EECS 461, Winter 2009, Problem Set 5[1]

issued: Wednesday March 11, 2009   due: Wednesday March 18, 2009

1. The Simulink diagram in Figure 1 is one way to model a virtual wall, where the special user code block switches the spring and damper forces on or off depending on whether the mass is in contact with ("inside") or outside the wall.
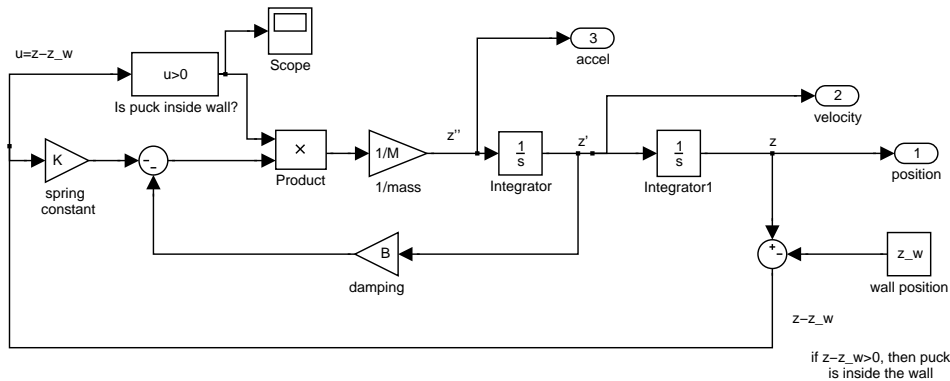

Figure 1: SIMULINK Model of Virtual Wall with Spring and Damping

An alternative way to simulate the virtual wall is by using Stateflow to model the fact that the puck is in either one of two discrete states: outside the wall, or inside the wall. This way of modelling the two discrete states may at first appear more complicated than the approach shown in Figure 1. Suppose, however, that the system we are trying to model and simulate has several discrete states. Then the simple trick used in Figure 1 will not work to distinguish between them. A simple example of a system with three discrete states is shown in Figure 2, which represents a one dimensional version of a Pong game. The virtual ball can be in one of three discrete states: in contact with the left paddle, in contact with the right paddle, or in contact with neither. If one were to model a two dimensional version of the Pong game, then the ball could be in one of several discrete states, depending on whether it was in contact with one of the paddles, or with the sides of the playing area, or in free flight. It is much more natural to model these situations using several discrete states.

In the remainder of this problem, we will develop a Stateflow model of the virtual wall, and then extend the model to a system with two virtual walls, and thus three discrete states. Be sure to use a fixed-step solver, ODE5 for your simulations.

(a) Modify the Simulink model in Figure 1 to that shown in Figure 3, with the Stateflow diagram shown in Figure 4. (Note: it is assumed that the puck has an initial position inside the wall, so you will have to specify the initial condition of the second integrator in your Simulink model). Use the Matlab file "prob1_PS5.m" to compare the results of the two simulations. They should be very similar, but if you zoom on the plots in the fifth plot, you should see some slight differences. Why do these differences exist? Hand in a copy of the third and fifth plots obtained from the Matlab file.

(b) Modify the Matlab, Simulink, and Stateflow files you used above to model what happens when the puck must move between *two* virtual walls, as depicted in Figure 5. When the puck is inside either wall, it will experience the restoring force of the appropriate spring and damper. When the puck is between the walls, it will experience zero force. It is assumed that the puck has initial condition inside the right wall. The Simulink file you will use should look like that in Figure 6. Hand in: a copy of the Stateflow block that goes with Figure 6 and a plot of the response of the puck for $B = 0$. (The latter response should look like that in Figure 7.)
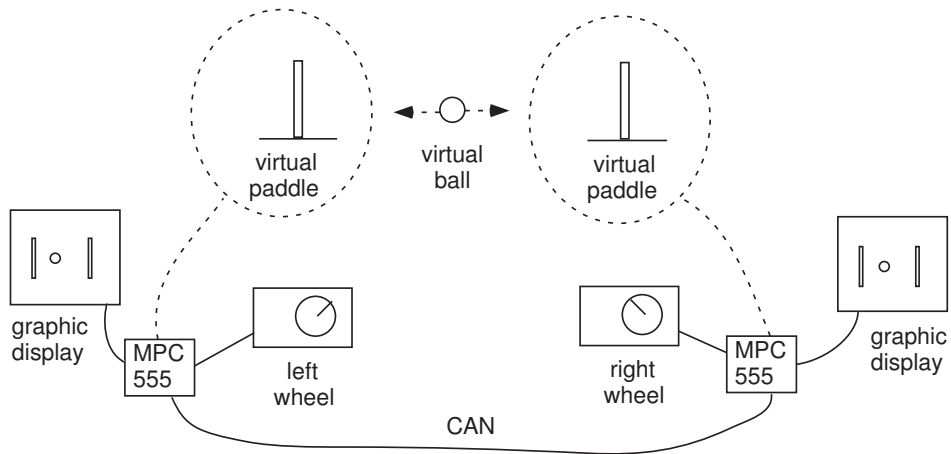
---

[1]Revised March 6, 2009.

