EECS 461, Winter 2009, Problem Set 4[1]

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1. We have seen that important properties of second order systems are described by the roots of the characteristic equation. If these roots are complex, it is useful to parameterize the location of these roots in the complex plane in terms of natural frequency and damping coefficient.

   The time response of a second order system to a unit step input (a constant input with magnitude equal to one) for various values of damping coefficient is plotted in Figure 1. Two measures are frequently used to describe this time response. One is the *rise time*, t_r, defined by the time it takes the system to travel

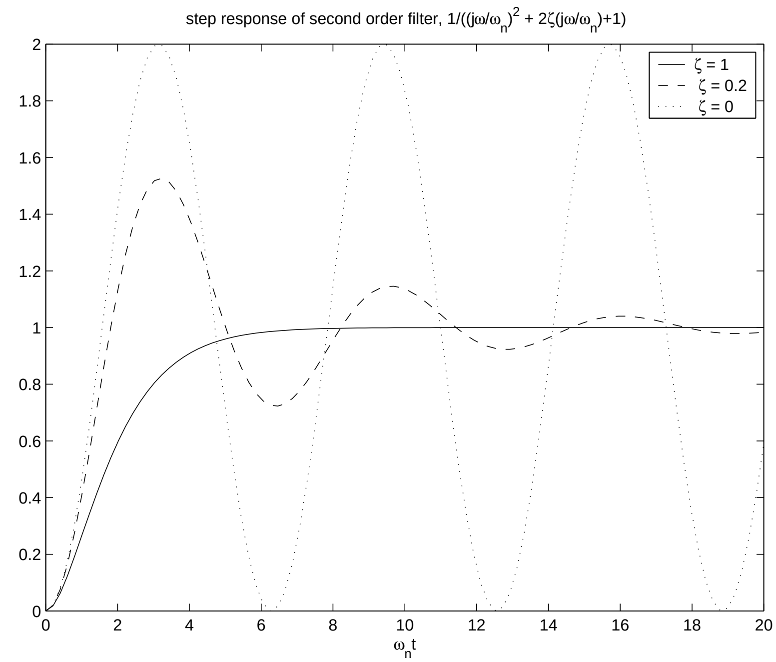

Figure 1: Step Response as a Function of Natural Frequency and Damping

   from 10% to 90% of its steady state value. It is possible to approximate the rise time by

$$t_r \approx \frac{1.8}{\omega_n}.$$

   A more accurate approximation can be obtained by incorporating the effect of damping on t_r; however, this is usually unnecessary. The trend is clear, however: rise time is inversely proportional to ω_n, and thus to the bandwidth of the frequency response of the second order system.

   Another important measure of system response is the *overshoot*, defined by the amount that the peak in the step response exceeds its steady state value. Note from the figure that $\zeta = 1$ implies there is no overshoot and $\zeta = 0.2$ implies an overshoot of about 50%. Overshoot is undefined for $\zeta = 0$ because the system never reaches steady state. In general, the relation between damping and overshoot for a second order system is shown in Figure 2. Many systems of interest have more than two integrators, and are thus of order greater than two. In many cases, these systems can be approximated by a second order system. One example is the DC motor we have been studying. Consider the speed control feedback system for the DC motor, depicted in Figure 3. The controller in this case is an integral controller K/s. With such a controller, the steady state response to a step change in the speed command has zero steady state error. The characteristic equation has three roots: one of these corresponds to the circuit dynamics and is very fast. The dominant response is due to a complex pair of roots.

[1] Revised March 1, 2009.

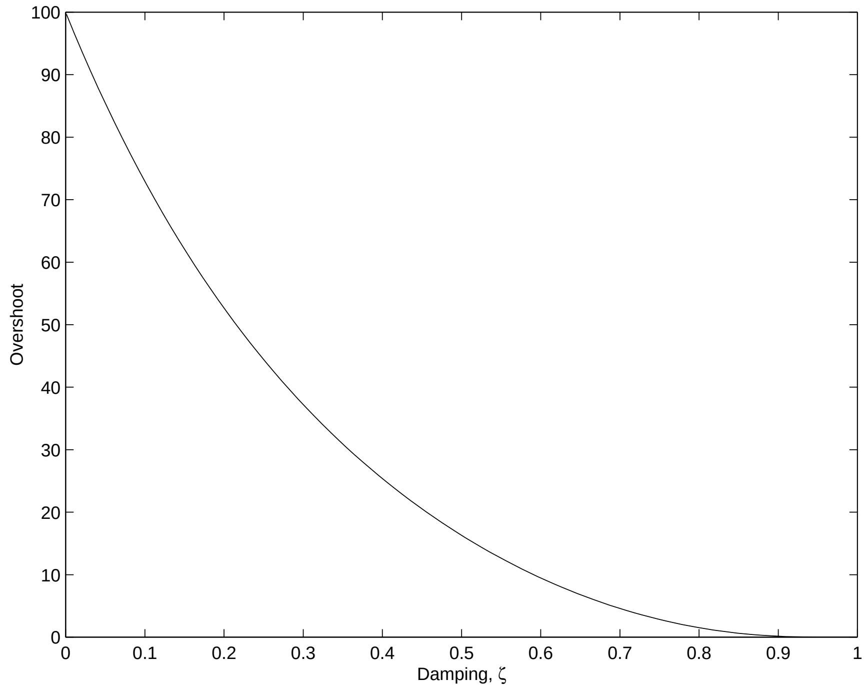

Figure 2: Overshoot vs. Damping

(a). Build a SIMULINK file as shown in Figure 3. Modify the m-file PS4_prob1.m to compute the natural frequency and damping for various values of K (including $K = 10$ and $K = 100$).

