



# Local Image Features

EECS 598-08 Fall 2014

Foundations of Computer Vision

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**Readings:** FP 5; SZ 4.2, 4.3

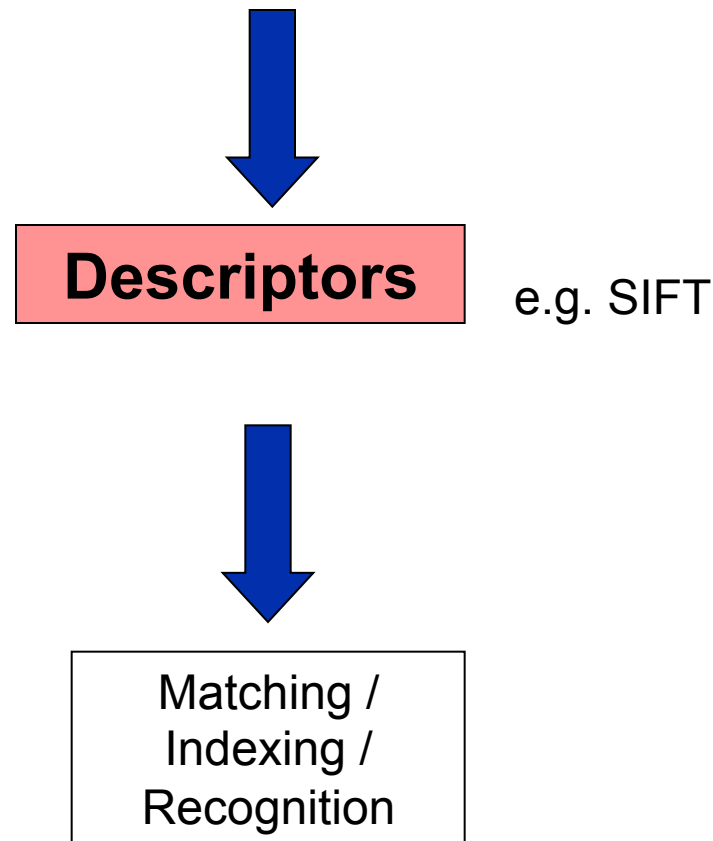
**Date:** 9/29/14

# Plan

- What are local image features and why are they useful.
- Local Image Feature Detection
- Invariance
- Local Image Feature Description

## Goal:

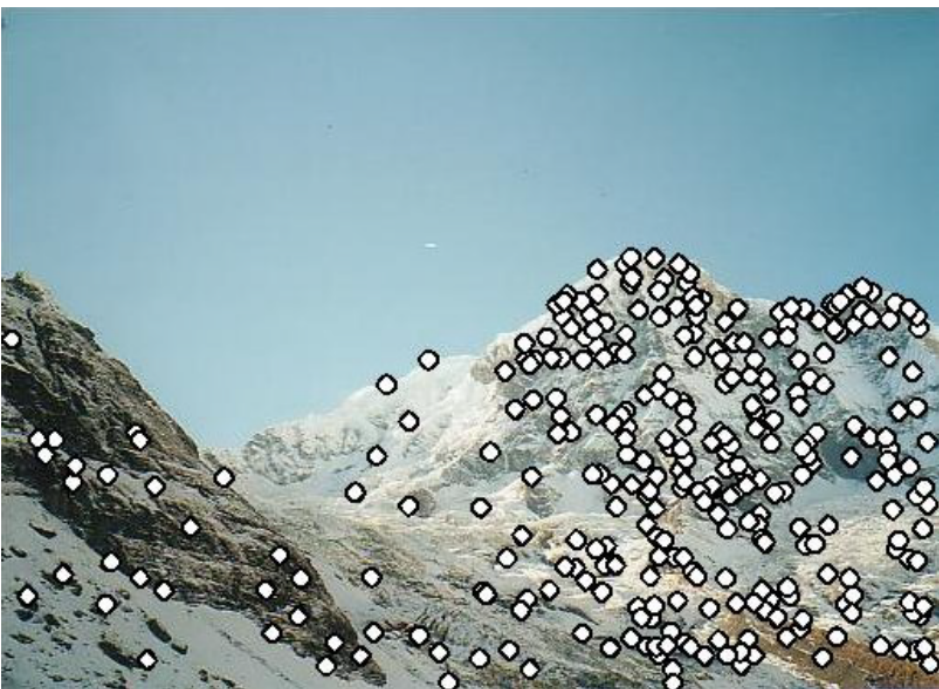
Identify interesting regions from the images (edges, corners, blobs...)



# Application: Image Stitching

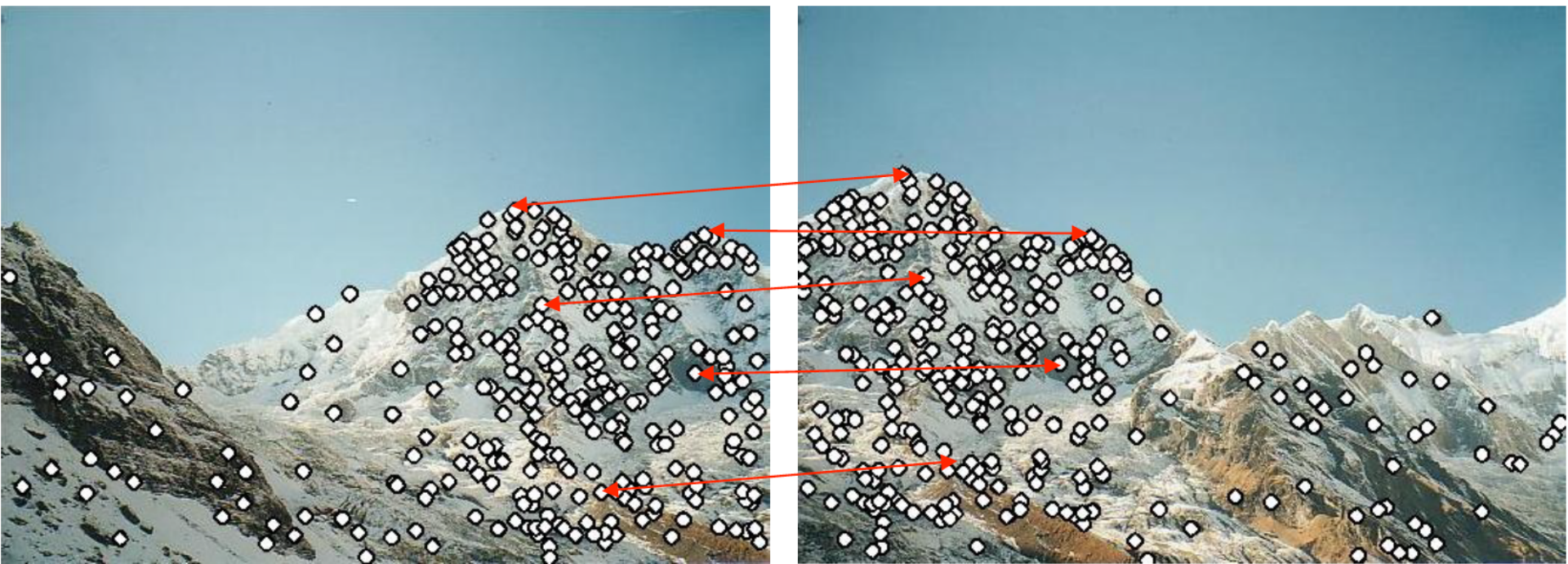


# Application: Image Stitching



1. Detect feature points in both images.

# Application: Image Stitching



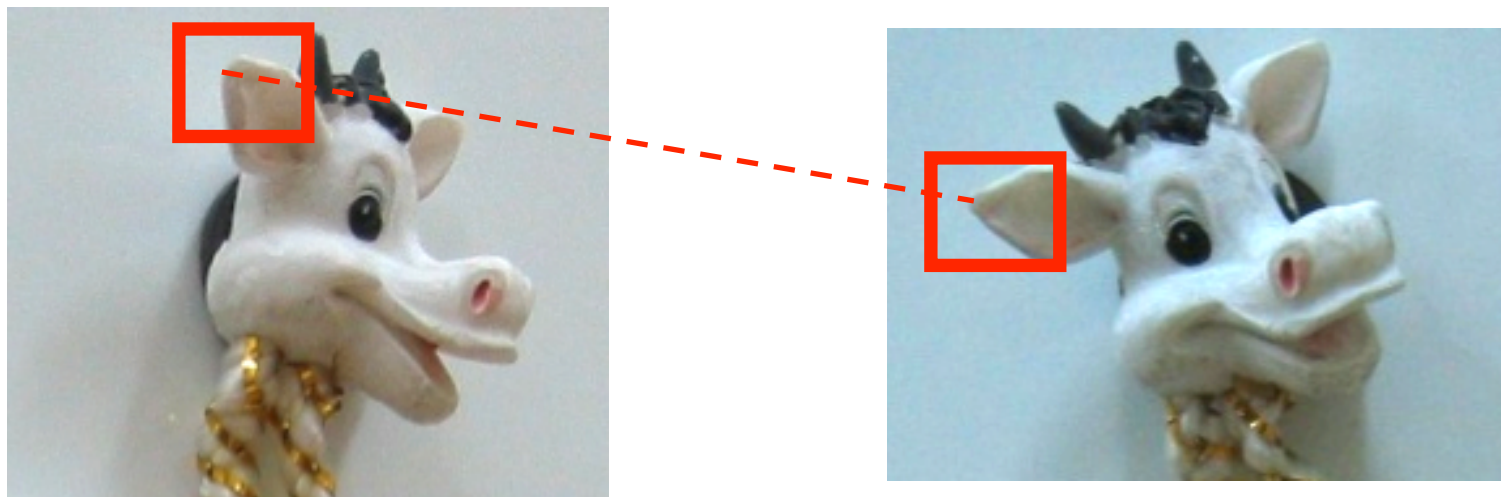
1. Detect feature points in both images.
2. Find corresponding pairs of feature points.

# Application: Image Stitching



1. Detect feature points in both images.
2. Find corresponding pairs of feature points.
3. Use the pairs to align the images.

# Application: Estimating Fundamental Matrix



1. Detect feature points in both images.
2. Find corresponding pairs of feature points.
3. Use the pairs to estimate epipolar geometry across images.



# Application: Detect Object Instances



1. Detect feature points in both images.
2. Find corresponding pairs of feature points.
3. Use the pairs to match object instances.

# Local Image Point Applications

- Image alignment (stitching, mosaics)
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval
- Robot navigation

# Advantages of local features

## Locality

- features are local, so robust to occlusion and clutter

## Distinctiveness:

- can differentiate a large database of objects

## Quantity

- hundreds or thousands in a single image

## Efficiency

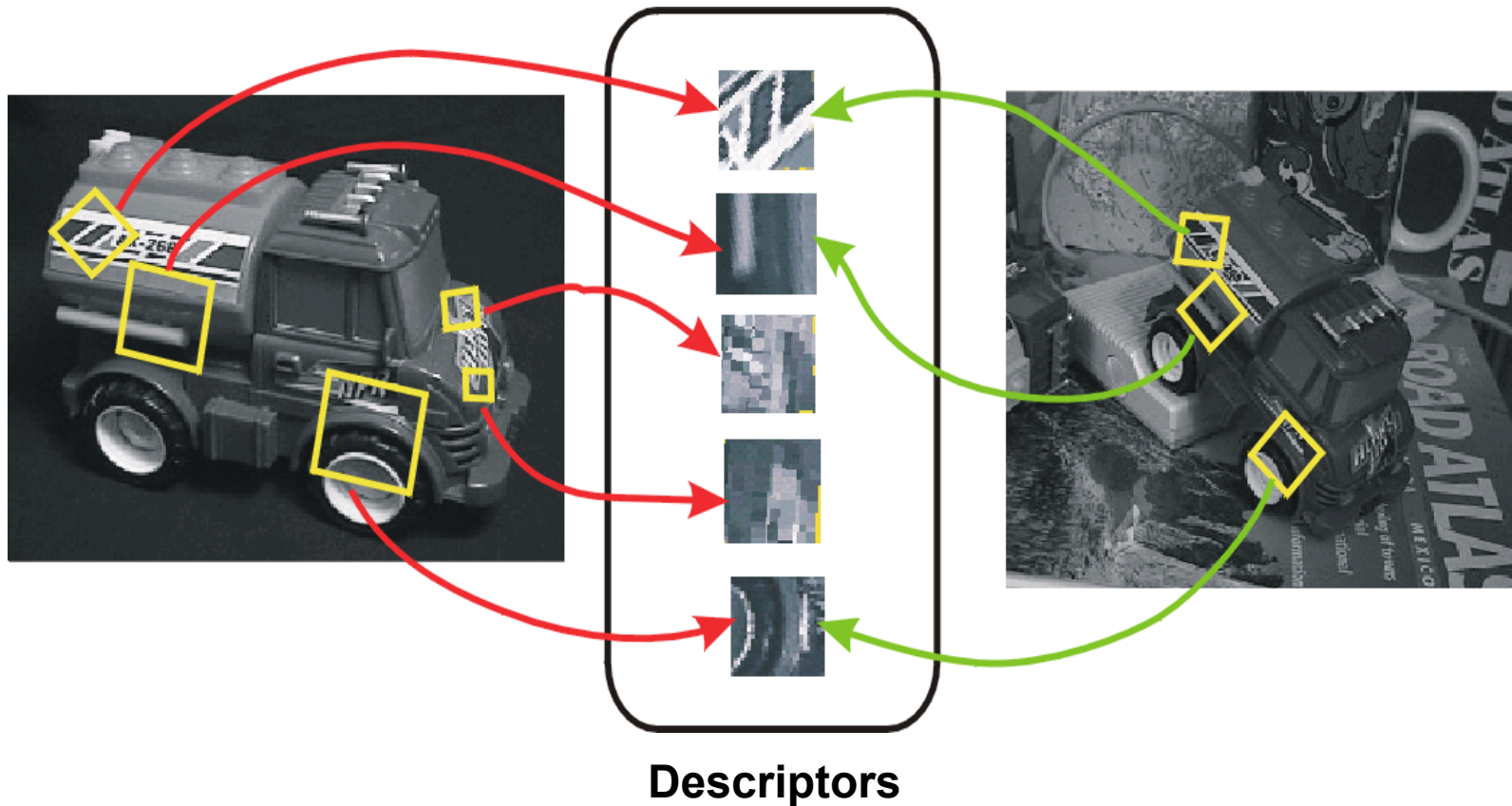
- real-time performance achievable

## Generality

- exploit different types of features in different situations

# Challenges

- Repeatability
- Uniqueness
- Invariance



# What makes a good feature?



# Repeatability



Illumination  
invariance



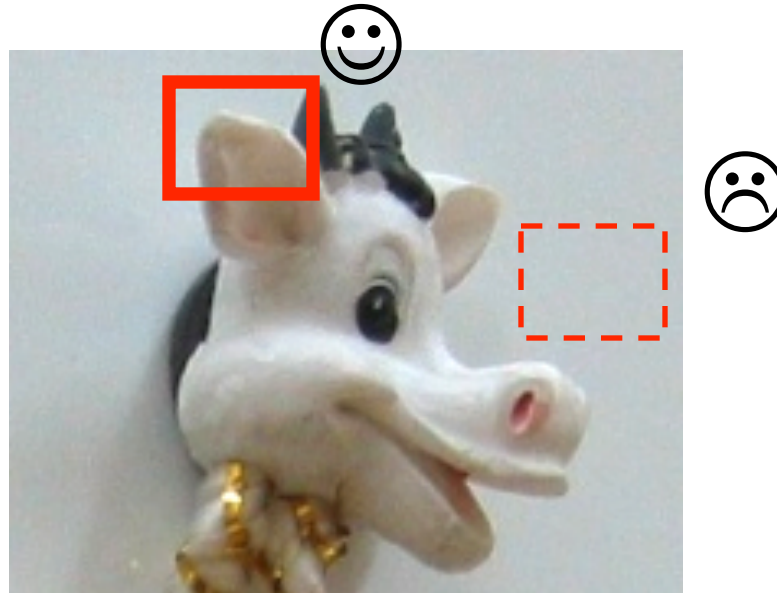
Scale  
invariance



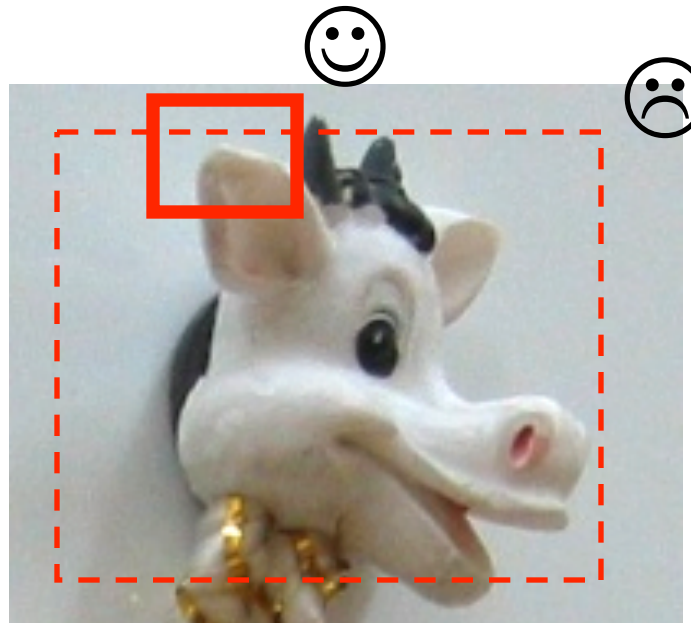
Pose invariance

- Rotation
- Affine

- Saliency



- Locality



# One criterion is uniqueness

Look for image regions that are unusual

- Lead to unambiguous matches in other images

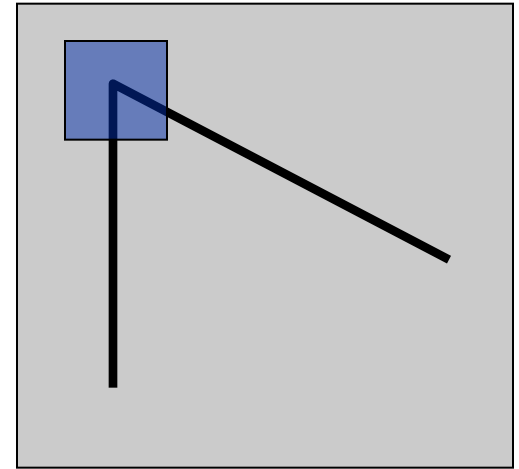
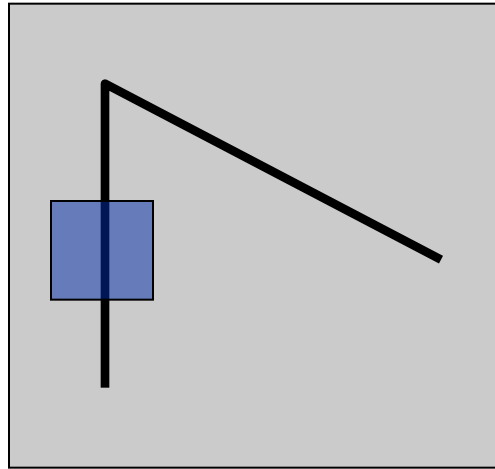
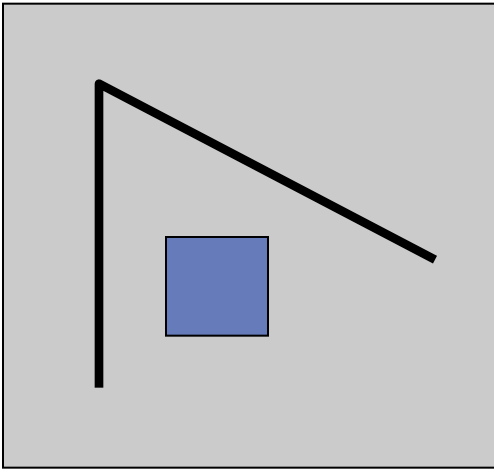
How to define “unusual”?



# Local measures of uniqueness

Suppose we only consider a small window of pixels

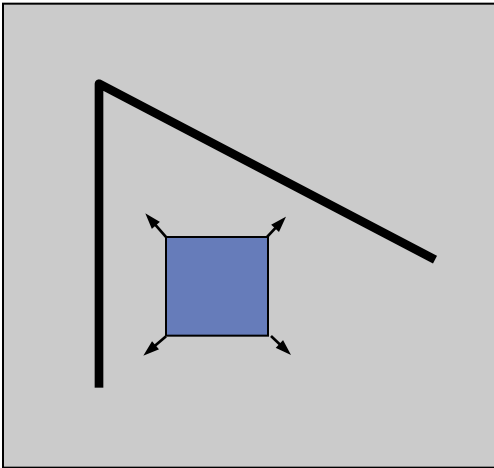
- What defines whether a feature is a good or bad candidate?



