitude and duty cycle of the PWM signal and the DC gains of each response. Verify that the steady state values of speed and torque satisfy the torque speed equation in part (2a) for an input  $V = 2.5$  volts.