(b). Use the discussion above to estimate the rise time and overshoot associated with these roots, and compare them to the values obtained from the time domain simulations.

(c). How does the time response correlate with the closed loop frequency response? Describe qualitatively. For example, compare the frequency response peak with the percent overshoot in the time response. As the frequency response bandwidth increases, what happens to the speed of response in the time domain? What's the relationship among the undamped natural frequency, bandwidth and speed of response?

2. Consider the mass/spring/damper system shown in Figures 4-5. For simplicity, set $M = 1$.

(a). Suppose that $B = 0$. What is the undamped natural frequency? What is the damping ratio? How does the natural frequency vary with the spring constant K? Evaluate step response and frequency response plots for a few values of K. How does the period of oscillation change as K increases?

(b). Now fix $K = 1$, and suppose that $B \neq 0$. How do the natural frequency and damping vary with B? For what values of B are the roots real? imaginary? Again, evaluate step and frequency response plots for a few values of B and describe the trends you see. In other words, as B increases, what happens to speed of response and overshoot?

You may use the Matlab file PS4_Prob2.m.

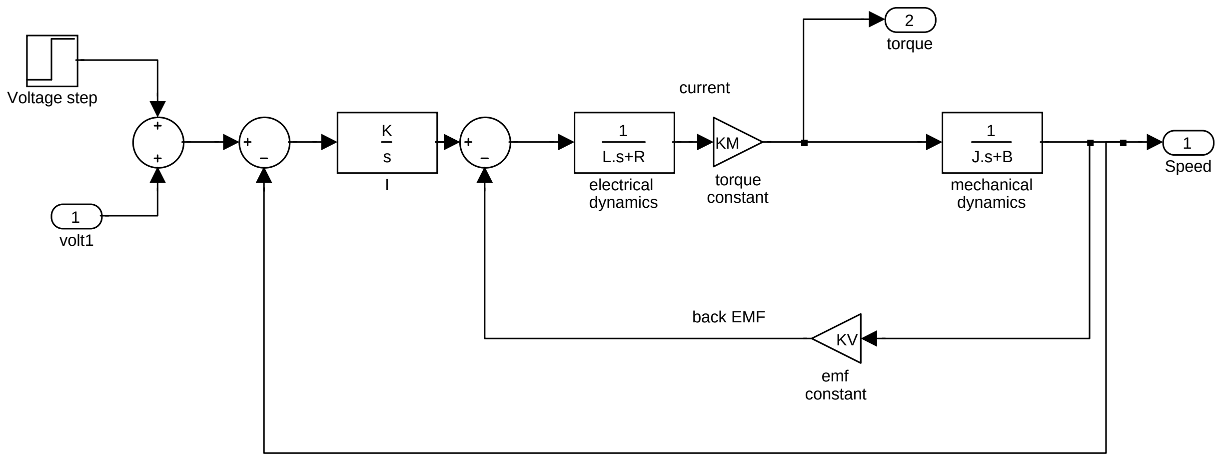

Figure 3: DC Motor with Speed Control

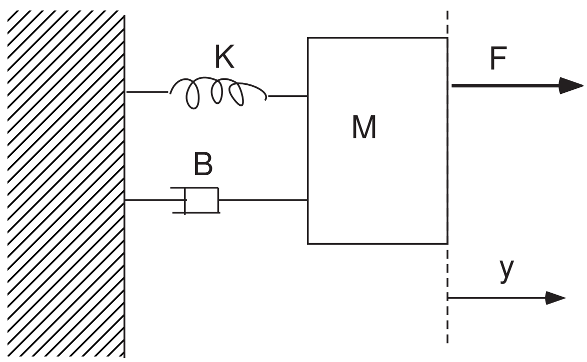

Figure 4: Spring Mass Damper System

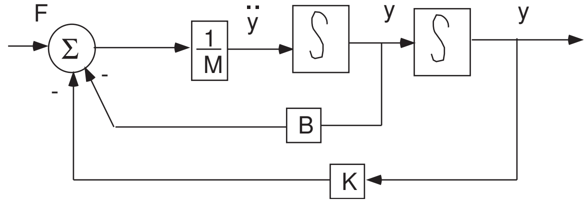

Figure 5: Block Diagram of Spring Mass Damper System