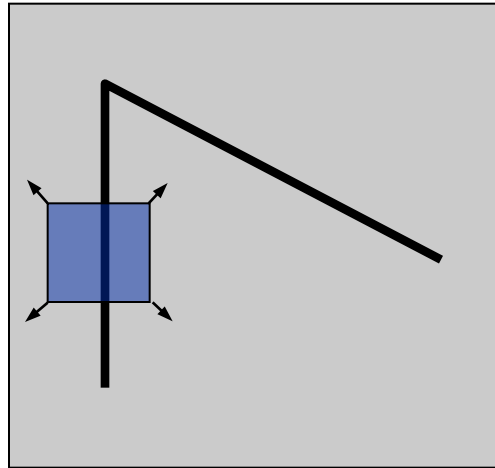
# Feature detection

Local measure of feature uniqueness

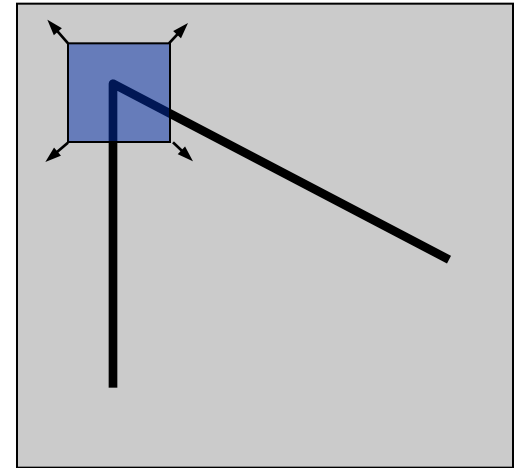
- How does the window change when you shift it?
- Shifting the window in *any direction* causes a *big change*



“flat” region:  
no change in all  
directions



“edge”:  
no change along  
the edge direction

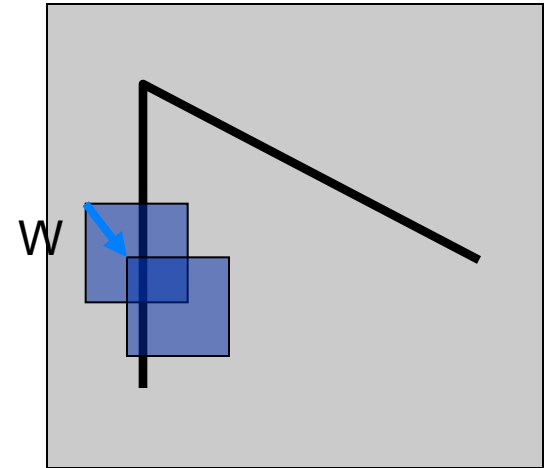


“corner”:  
significant change  
in all directions

# Feature detection: the math

Consider shifting the window  $W$  by  $(u,v)$

- how do the pixels in  $W$  change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD “error” of  $E(u,v)$ :



$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

# Small motion assumption

- Taylor Series expansion of  $I$

$$I(x + u, y + v) = I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \text{higher order terms}$$

- If the motion is small, then the first order approx. is good:

$$I(x + u, y + v) \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v$$

$$\approx I(x, y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

shorthand

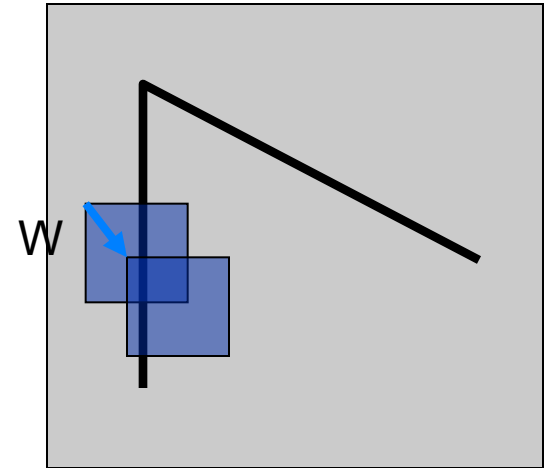
$$I_x = \frac{\partial I}{\partial x}$$

- Plug this back into the objective function.

# Feature detection: the math

Consider shifting the window  $W$  by  $(u,v)$

- how do the pixels in  $W$  change?
- compare each pixel before and after by summing up the squared differences
- this defines an “error” of  $E(u,v)$ :

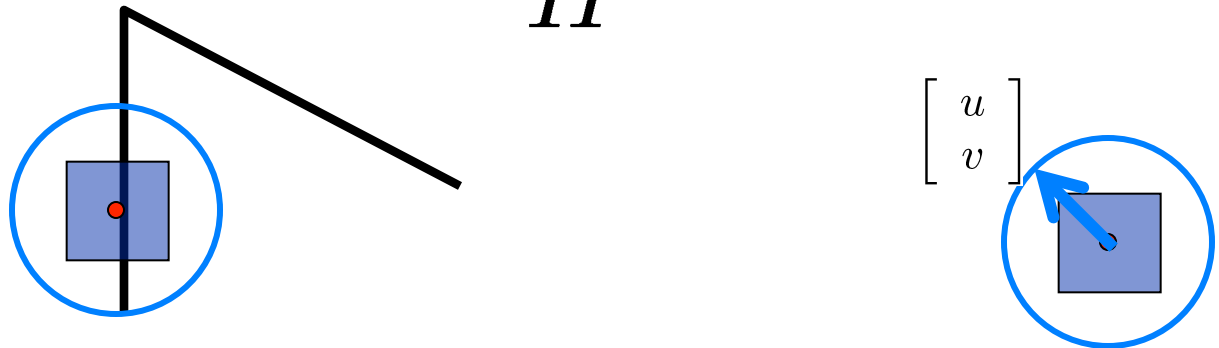


$$\begin{aligned}
 E(u, v) &= \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2 \\
 &\approx \sum_{(x,y) \in W} \left[ I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x, y) \right]^2 \\
 &\approx \sum_{(x,y) \in W} \left[ [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2
 \end{aligned}$$

# Feature detection: the math

This can be rewritten:

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$



For the example above

- You can move the center of the window to anywhere on the blue unit circle
- Which directions will result in the largest and smallest  $E$  values?
- We can find these directions by looking at the eigenvectors of  $H$

# Quick eigenvalue/eigenvector review

The **eigenvectors** of a matrix **A** are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar  $\lambda$  is the **eigenvalue** corresponding to **x**

- The eigenvalues are found by solving:

$$\det(A - \lambda I) = 0$$

- In our case, **A = H** is a 2x2 matrix, so we have

$$\det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

- The solution:

$$\lambda_{\pm} = \frac{1}{2} \left[ (h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2} \right]$$

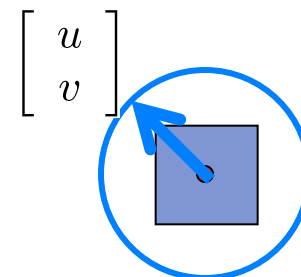
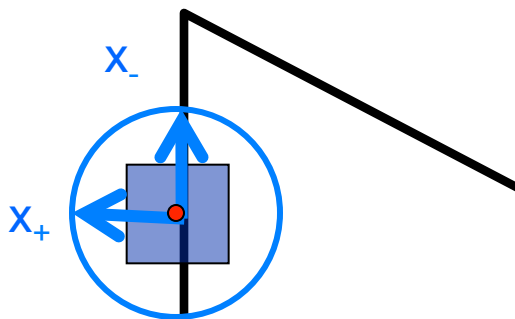
Once you know  $\lambda$ , you find **x** by solving

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

# Feature detection: the math

This can be rewritten:

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$



Eigenvalues and eigenvectors of  $H$

- Define shifts with the smallest and largest change (E value)
- $x_+$  = direction of largest increase in E.
- $\lambda_+$  = amount of increase in direction  $x_+$
- $x_-$  = direction of smallest increase in E.
- $\lambda_-$  = amount of increase in direction  $x_+$

$$H x_+ = \lambda_+ x_+$$

$$H x_- = \lambda_- x_-$$



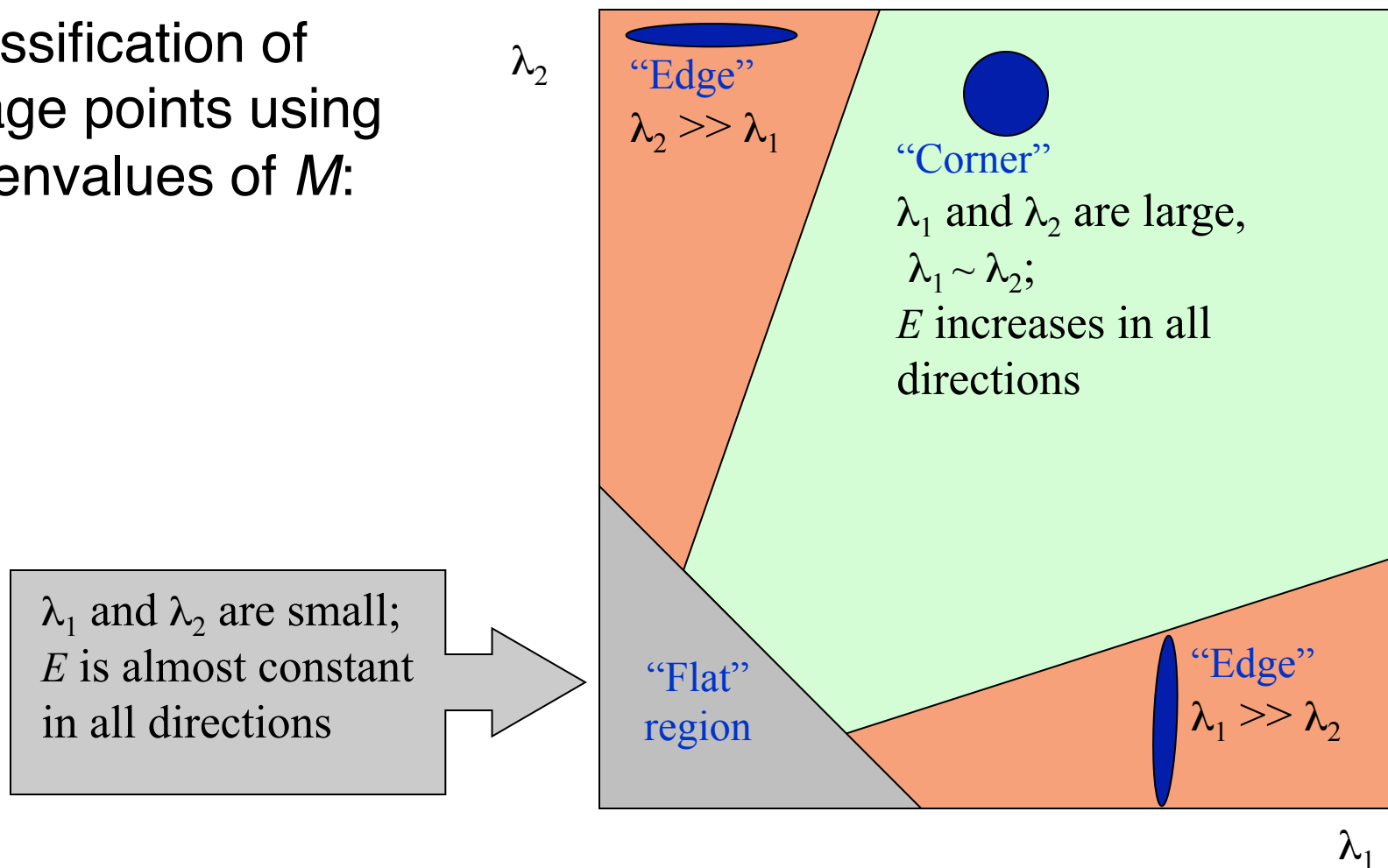
# Feature detection: the math

How are  $\lambda_+$ ,  $x_+$ ,  $\lambda_-$ , and  $x_-$  relevant for feature detection?

- What's our feature scoring function?

# Feature detection: the math

Classification of image points using eigenvalues of  $M$ :



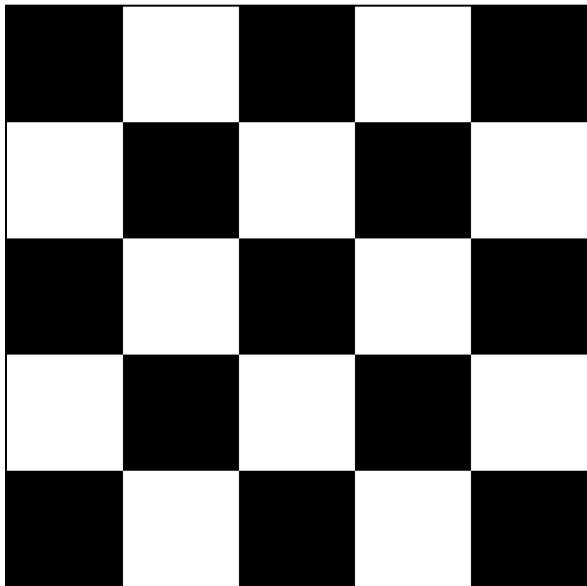
# Feature detection: the math

How are  $\lambda_+$ ,  $x_+$ ,  $\lambda_-$ , and  $x_-$  relevant for feature detection?

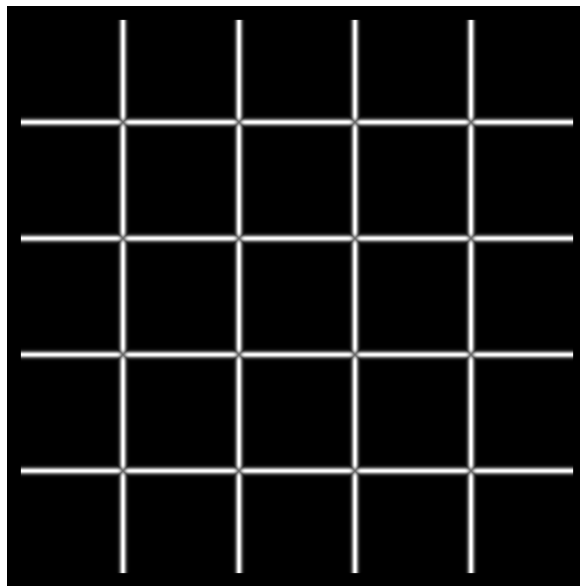
- What's our feature scoring function?

Want  $E(u,v)$  to be *large* for small shifts in *all* directions

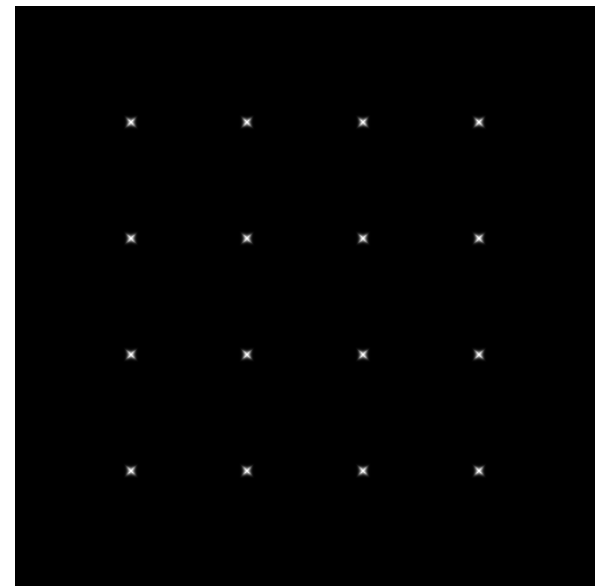
- the *minimum* of  $E(u,v)$  should be large, over all unit vectors  $[u \ v]$
- this minimum is given by the smaller eigenvalue ( $\lambda_-$ ) of  $H$



$I$



$\lambda_+$

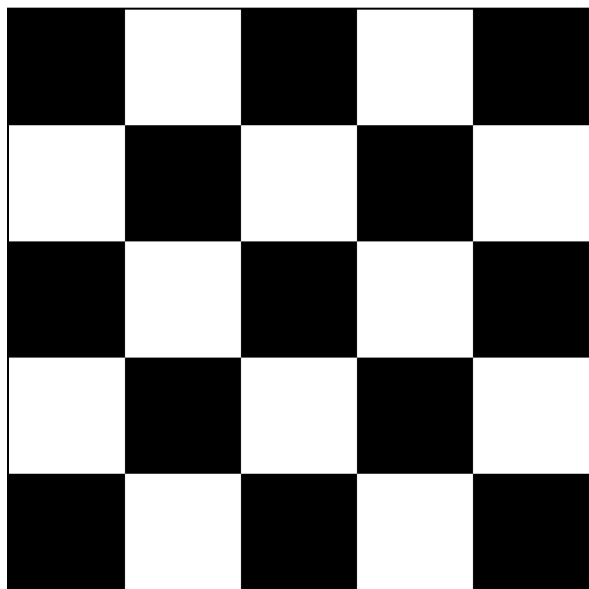


$\lambda_-$

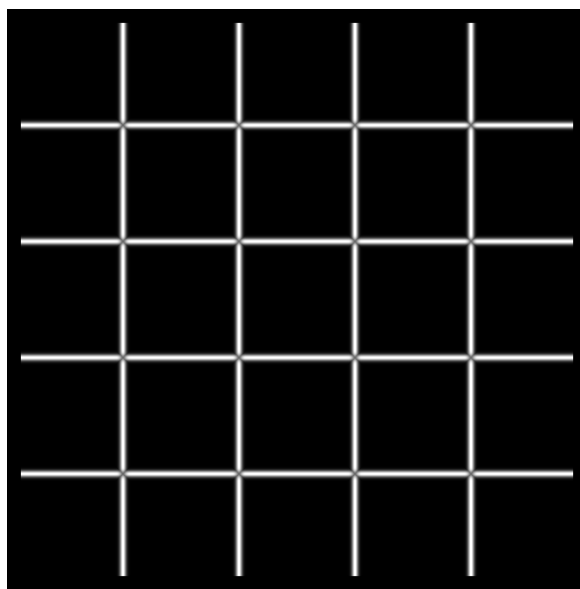
# Feature detection summary

Here's what you do

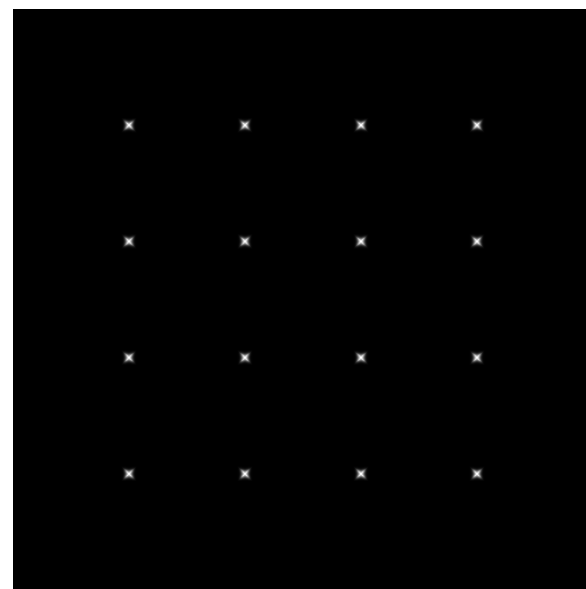
- Compute the gradient at each point in the image
- Create the  $H$  matrix from the entries in the gradient
- Compute the eigenvalues.
- Find points with large response ( $\lambda_- > \text{threshold}$ )
- Choose those points where  $\lambda_-$  is a local maximum as features



$I$



$\lambda_+$

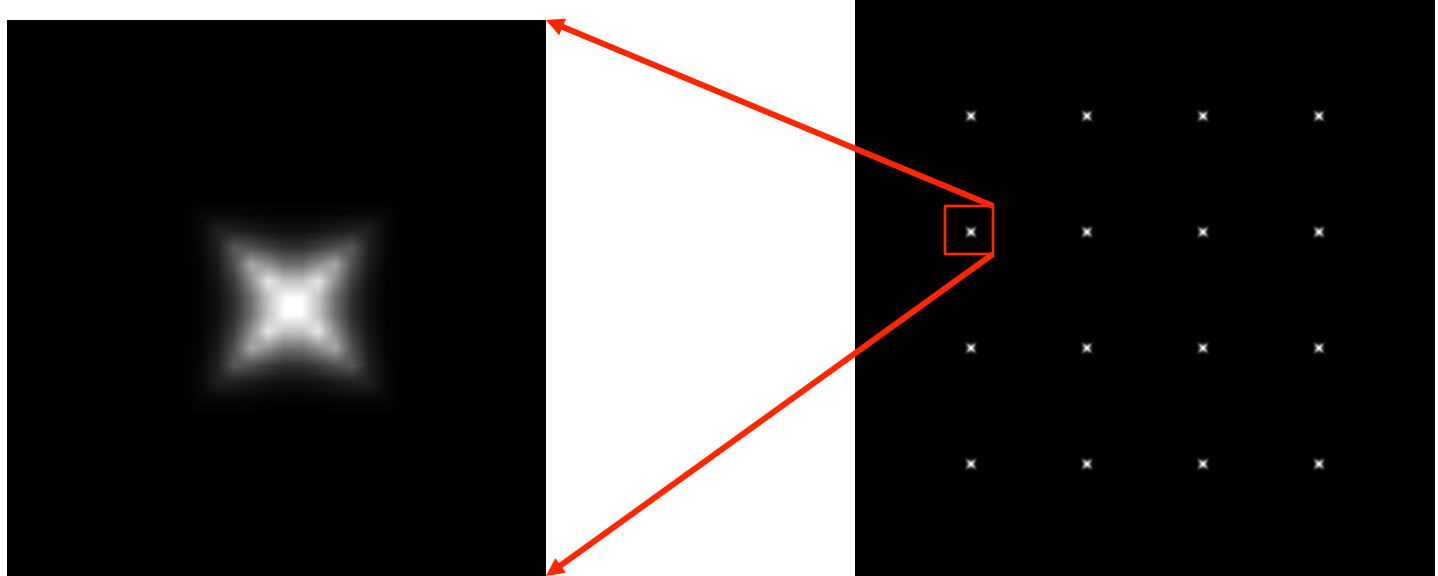


$\lambda_-$

# Feature detection summary

Here's what you do

- Compute the gradient at each point in the image
- Create the  $H$  matrix from the entries in the gradient
- Compute the eigenvalues.
- Find points with large response ( $\lambda_- > \text{threshold}$ )
- Choose those points where  $\lambda_-$  is a local maximum as features



$\lambda_-$

# The Harris operator

$\lambda_+$  is a variant of the “Harris operator” for feature detection

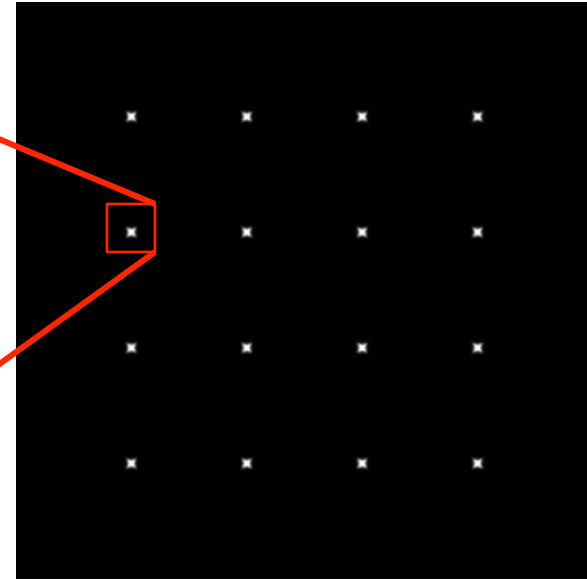
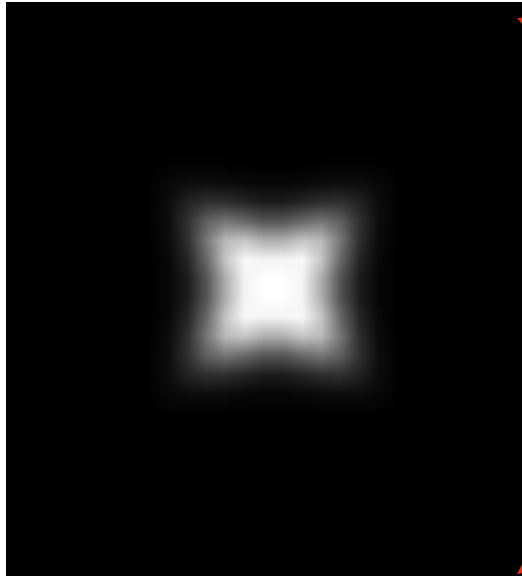
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

$$= \frac{\mathit{determinant}(H)}{\mathit{trace}(H)}$$

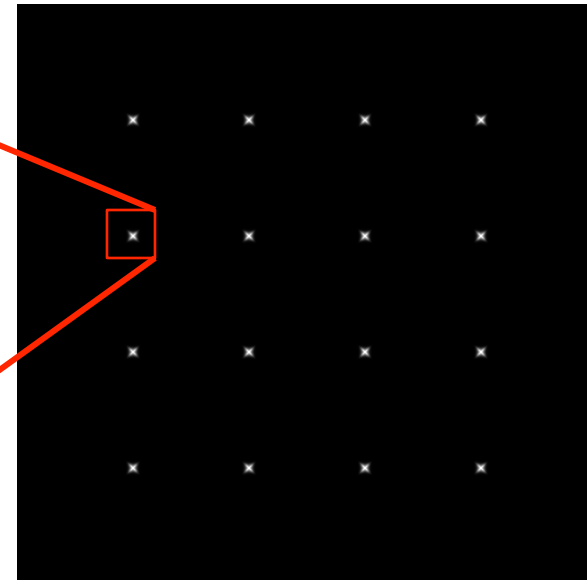
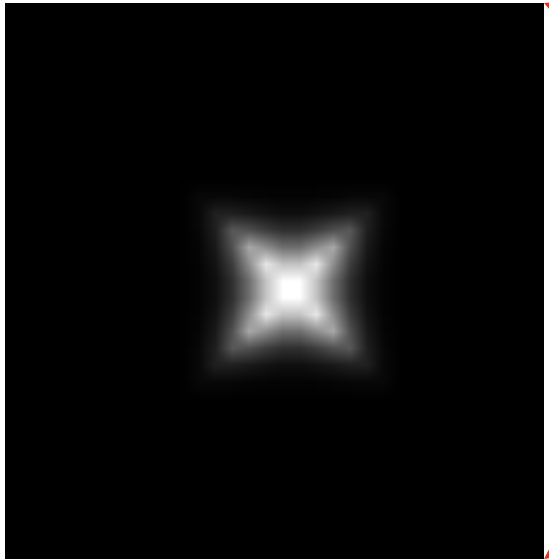
- The *trace* is the sum of the diagonals, i.e.,  $\mathit{trace}(H) = h_{11} + h_{22}$
- Very similar to  $\lambda_+$  but less expensive (no square root)
- Called the “Harris Corner Detector” or “Harris Operator”
- Lots of other detectors, this is one of the most popular

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)  
*Proceedings of the 4th Alvey Vision Conference*: pages 147--151. 1988.