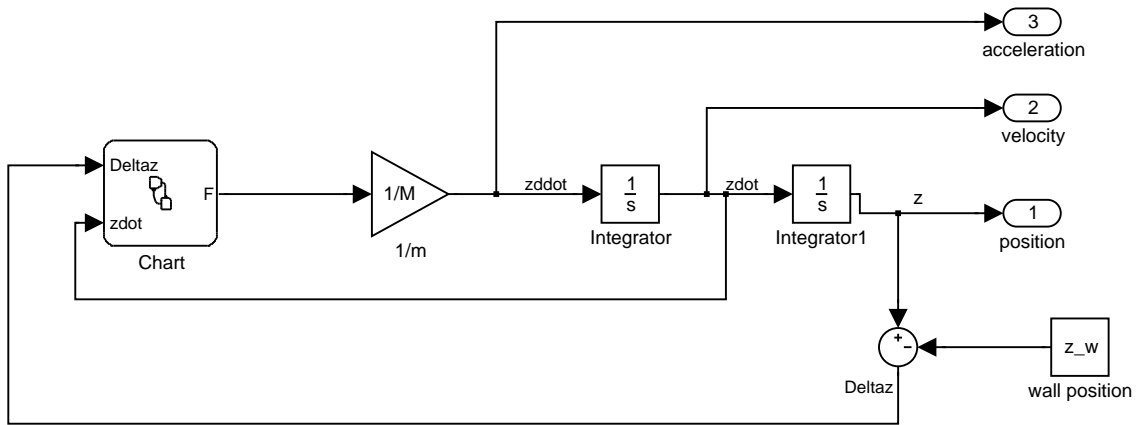Figure 2: A virtual Pong game with three discrete states

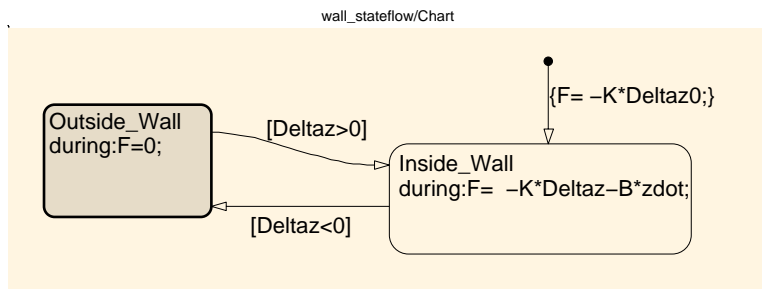Figure 3: SIMULINK model of virtual wall with Stateflow used to model two discrete states
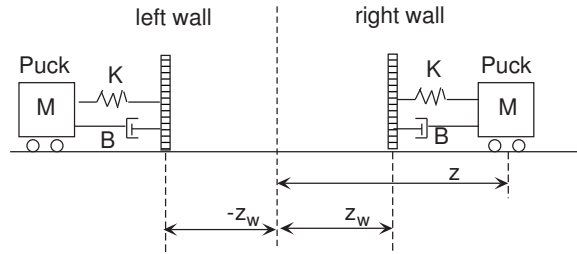

Figure 4: Details of Stateflow block
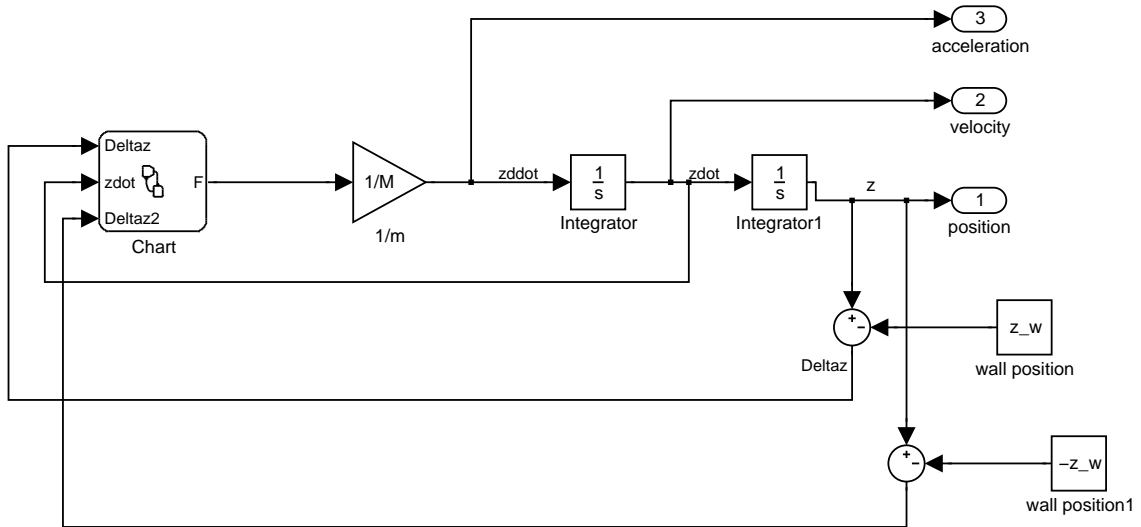
Figure 5: Two Virtual Walls