# The Harris operator



Harris  
operator



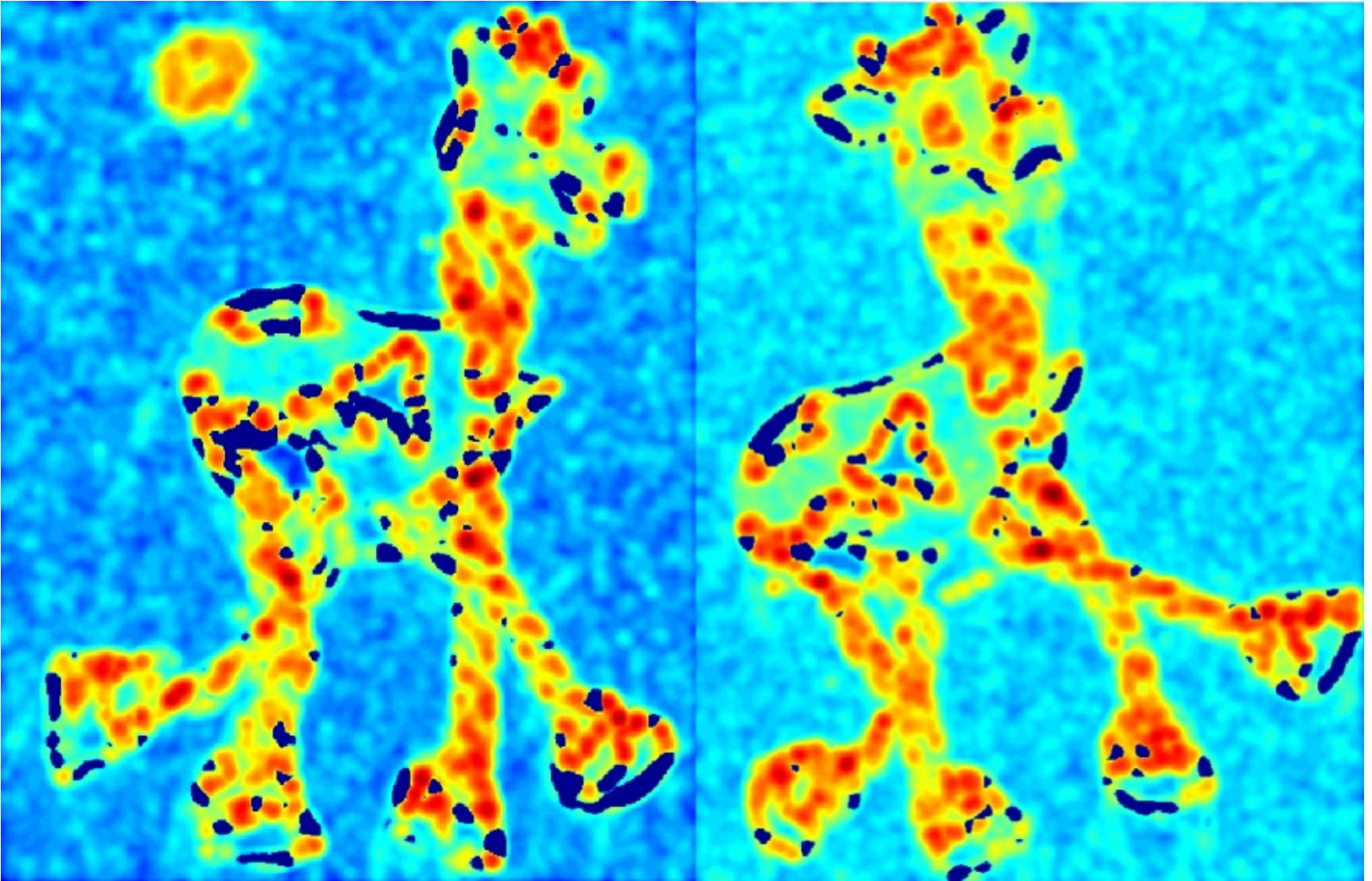
$\lambda_-$

# Harris detector example

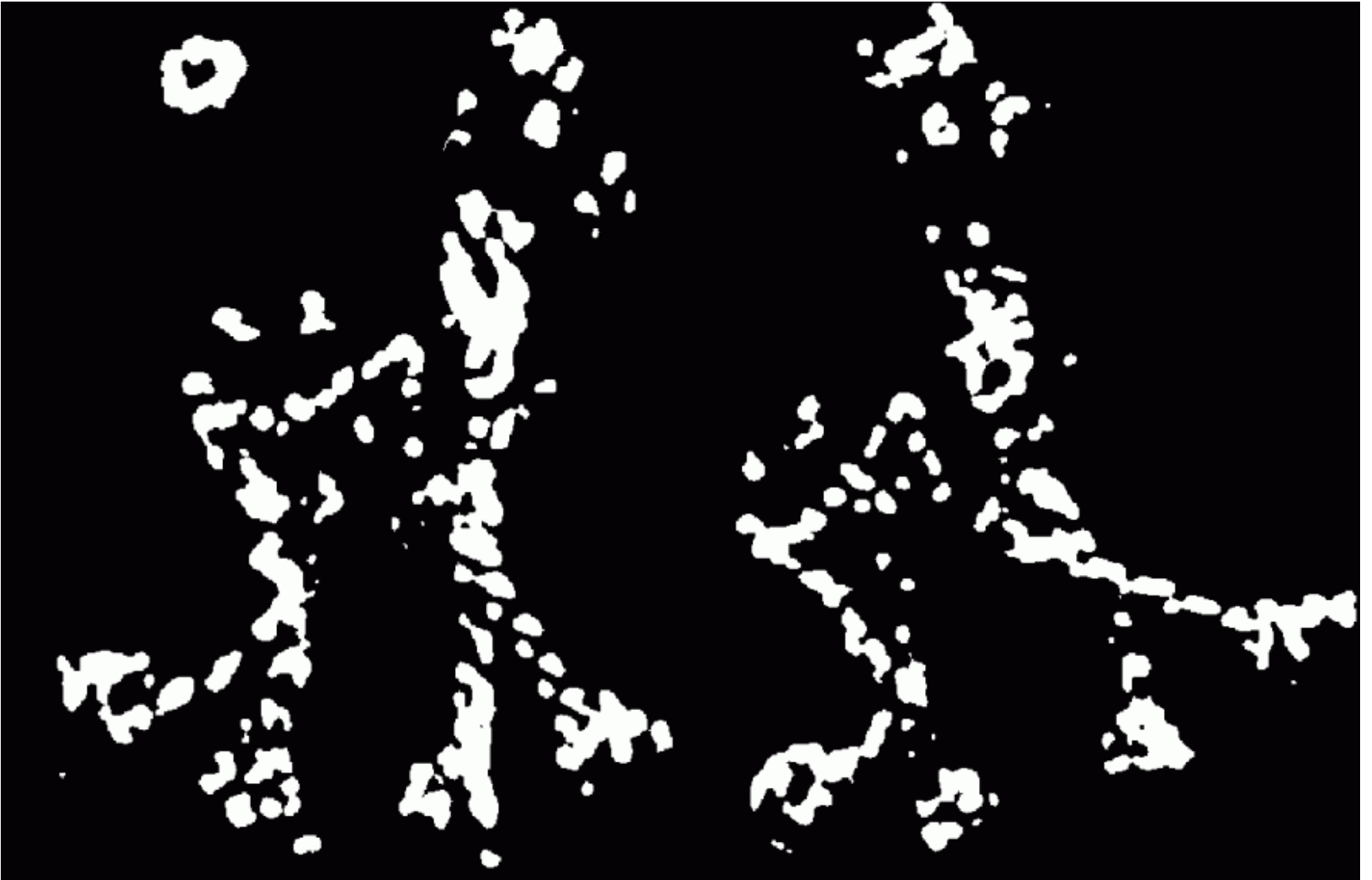




f value (red high, blue low)



# Threshold ( $f > \text{value}$ )



# Find local maxima of $f$



# Harris features (in red)



# Towards Invariance

Suppose you **rotate** the image by some angle

- Will you still pick up the same features?

What if you change the brightness?

Scale?

**Invariance** defined:

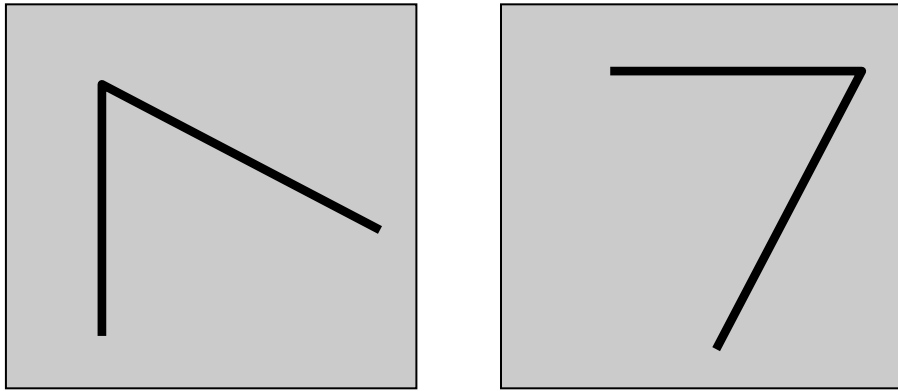
Suppose we are comparing two images  $I$  and  $J$ .

$J$  may be a transformed version of  $I$

We want to detect the same features from  $I$  and  $J$  regardless of the transformation: this is **transformational invariance**.

# Harris Detector: Some Properties

- Is the Harris detector rotationally invariant?



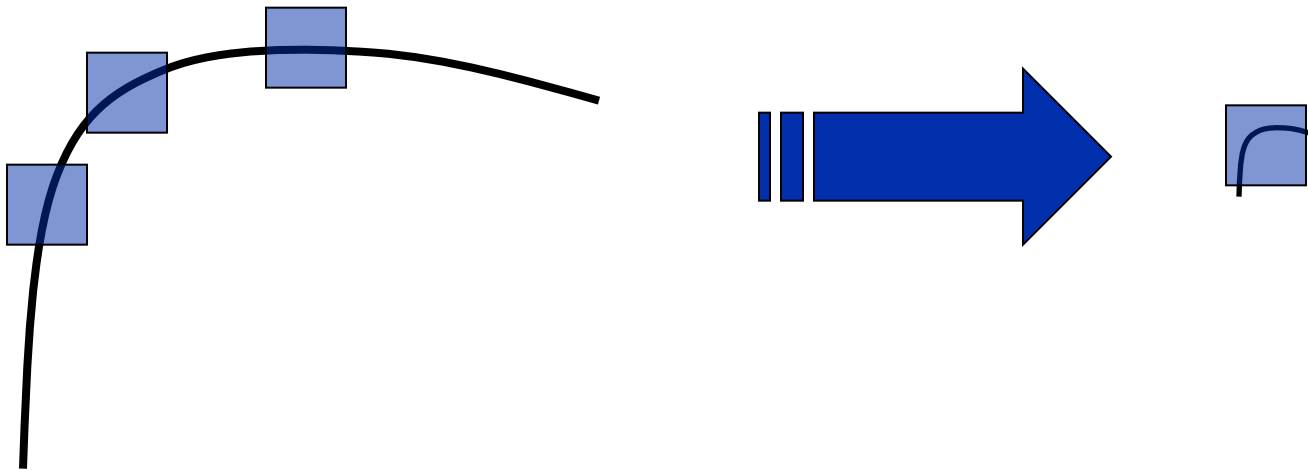
**Corner response  $R$  is invariant to image rotation**

$$H = U^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} U \rightarrow f(\lambda_1, \lambda_2) \quad \text{doesn't change!}$$

# Harris Detector: Some Properties

- Is it scale invariant?

*Corner response  $R$  is not scale invariant!*



All points will be  
classified as **edges**

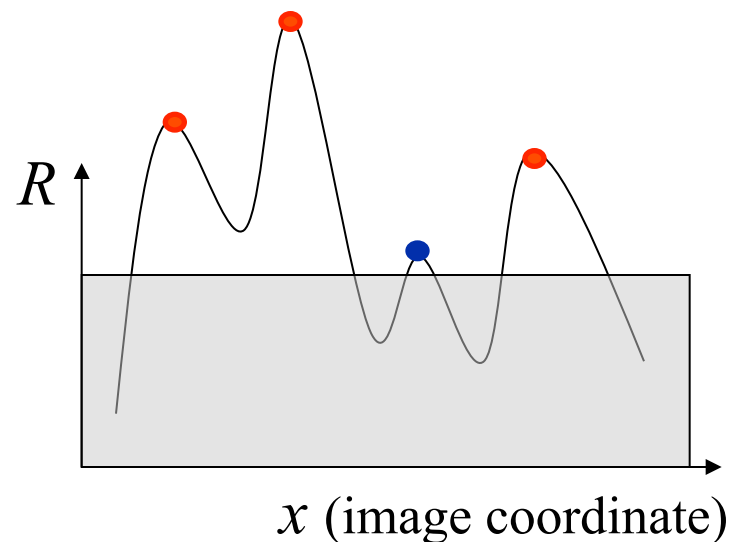
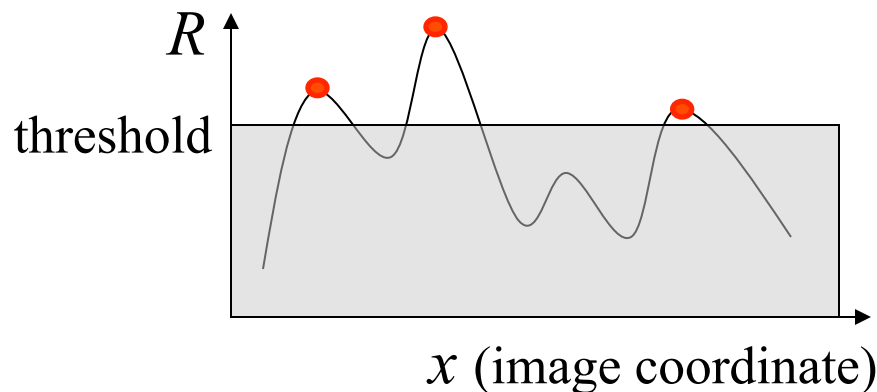
**Corner !**

# Harris Detector: Some Properties

- Partial invariance to *affine intensity* changes

$$I \rightarrow s I + b$$

- invariance to intensity shift  $I \rightarrow I + b$  (*why?*)  
(only derivatives are used)
- Not invariant to intensity scale:  $I \rightarrow a I$



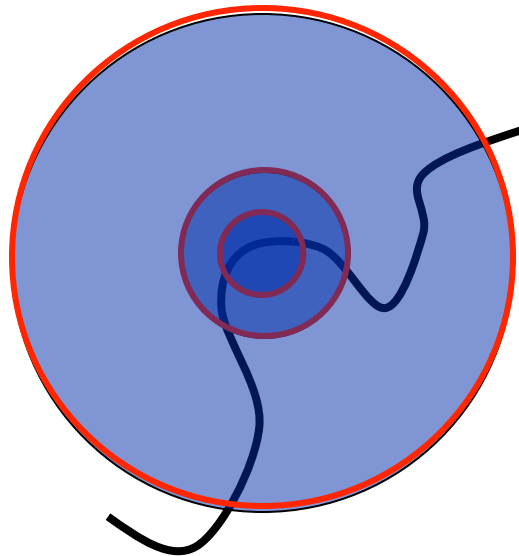


# Invariance

Detector	Illumination	Rotation	Scale	View point
Harris corner	partial	Yes	No	No

# Scale invariant detection

Suppose you're looking for corners

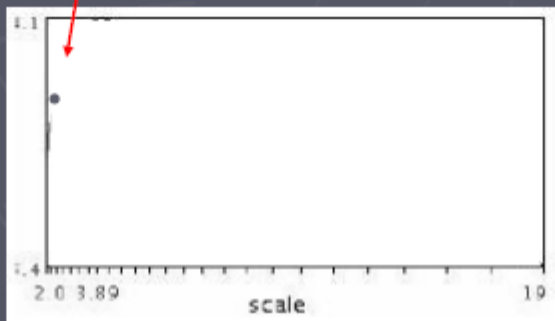


Key idea: find scale that gives local maximum of  $f$

- $f$  is a local maximum in both position and scale
- Common definition of  $f$ : Laplacian  
(or difference between two Gaussian filtered images with different sigmas)

# Automatic scale selection

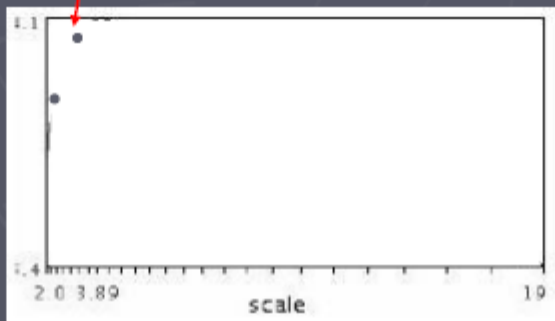
Lindeberg et al., 1996



$$f(I_{i_1 \dots i_m}(x, \sigma))$$

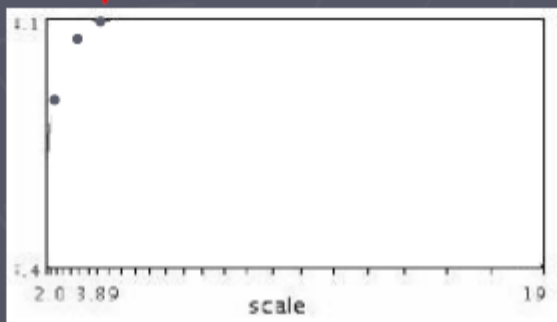
Slide from Tinne Tuytelaars

# Automatic scale selection



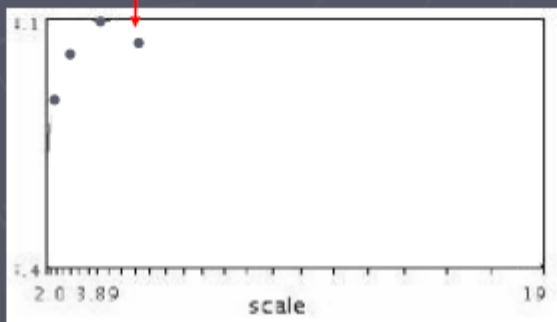
$$f(I_{i_1..i_m}(x, \sigma))$$

# Automatic scale selection



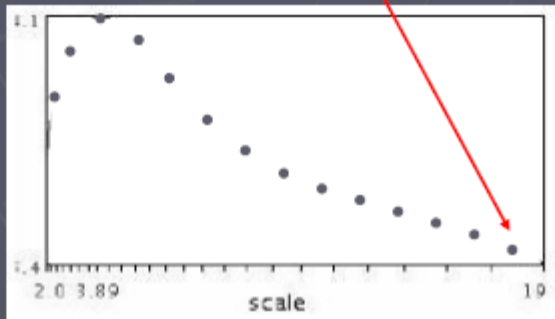
$$f(I_{i_1..i_m}(x, \sigma))$$

# Automatic scale selection



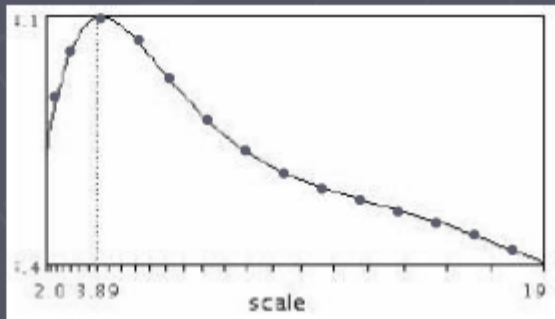
$$f(I_{i_1..i_m}(x, \sigma))$$

# Automatic scale selection



$$f(I_{i_1 \dots i_m}(x, \sigma))$$

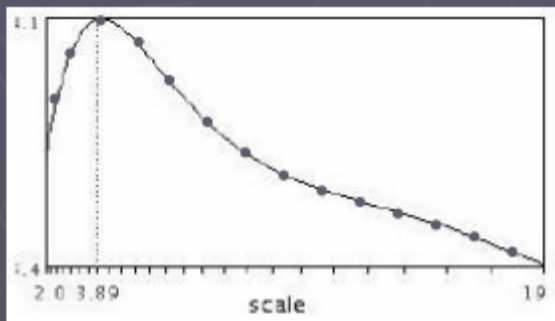
# Automatic scale selection



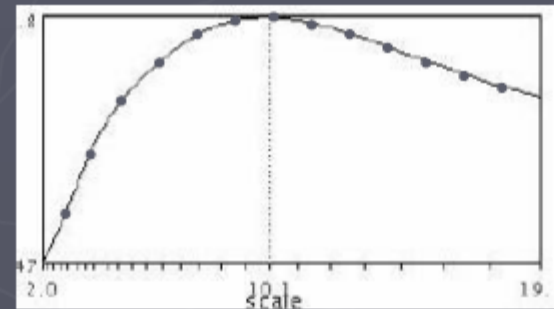
$$f(I_{i_1 \dots i_m}(x, \sigma))$$



# Automatic scale selection



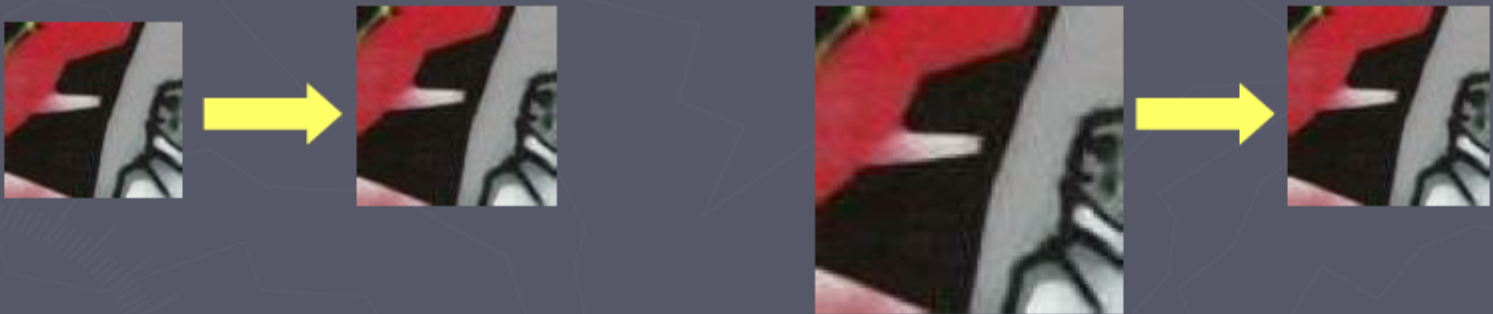
$$f(I_{i_1 \dots i_m}(x, \sigma))$$



$$f(I_{i_1 \dots i_m}(x', \sigma'))$$

# Automatic scale selection

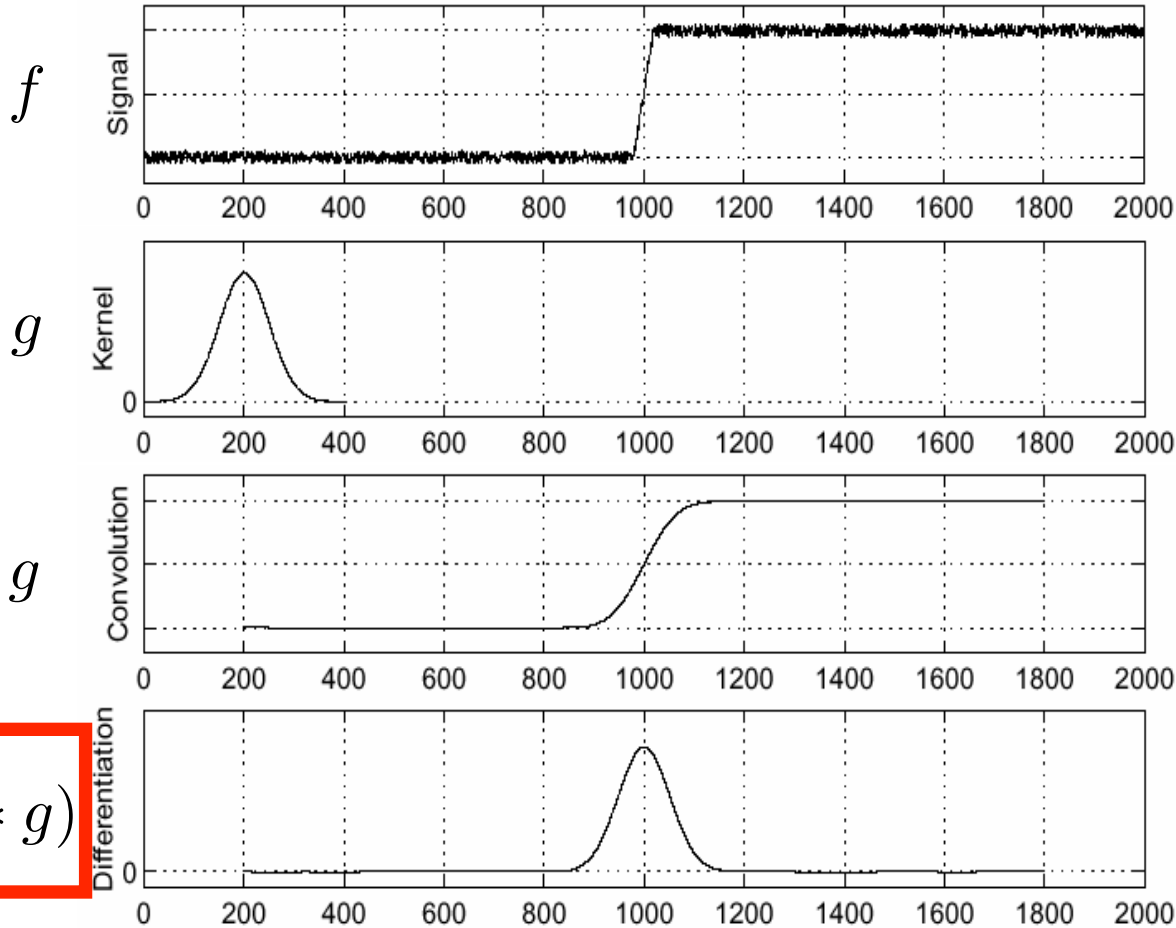
Normalize: rescale to fixed size



# Scale-Invariant Feature Detection Example

- Recall: Edges...

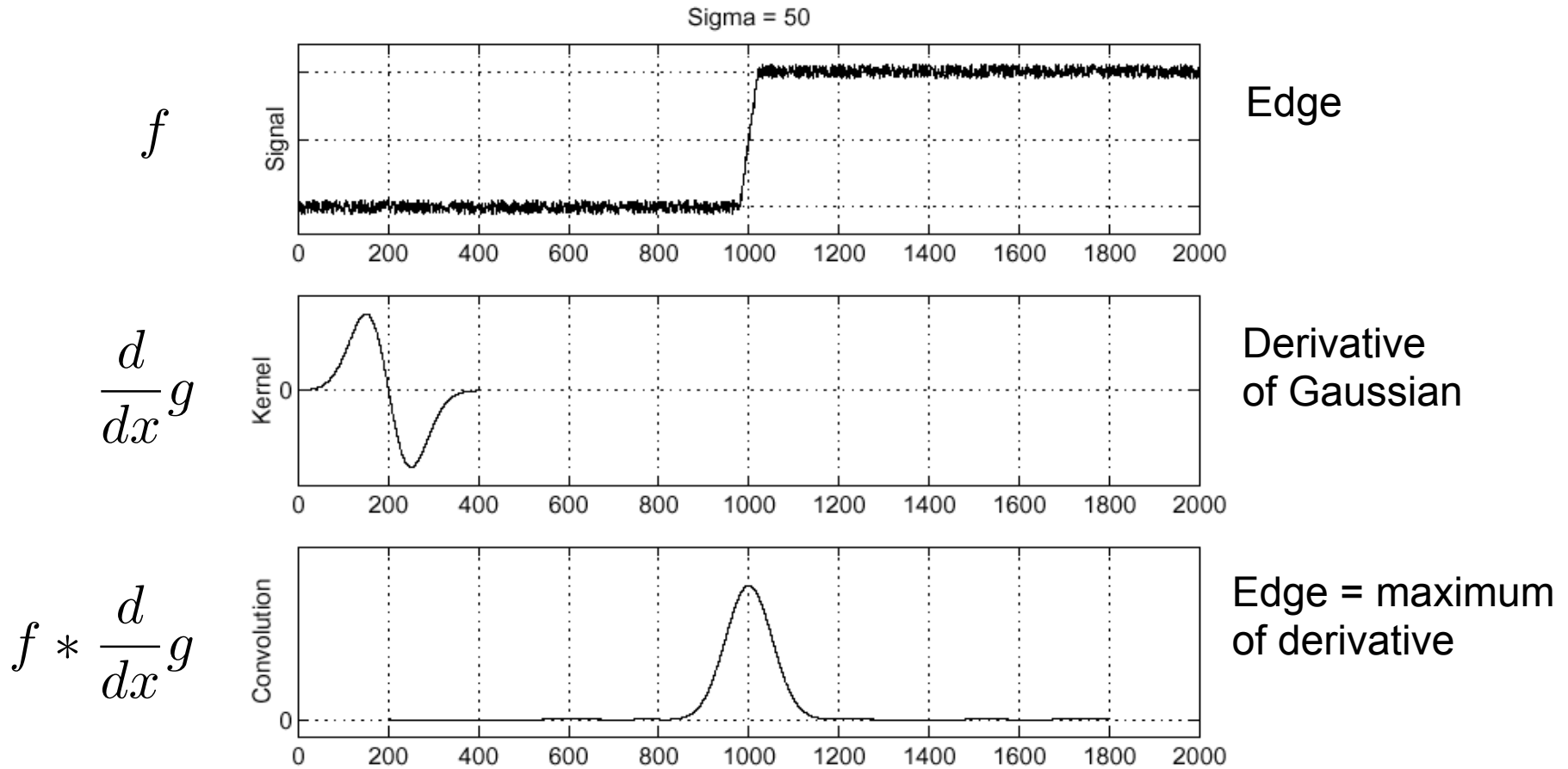
Sigma = 50



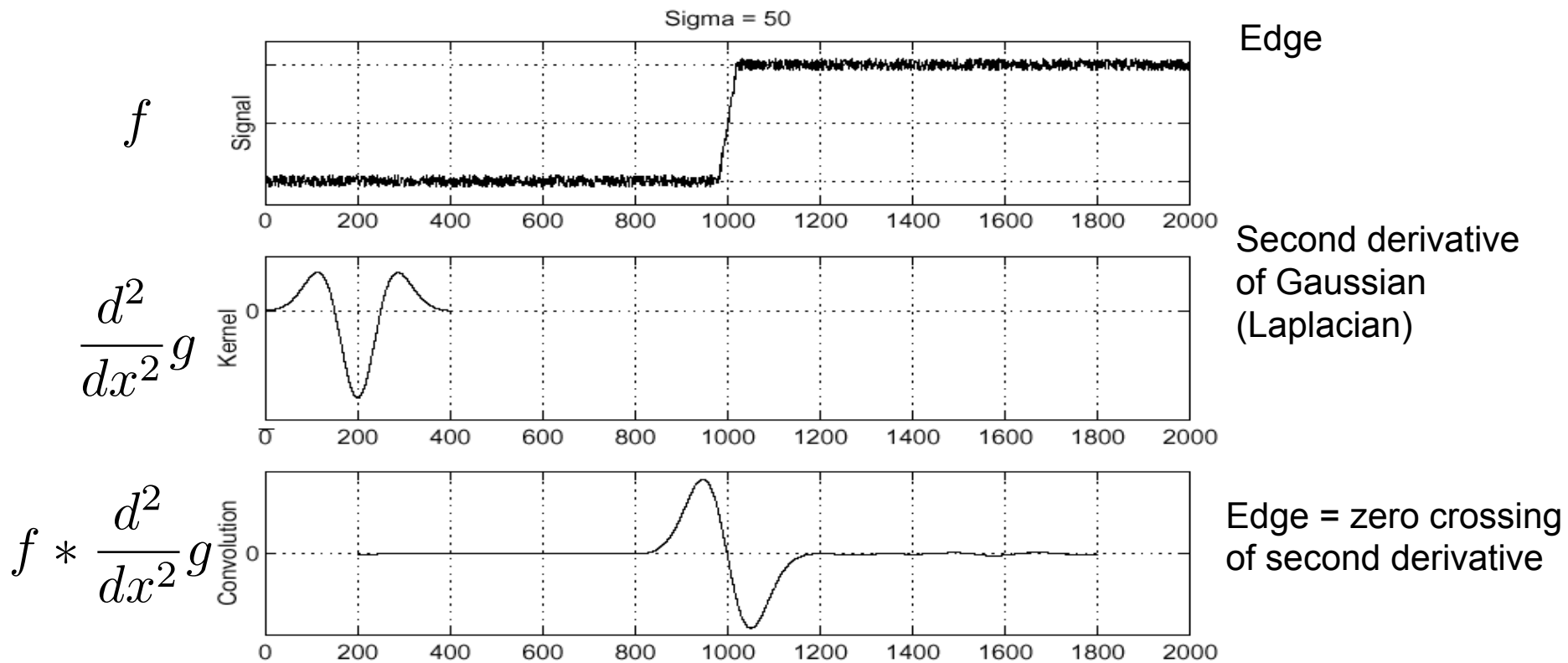
$$\frac{d}{dx}(f * g)$$

$$= \frac{dg}{dx} * f \quad \text{DoG filter}$$

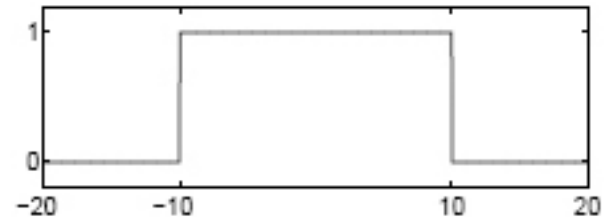
# Edge detection



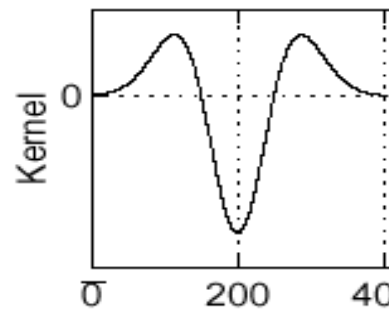
# Edge detection as zero crossing



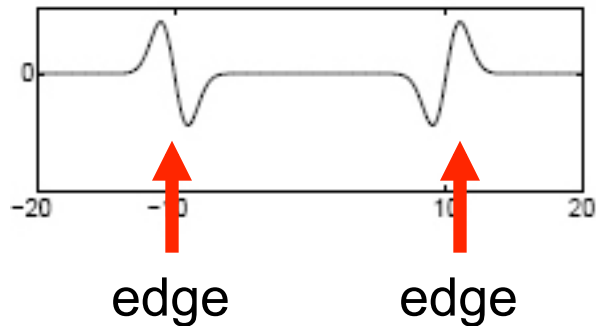
# Edge detection as zero crossing



\*

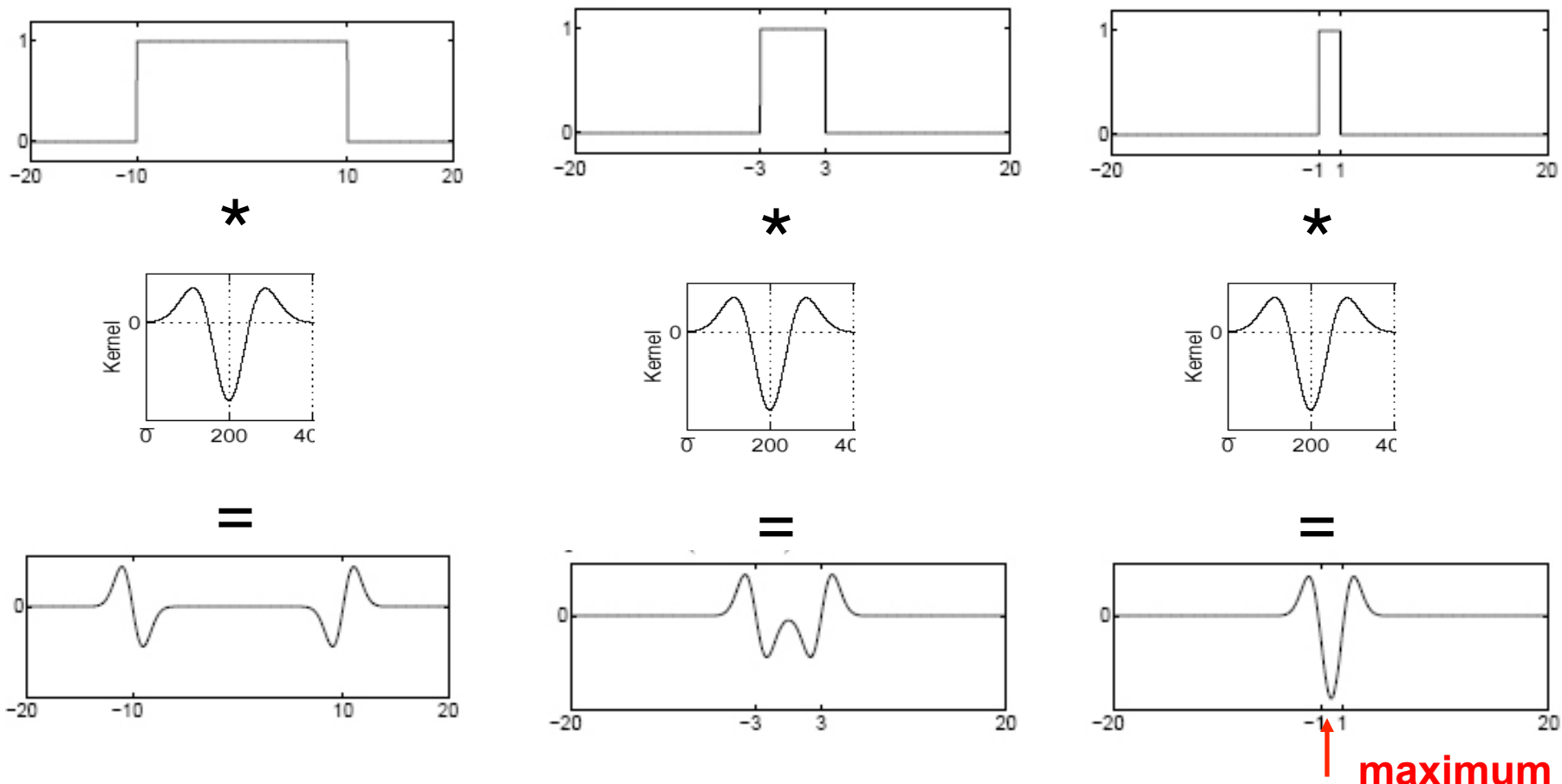


=



# From edges to blobs

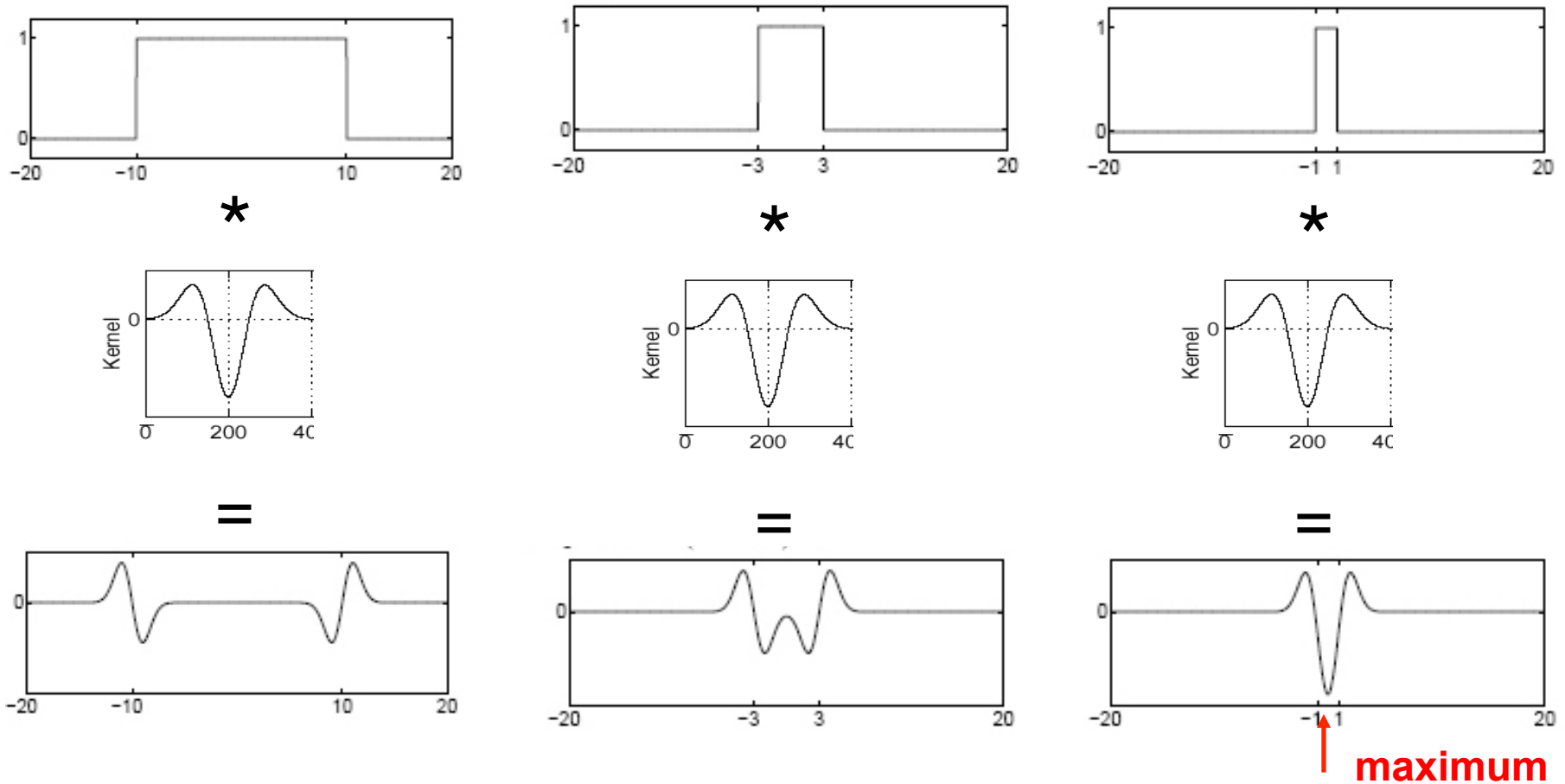
- Blob = superposition of nearby edges



Magnitude of the Laplacian response achieves a maximum at the center of the blob, provided the scale of the Laplacian is “matched” to the scale of the blob