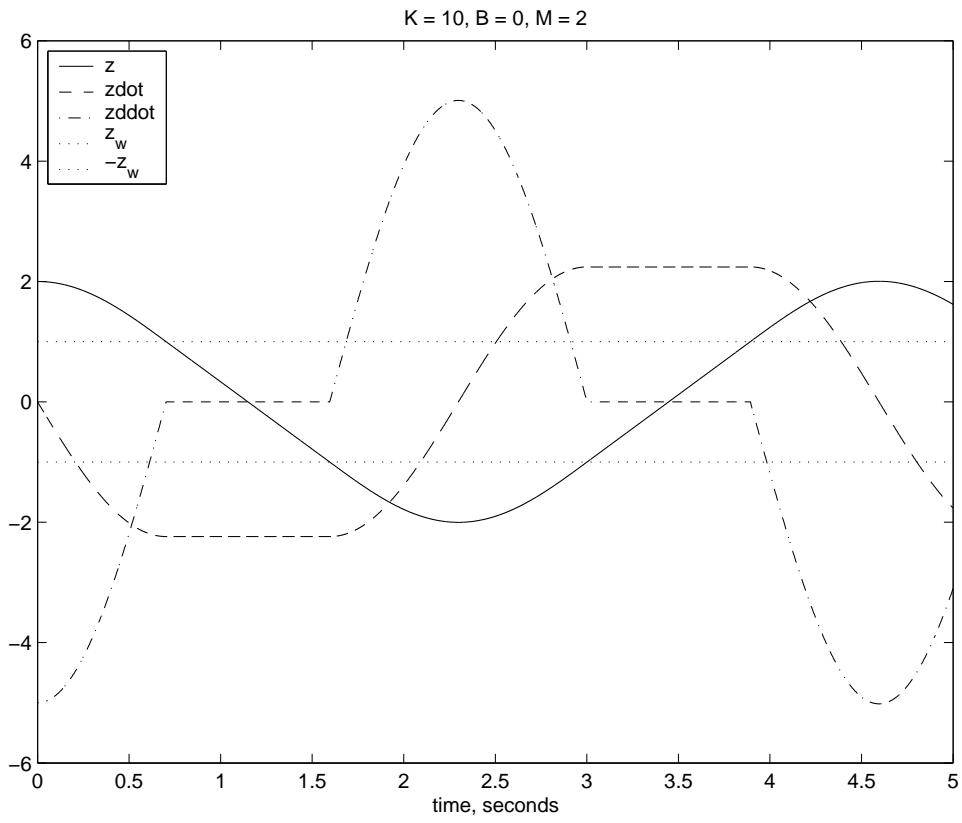Figure 6: SIMULINK model of puck trapped between two virtual walls

3

Figure 7: Response of puck to two walls

2. State variable models can be applied in many situations. When modelling a mechanical system the state variables (or states) are generally chosen to be the position and velocity of each mass in the system (if the system exhibits rotary motion, the states are chosen to be the angular position and angular velocity of each inertia). If the system to be modelled is an electric circuit, such as the RLC circuit depicted in Figure 8, then the states may be chosen as the current through each inductor and the voltage across each capacitor
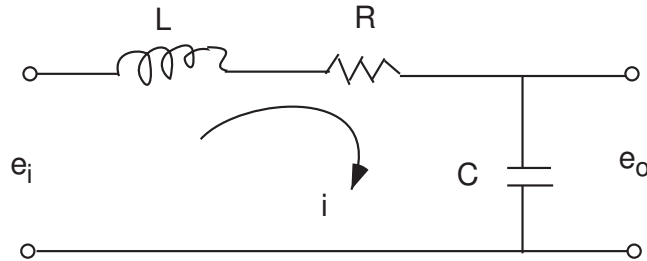

Figure 8: RLC circuit.

(a) Consider the RLC circuit shown in Figure 8. Kirchoff's laws state that the voltages e_o and e_i and the current i are related by the equations:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\, dt = e_i \tag{1}$$

$$\frac{1}{C}\int i\, dt = e_o. \tag{2}$$

Find the transfer function from input voltage e_i to output voltage e_o. HINT: First find the transfer functions from e_i to i and from i to e_o.

(b) Find a second order differential equation relating input voltage e_i to output voltage e_o.

(c) Define state variables $x_1 = e_o$, $x_2 = \dot{e}_o$, input $u = e_i$, and output $y = e_o$. Find a state variable representation of the system in the form

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

(d) For what values of the parameters R, L, and C is the circuit an oscillator?

(e) Let parameter values be chosen so that the circuit is an oscillator with natural frequency $\omega_n = 10$ radians/second. Suppose that the circuit is simulated on a computer using Forward Euler integration with simulation step size T seconds. Where do the characteristic roots (the "eigenvalues") of the discrete system lie, as a function of T, for the specified natural frequency?

(f) How should we change the values of the circuit parameters if we wish its discrete approximation to be an oscillator?