# From edges to blobs

- Blob = superposition of nearby edges

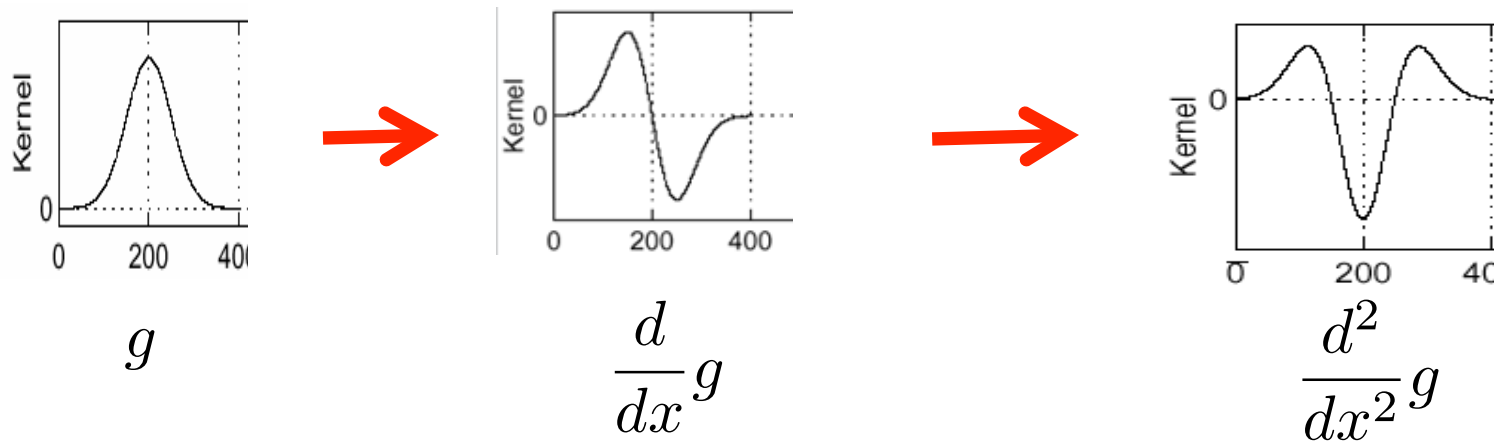
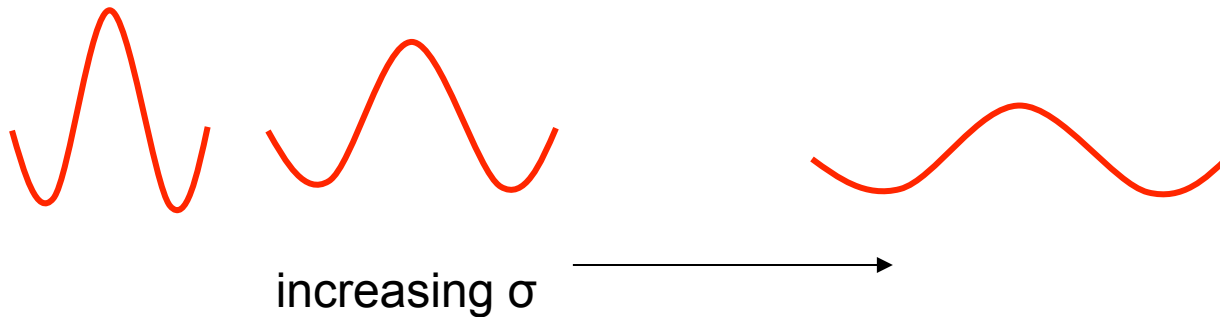


**What if the blob is slightly thicker or slimmer?**



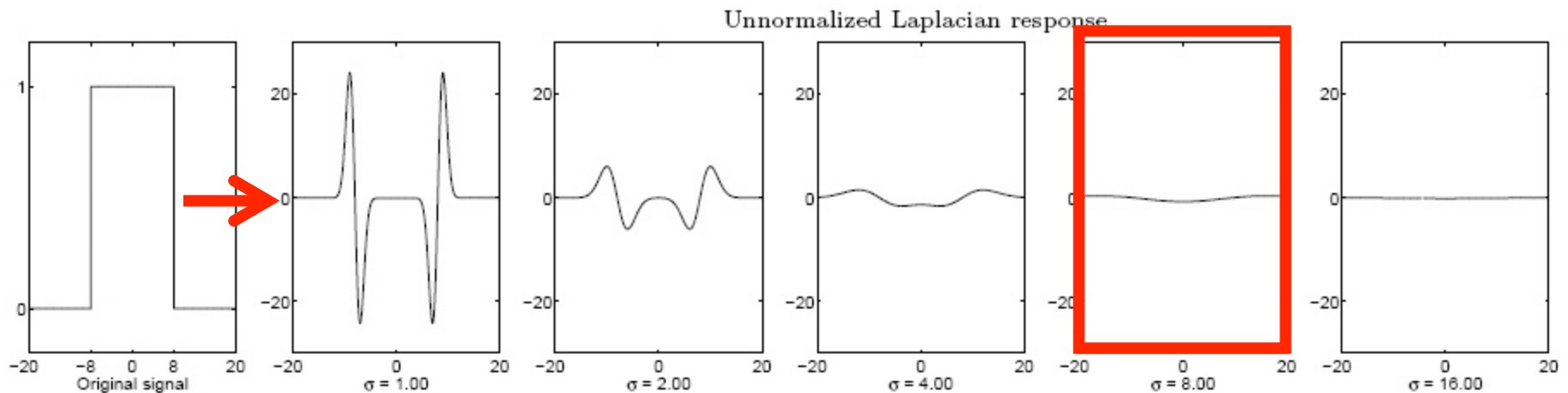
# Scale selection

- We want to find the **characteristic scale** of the blob by convolving it with Laplacians at several scales and looking for the maximum response



# Scale selection

- We want to find the **characteristic scale** of the blob by convolving it with Laplacians at several scales and looking for the maximum response
- However, Laplacian response decays as scale increases:



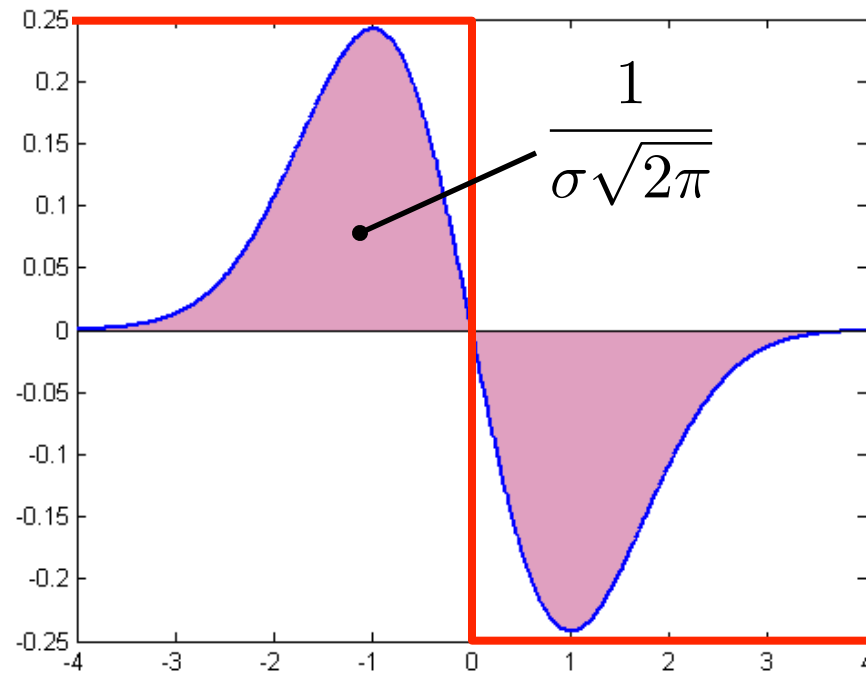
original signal  
(radius=8)

This should  
give the max  
response ☹️



# Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as  $\sigma$  increases

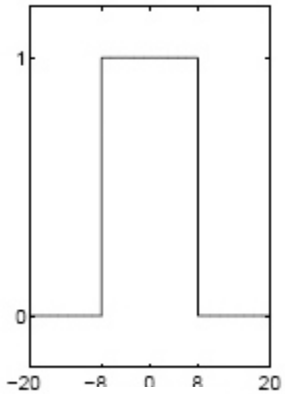


# Scale normalization

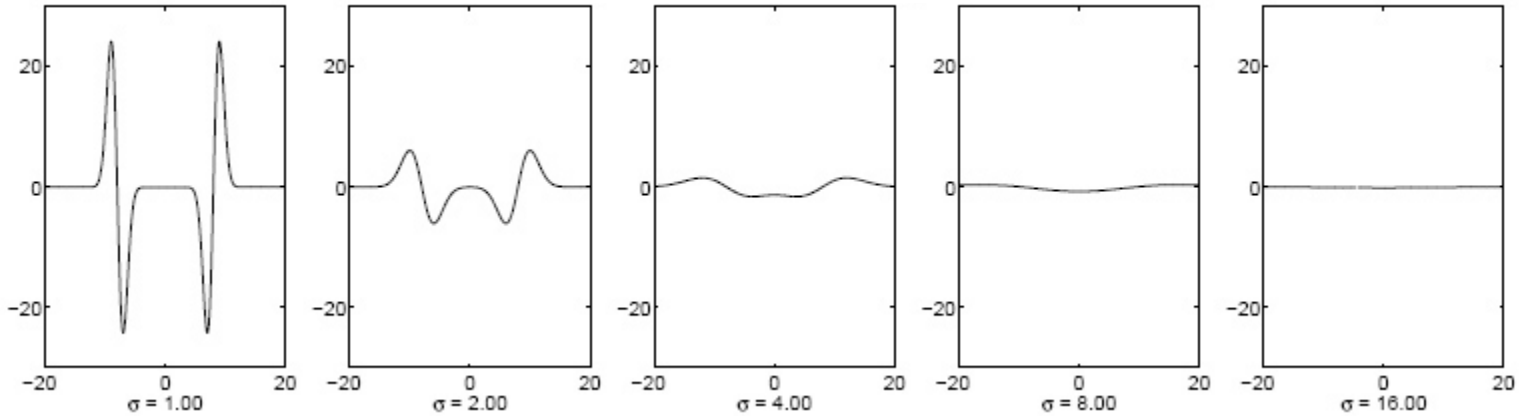
- To keep response the same (scale-invariant), must multiply Gaussian derivative by  $\sigma$
- Laplacian is the second Gaussian derivative, so it must be multiplied by  $\sigma^2$

# Effect of scale normalization

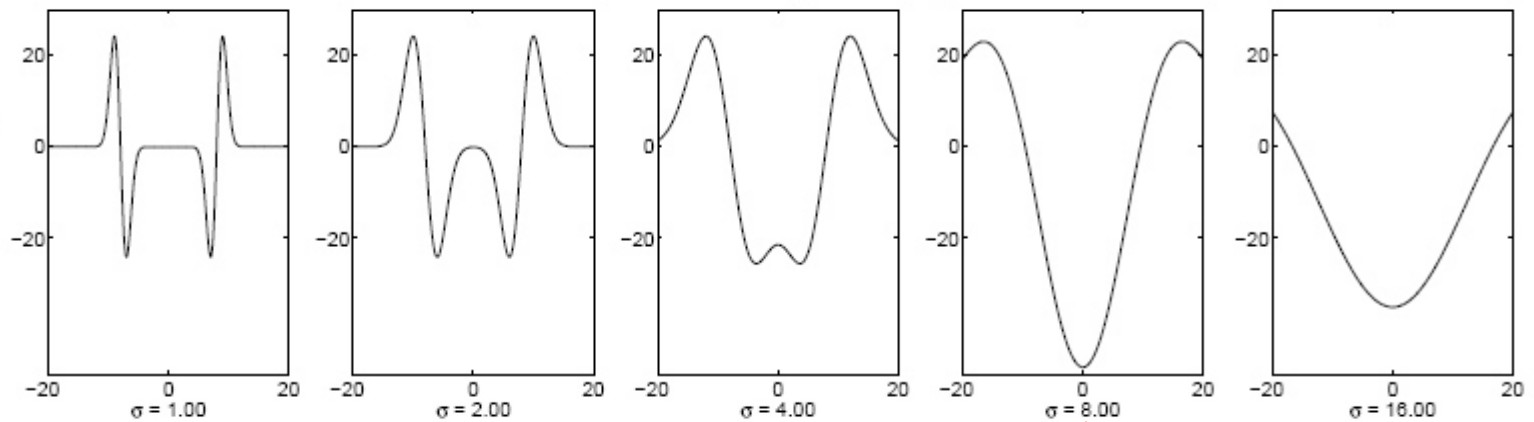
Original signal



Unnormalized Laplacian response



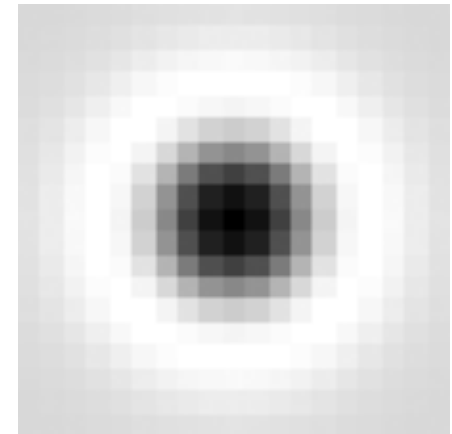
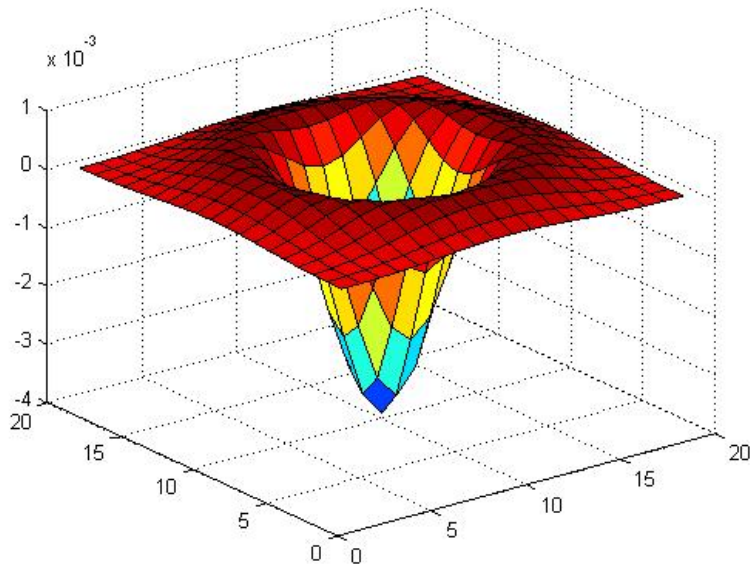
Scale-normalized Laplacian response



↑  
**Maximum 😊**

# Blob detection in 2D

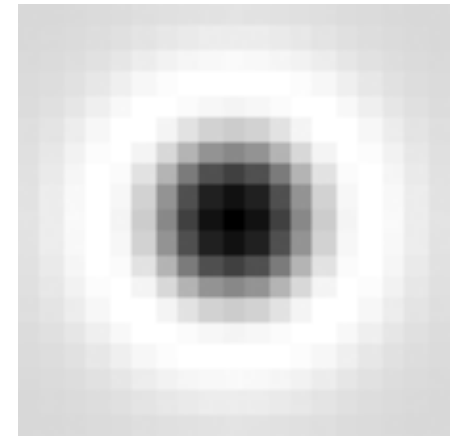
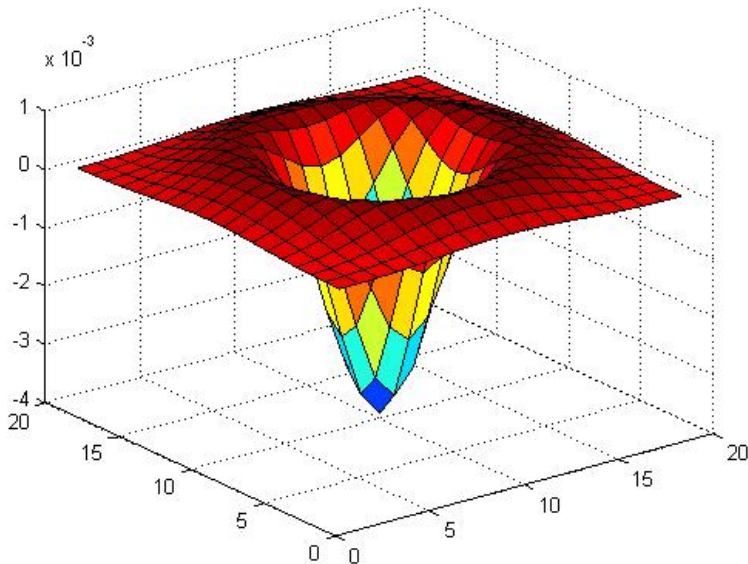
- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D



$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

# Blob detection in 2D

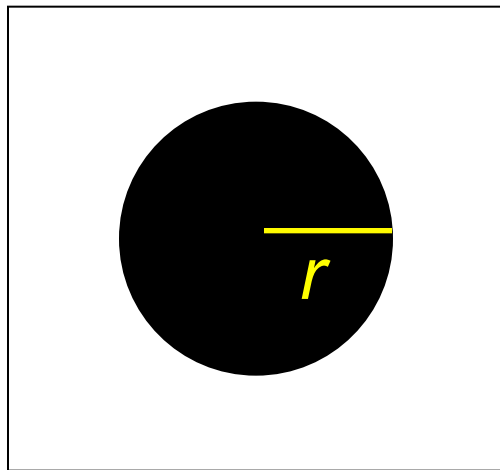
- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D



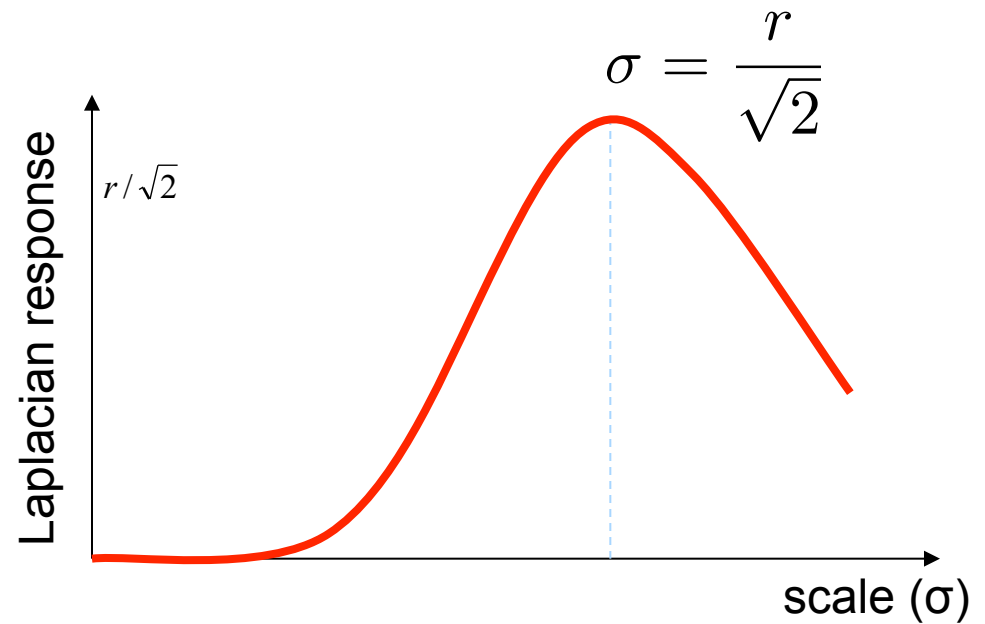
Scale-normalized: 
$$\nabla_{\text{norm}}^2 g = \sigma^2 \left( \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

# Scale selection

- For a binary circle of radius  $r$ , the Laplacian achieves a maximum at



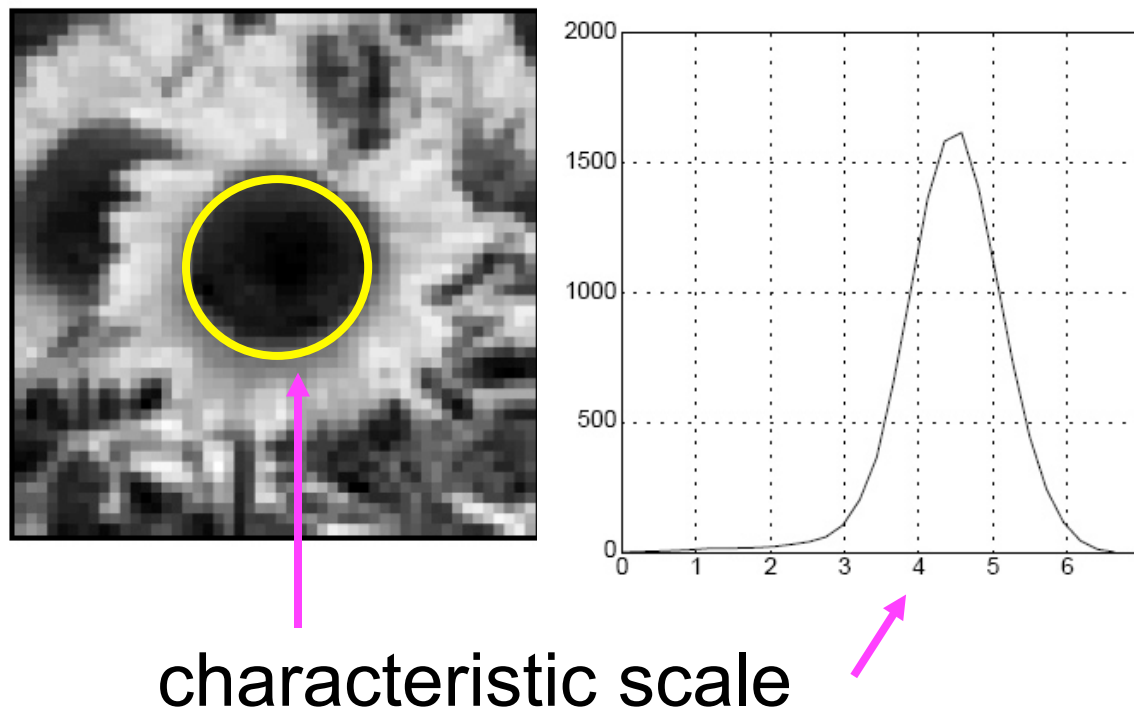
image





# Characteristic scale

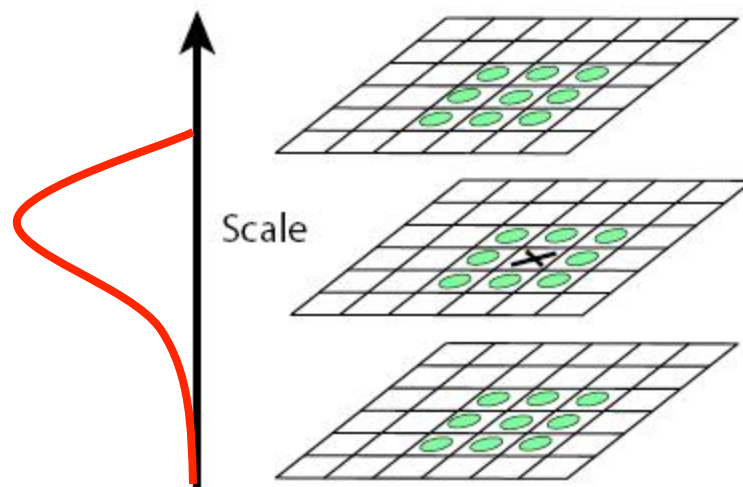
- We define the **characteristic scale** as the scale that produces peak of Laplacian response



T. Lindeberg (1998). ["Feature detection with automatic scale selection."](#) *International Journal of Computer Vision* **30** (2): pp 77--116.

# Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space
3. This indicates if a blob has been detected
4. And what is its intrinsic scale



# Scale-space blob detector: example

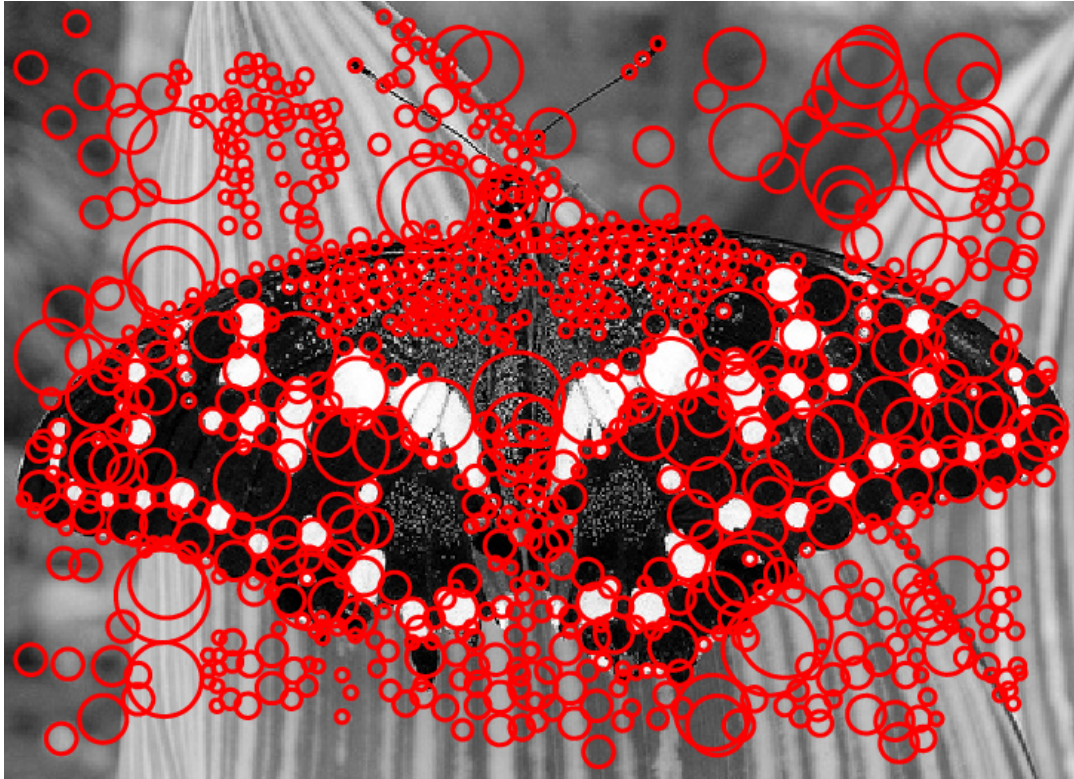


# Scale-space blob detector: example



sigma = 11.9912

# Scale-space blob detector: example



# Difference of Gaussians Approximations to Laplacian

David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" *IJCV* 60 (2), 04

- Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left( G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

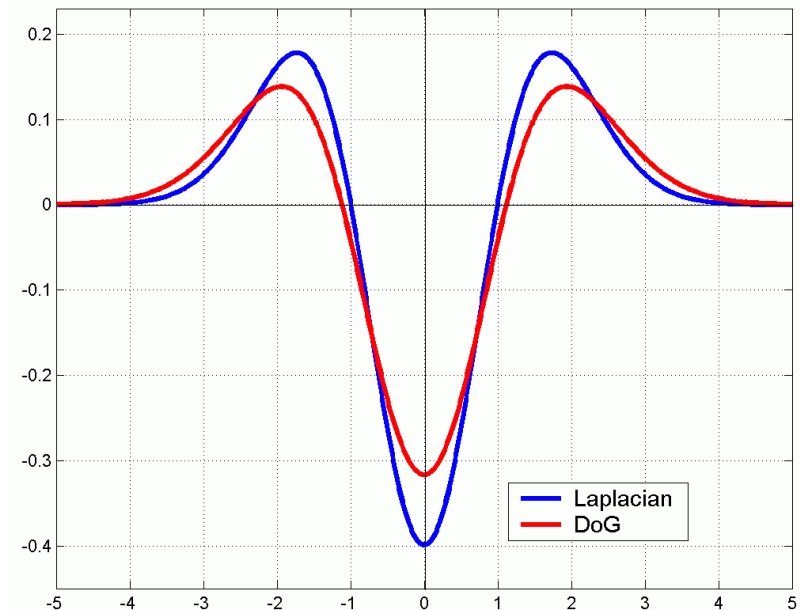
Laplacian

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

Difference of Gaussians

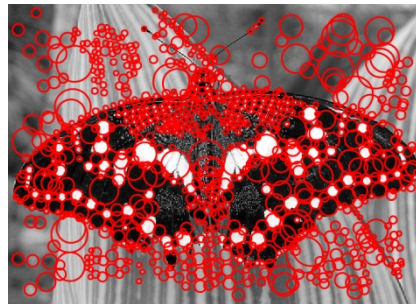
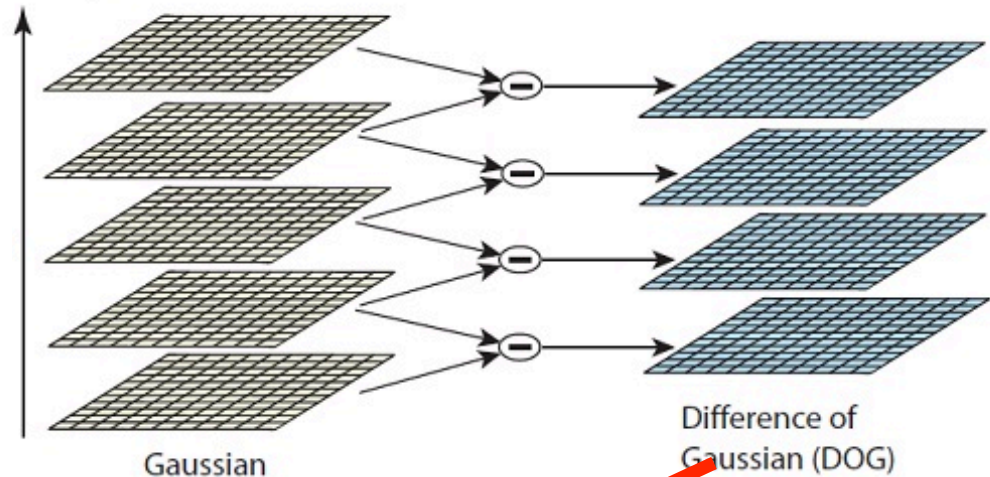
or

Difference of gaussian blurred images at scales  $k\sigma$  and  $\sigma$



$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k - 1)\sigma^2 L$$

# Difference of Gaussians (DoG)



**Output:** location, scale, orientation (more later)

# Invariance

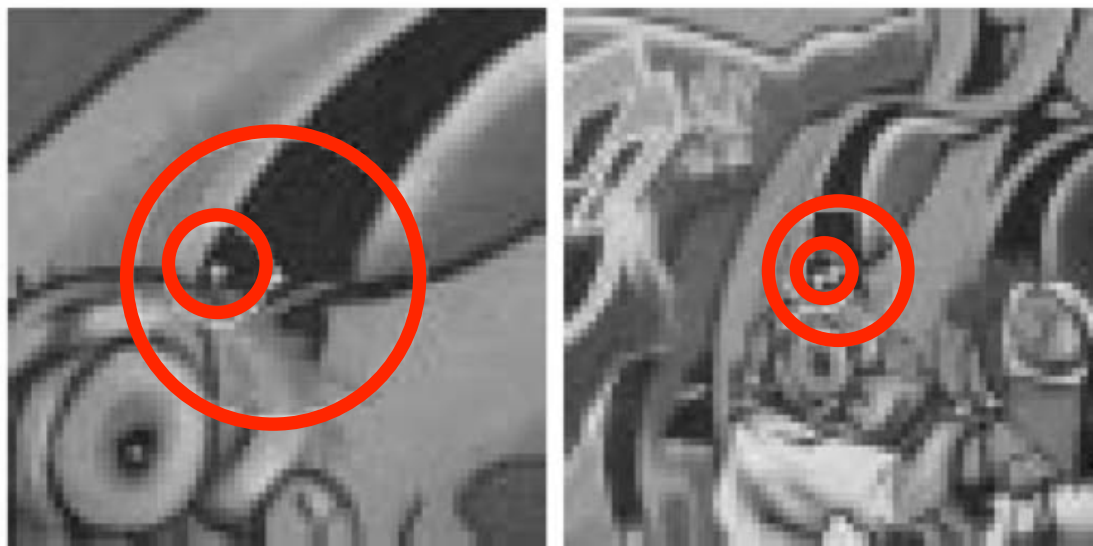
Detector	Illumination	Rotation	Scale	View point
Harris corner	Yes	Yes	No	No
Lowe '99 (DoG)	Yes	Yes	Yes	No



# Harris-Laplace

[Mikolajczyk & Schmid '01]

- Collect locations  $(x,y)$  of detected Harris features for  $\sigma = \sigma_1 \dots \sigma_2$  (the sigma is here comes from  $g_x, g_y$ )
- For each detected location  $(x,y)$  and for each  $\sigma$ , reject detection if  $\text{Laplacian}(x,y, \sigma)$  is not a local maximum



**Output:** location, scale

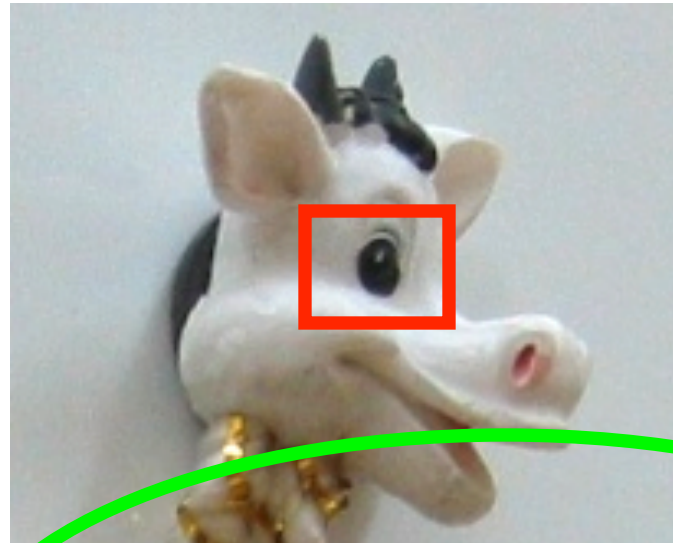
# Invariance

<b>Detector</b>	<b>Illumination</b>	<b>Rotation</b>	<b>Scale</b>	<b>View point</b>
Harris corner	Yes	Yes	No	No
Lowe '99 (DoG)	Yes	Yes	Yes	No
Mikolajczyk & Schmid '01	Yes	Yes	Yes	No

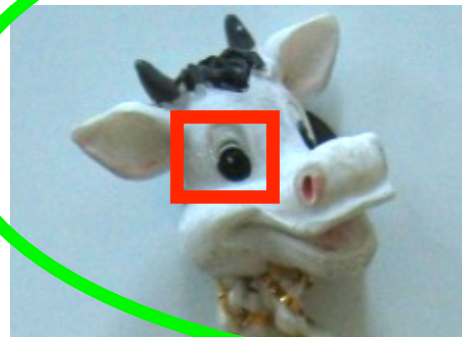
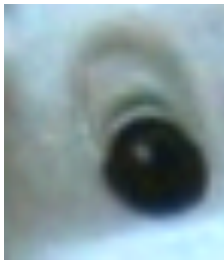
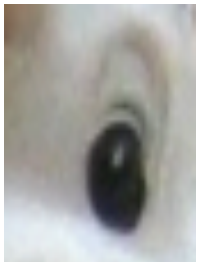
# Repeatability



Illumination  
invariance

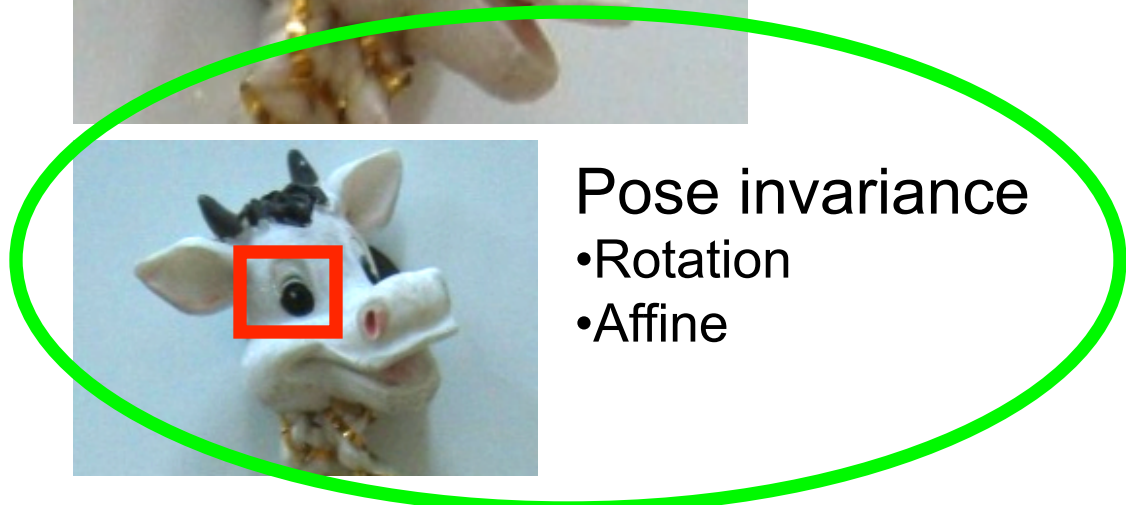


Scale  
invariance



Pose invariance

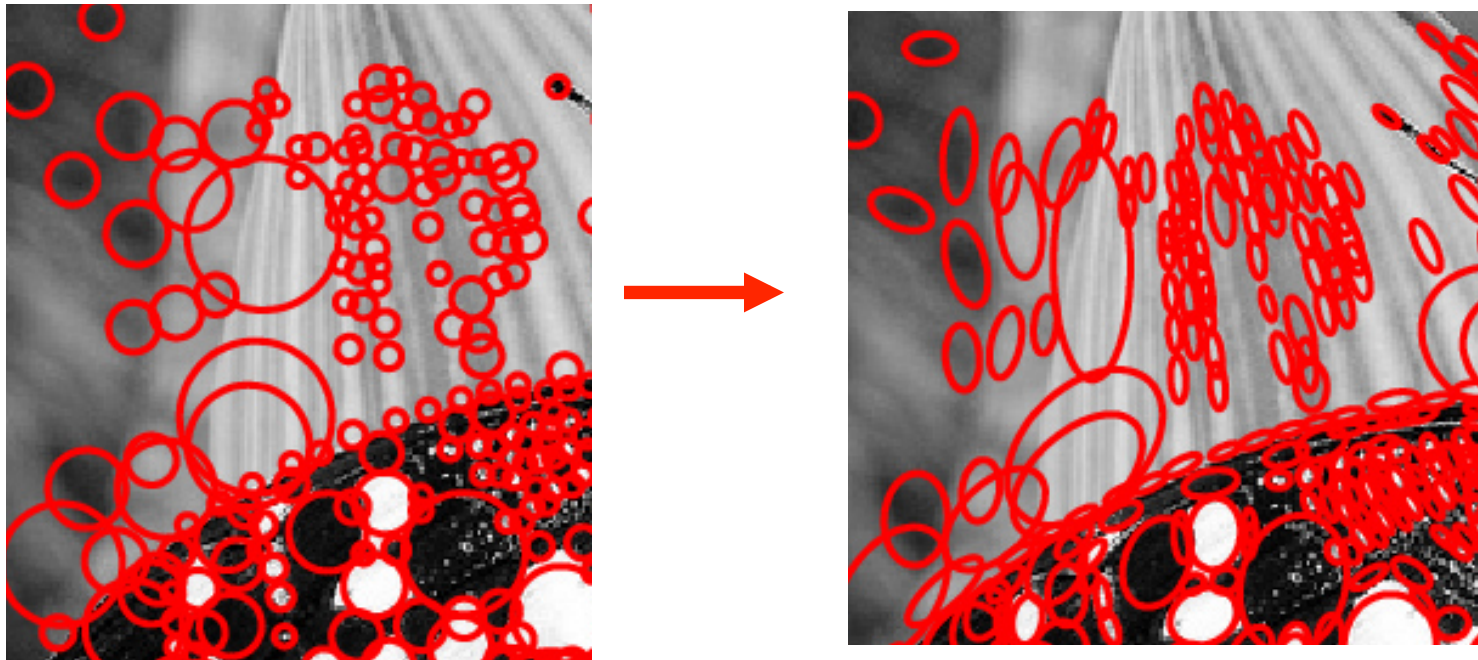
- Rotation
- Affine



# Affine invariance

K. Mikolajczyk and C. Schmid,  
[Scale and Affine invariant interest point detectors](#), IJCV 60(1):63-86, 2004.

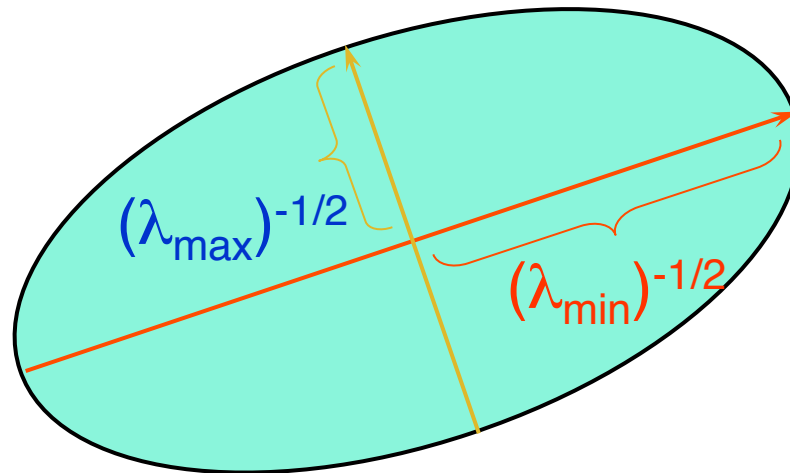
Similarly to characteristic scale selection, detect the **characteristic shape** of the local feature



# Affine invariance

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

We can visualize  $M$  as an ellipse with axis lengths determined by the eigenvalues and orientation determined by  $R$



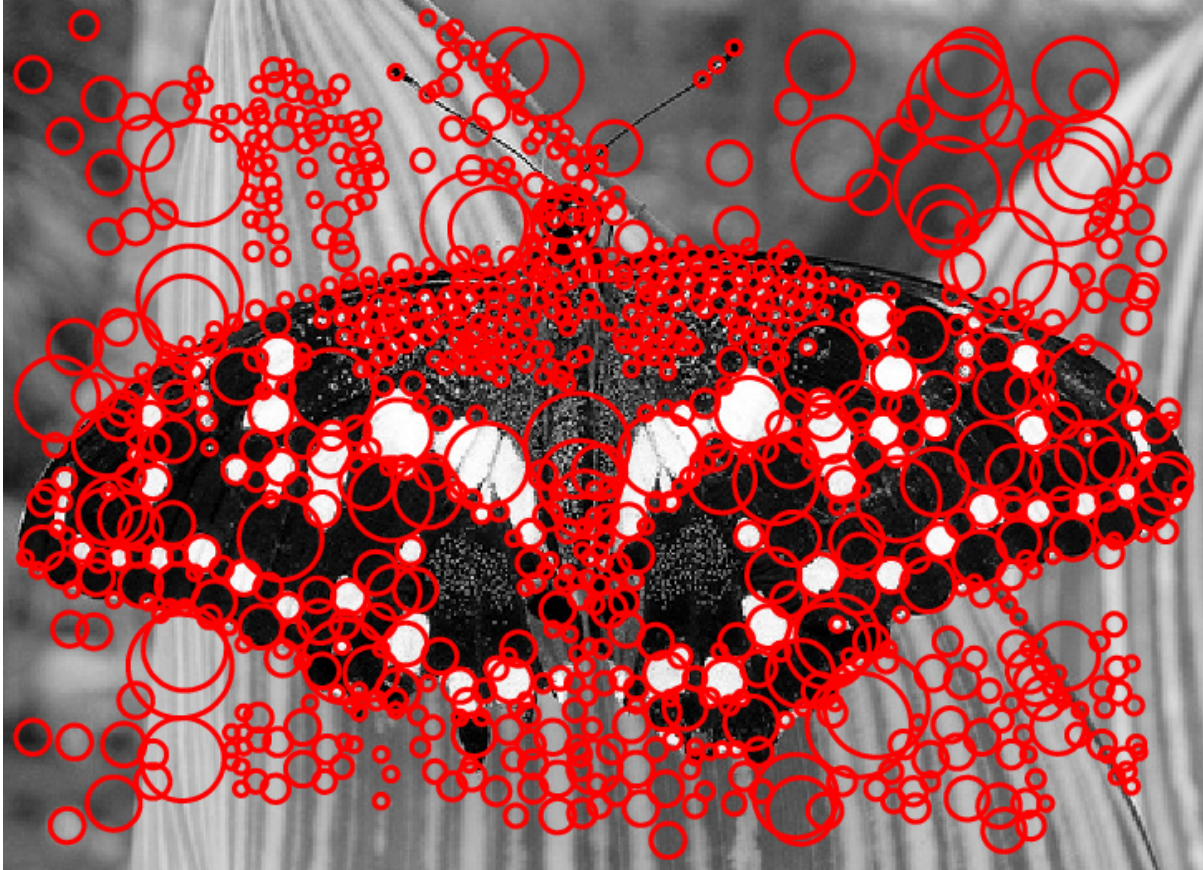
The second moment ellipse can be viewed as the “characteristic shape” of a region

# Affine adaptation

1. Detect initial region with Harris Laplace
2. Estimate affine shape with  $M$
3. Normalize the affine region to a circular one
4. Re-detect the new location and scale in the normalized image
5. Go to step 2 if the eigenvalues of the  $M$  for the new point are not equal [detector not yet adapted to the characteristic shape]

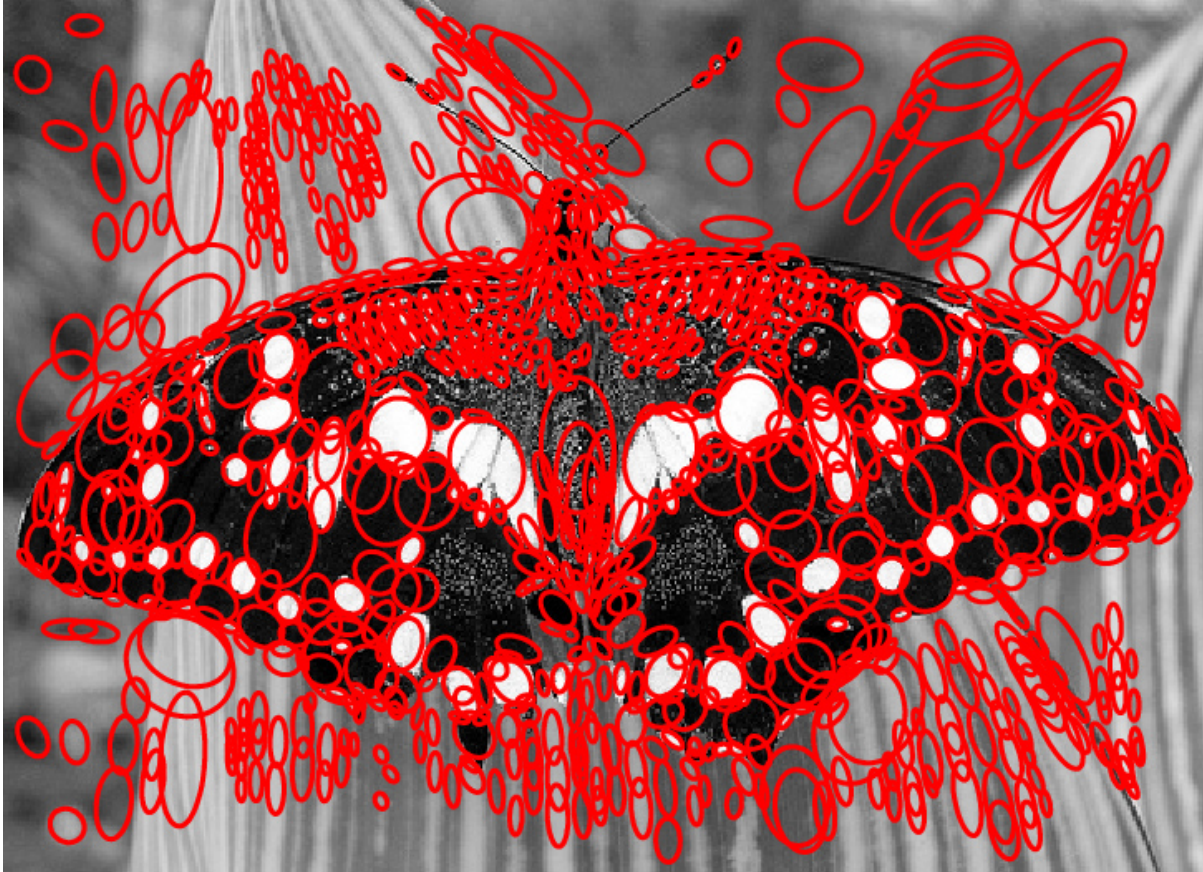


# Without affine invariance



Scale-invariant regions (blobs)

# With affine invariance



Affine-adapted blobs



# Invariance

Detector	Illumination	Rotation	Scale	View point
Harris corner	Yes	Yes	No	No
Lowe '99 (DoG)	Yes	Yes	Yes	No
Mikolajczyk & Schmid '01	Yes	Yes	Yes	No
Mikolajczyk & Schmid '02	Yes	Yes	Yes	Yes

Detector	Illumination	Rotation	Scale	View point
Harris corner	Yes	Yes	No	No
Lowe '99 (DoG)	Yes	Yes	Yes	Yes
Mikolajczyk & Schmid '01, '02	Yes	Yes	Yes	Yes
Tuytelaars, '00	Yes	Yes	No (Yes '04 )	Yes
Kadir & Brady, 01	Yes	Yes	Yes	no
Matas, '02	Yes	Yes	Yes	no

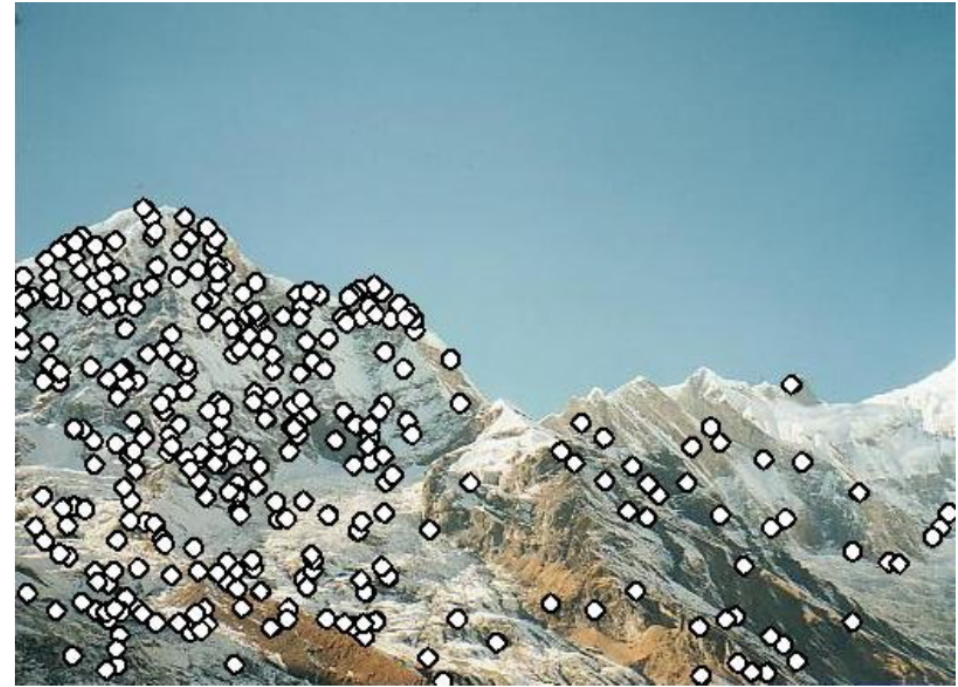
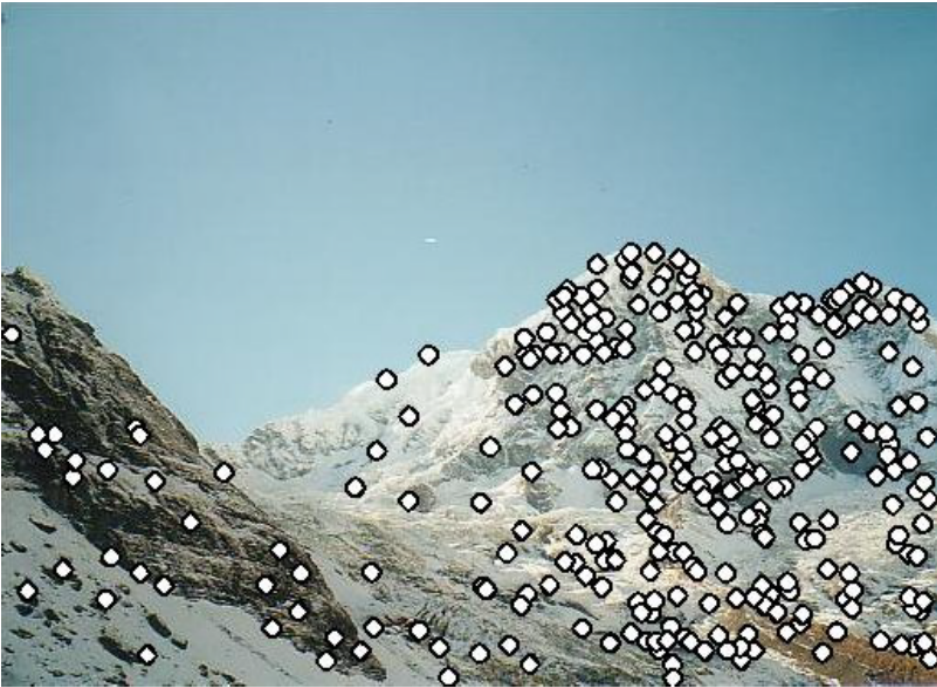
# Feature Descriptors

Overview

# Application: Image Stitching

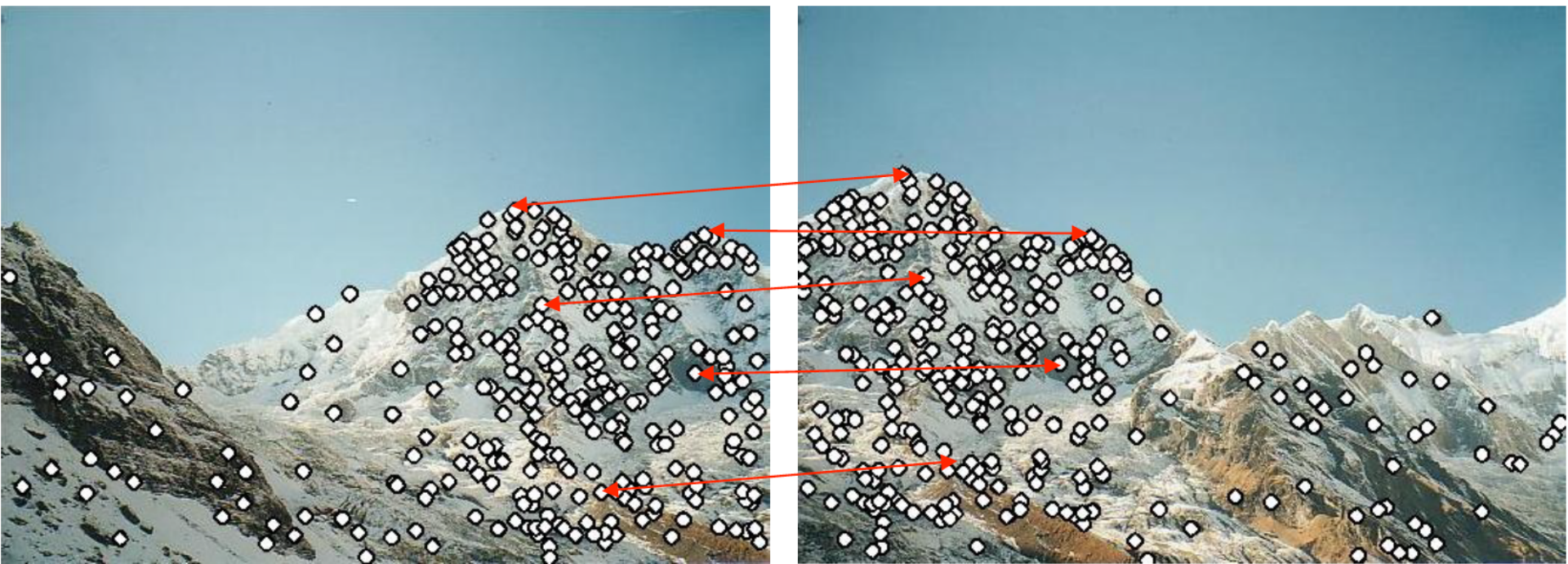


# Application: Image Stitching



1. Detect feature points in both images.

# Application: Image Stitching



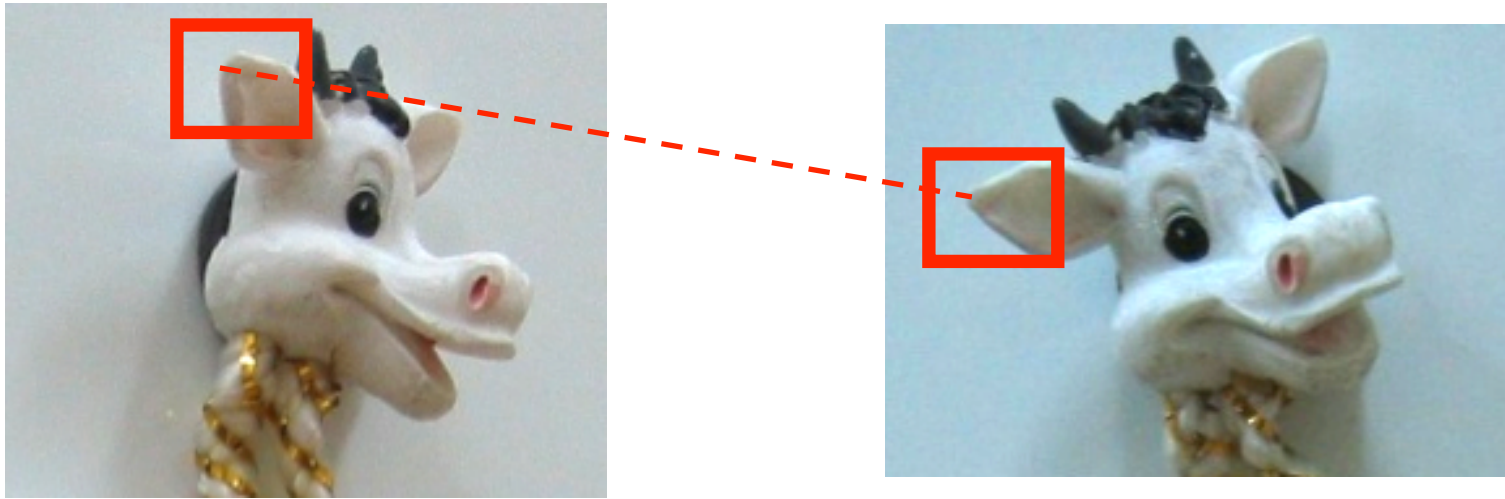
1. Detect feature points in both images.
2. Find corresponding pairs of feature points.

# Application: Image Stitching



1. Detect feature points in both images.
2. Find corresponding pairs of feature points.
3. Use the pairs to align the images.

# Application: Estimating Fundamental Matrix



1. Detect feature points in both images.
2. Find corresponding pairs of feature points.
3. Use the pairs to estimate epipolar geometry across images.



# Application: Detect Object Instances



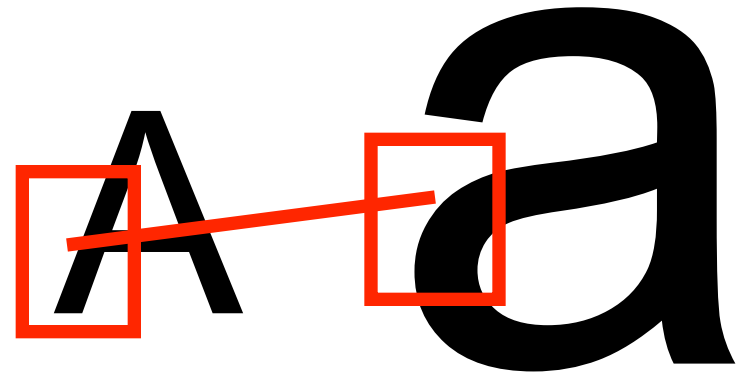
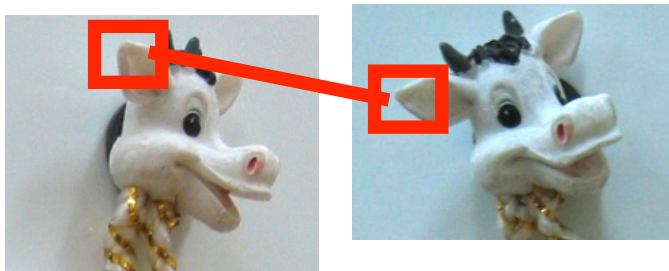
1. Detect feature points in both images.
2. Find corresponding pairs of feature points.
3. Use the pairs to match object instances.

# Challenges

Depending on the application a descriptor must incorporate information that is:

- Invariant w.r.t:

- Illumination
- Pose
- Scale
- Intraclass variability



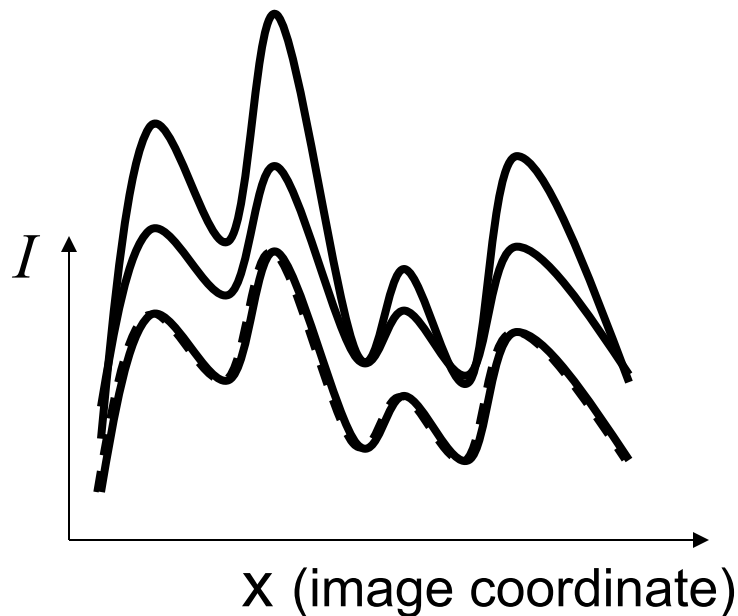
- **Highly distinctive** (allows a single feature to find its correct match with good probability in a large database of features)

# Illumination normalization

- *Affine intensity change:*

$$I \rightarrow I + b$$

$$\rightarrow a I + b$$



- Make each patch zero mean:

$$\mu = \frac{1}{N} \sum_{x,y} I(x, y)$$

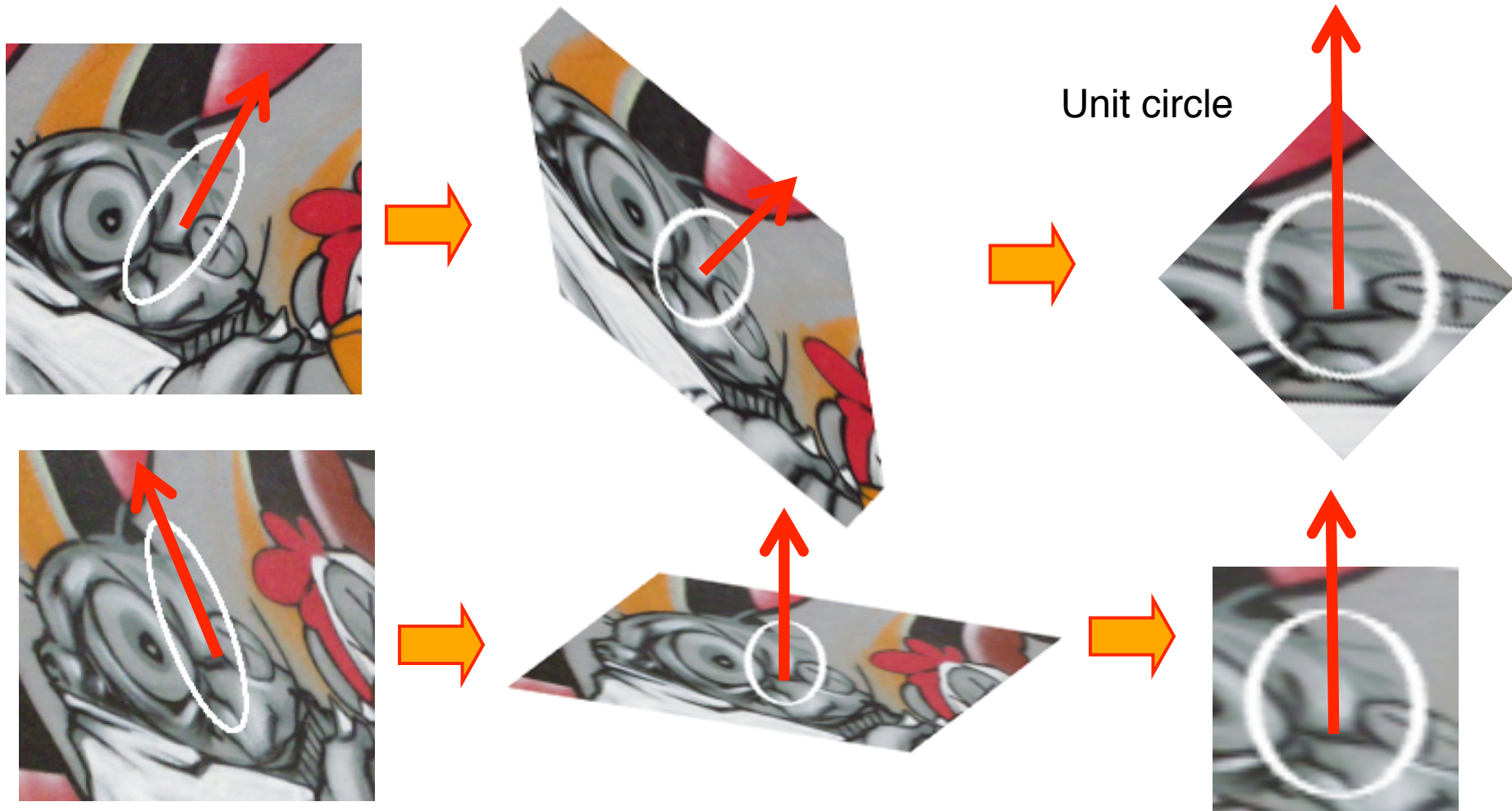
$$Z(x, y) = I(x, y) - \mu$$

- Then make unit variance:

$$\sigma^2 = \frac{1}{N} \sum_{x,y} Z(x, y)^2$$

$$ZN(x, y) = \frac{Z(x, y)}{\sigma}$$

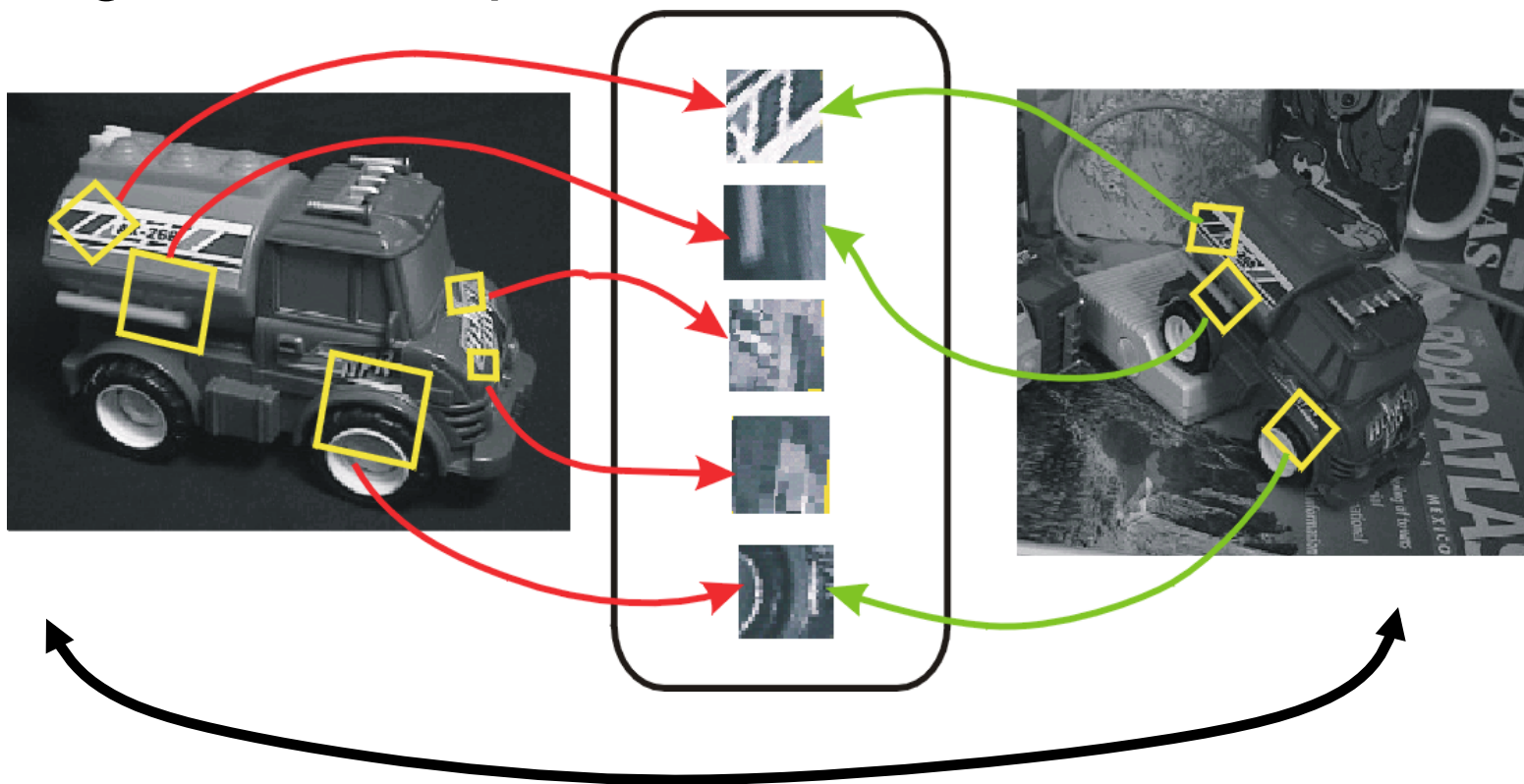
# Pose normalization



NOTE: location, scale, rotation & affine pose are given by the detector or calculated within the detected regions

# Pose normalization

- Keypoints are transformed in order to be invariant to translation, rotation, scale, and other geometrical parameters [Lowe 2000]



Change of scale, pose, illumination...

Courtesy of D. Lowe

# The simplest descriptor



1 x NM vector of pixel intensities

$$W = [ \text{[blurred row of pixels]} \quad \dots \quad \text{[blurred row of pixels]} ]$$

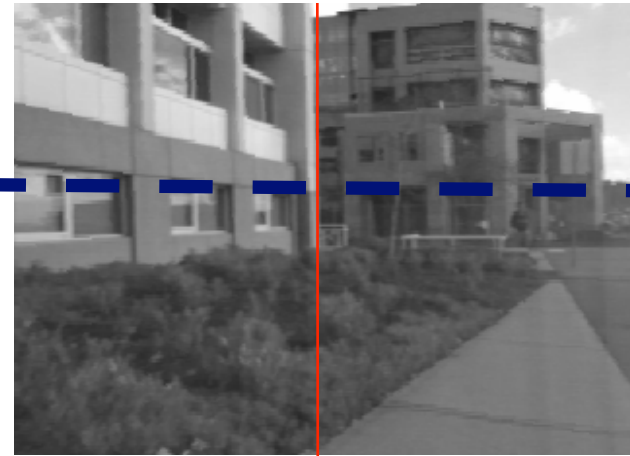
$$W_n = \frac{(w - \bar{w})}{\| (w - \bar{w}) \|}$$

Makes the descriptor invariant with respect to affine transformation of the illumination condition

# Why not?

- Sensitive to small variation of:
  - location
  - Pose
  - Scale
  - intra-class variability
- Poorly distinctive

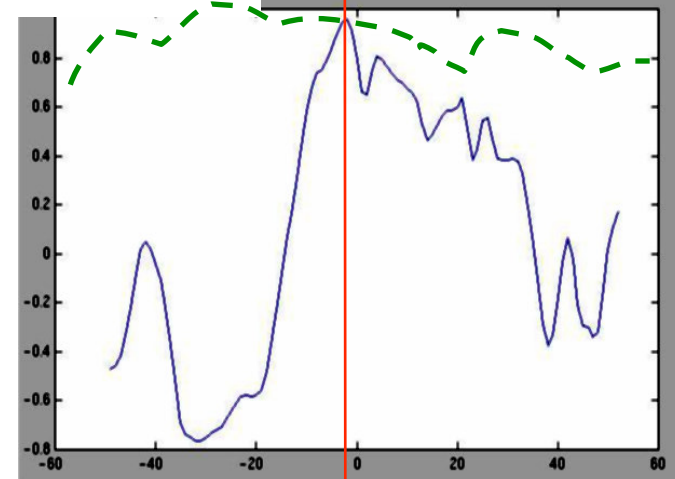
# Sensitive to pose variations



Normalized Correlation:

$$W_n \cdot W'_n = \frac{(w - \bar{w})(w' - \bar{w}')}{\| (w - \bar{w})(w' - \bar{w}') \|}$$

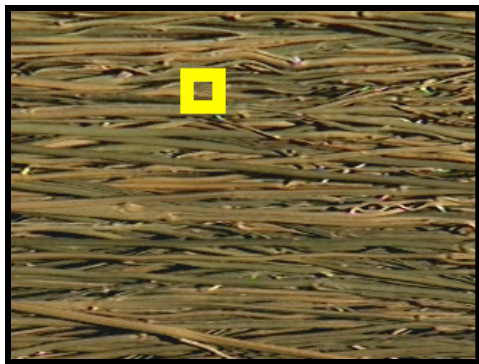
Norm. corr



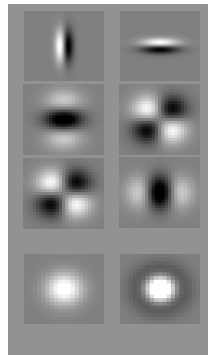


<b>Detector</b>	<b>Illumination</b>	<b>Pose</b>	<b>Intra-class variab.</b>
PATCH	Good	Poor	Poor

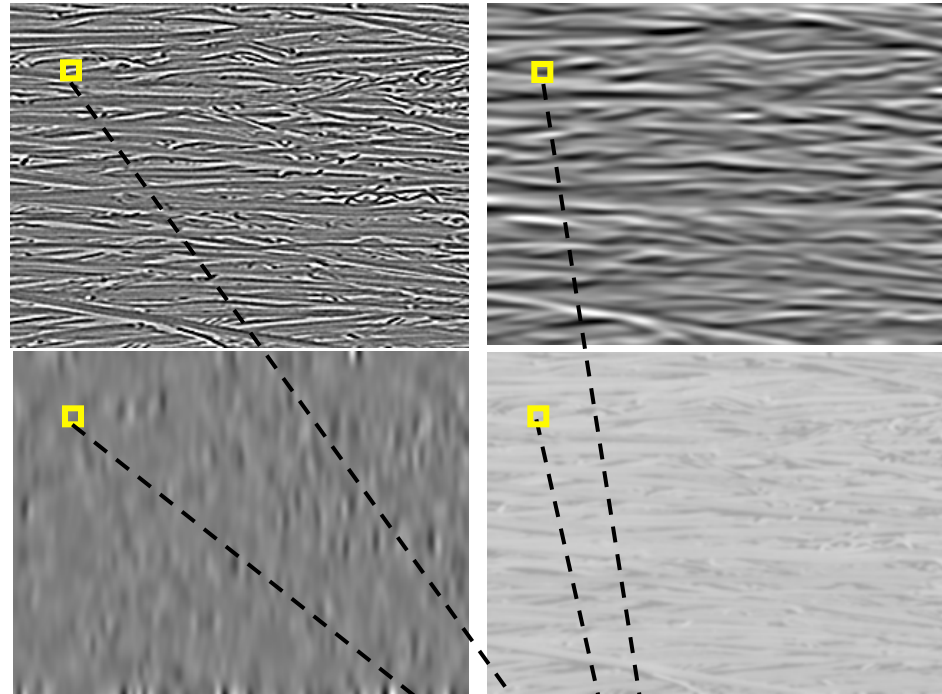
# Bank of filters



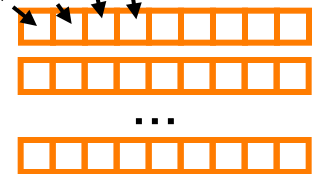
image



filter bank



filter responses



descriptor

More robust but still quite sensitive to pose variations

Detector	Illumination	Pose	Intra-class variab.
PATCH	Good	Poor	Poor
FILTERS	Good	Medium	Medium

# SIFT descriptor

David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" *IJCV* 60 (2), 04

- Alternative representation for image patches
- Location and characteristic scale  $s$  given by DoG detector

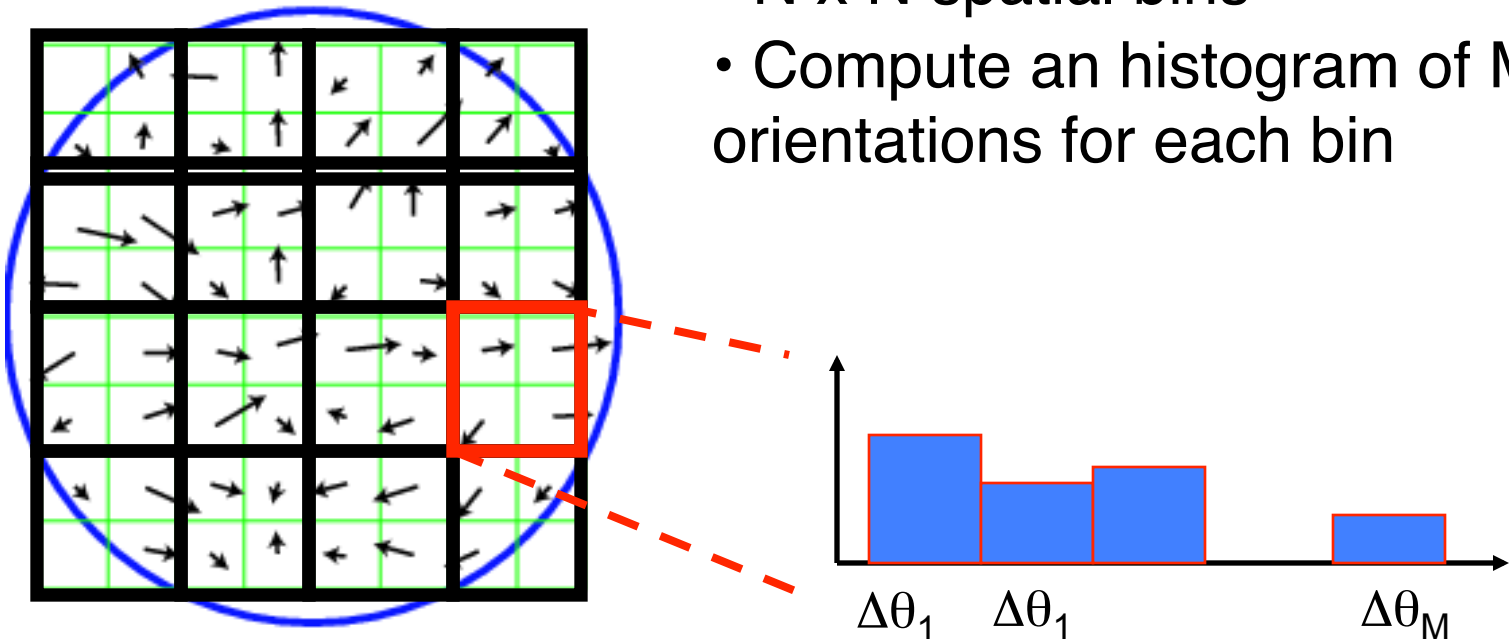


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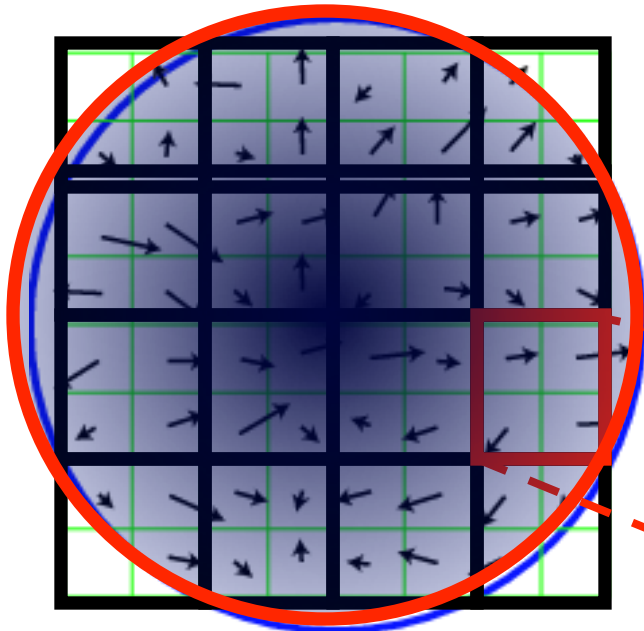
- Compute gradient at each pixel
- $N \times N$  spatial bins
- Compute an histogram of  $M$  orientations for each bin



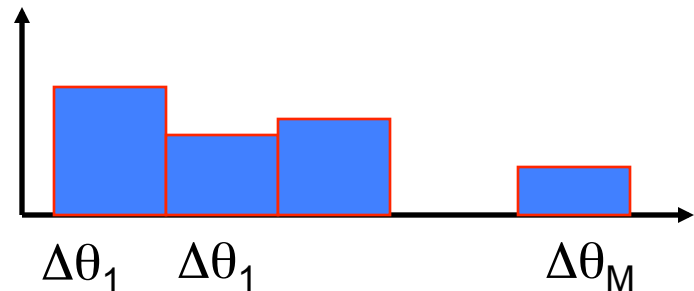
# SIFT descriptor

David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" *IJCV* 60 (2), 04

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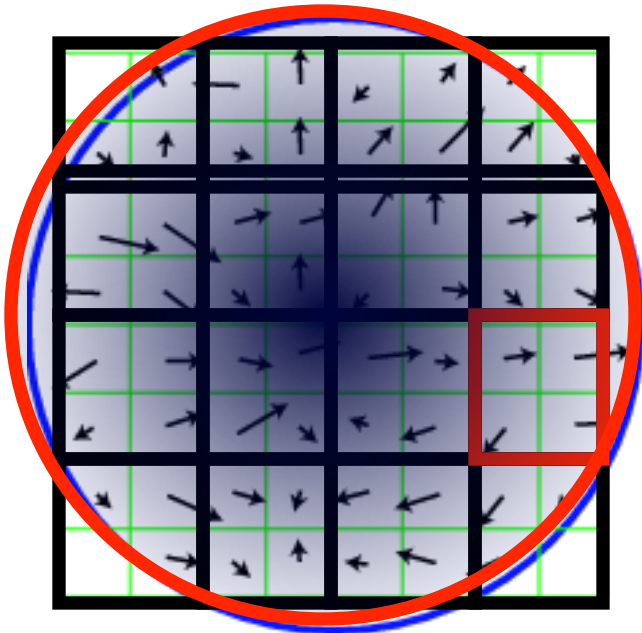
- Compute gradient at each pixel
- $N \times N$  spatial bins
- Compute an histogram of  $M$  orientations for each bin
- Gaussian center-weighting



# SIFT descriptor

David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" *IJCV* 60 (2), 04

- Alternative representation for image patches
- Location and characteristic scale  $s$  given by DoG detector



- Compute gradient at each pixel
- $N \times N$  spatial bins
- Compute an histogram of  $M$  orientations for each bin
- Gaussian center-weighting
- Normalized unit norm

Typically  $M = 8$ ;  $N = 4$   
1 x 128 descriptor

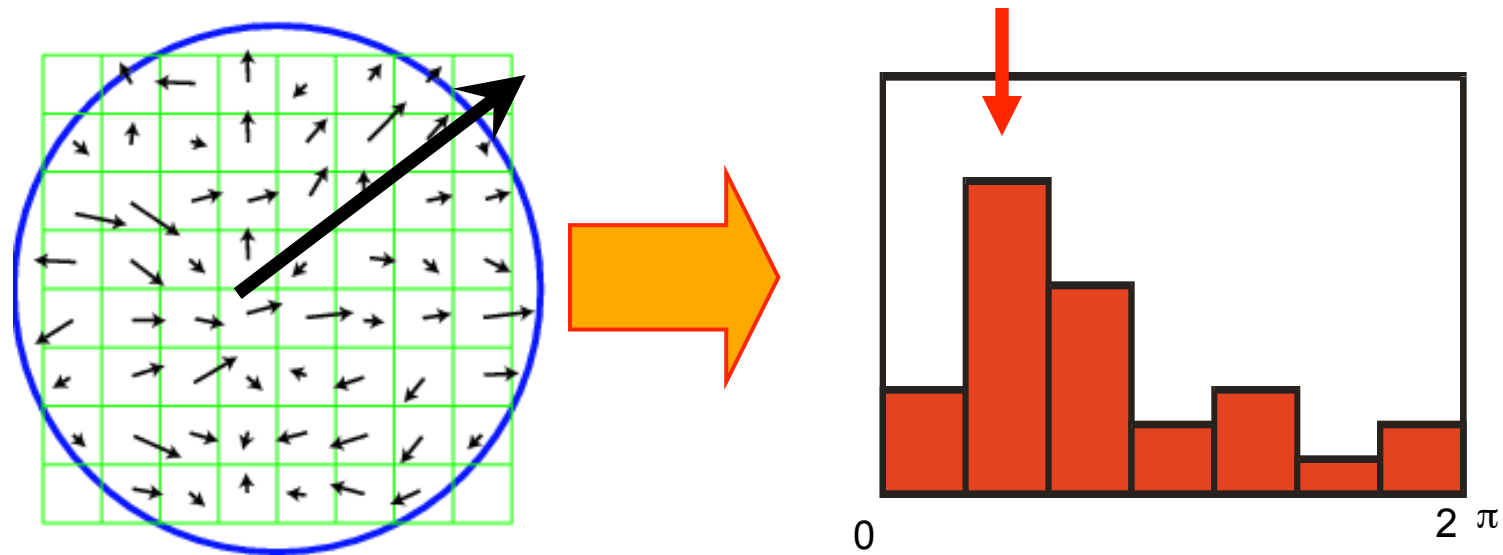
# SIFT Descriptor

- Robust w.r.t. small variation in:
  - Illumination (thanks to gradient & normalization)
  - Pose (small affine variation thanks to orientation histogram )
  - Scale (scale is fixed by DOG)
  - Intra-class variability (small variations thanks to histograms)



# Rotational Invariance

- Find dominant orientation by building smoothed orientation histogram
- Rotate all orientations by the dominant orientation



This makes the SIFT descriptor rotational invariant

# SIFT Rotational Invariance Example



# Rotation invariance (Alternate)

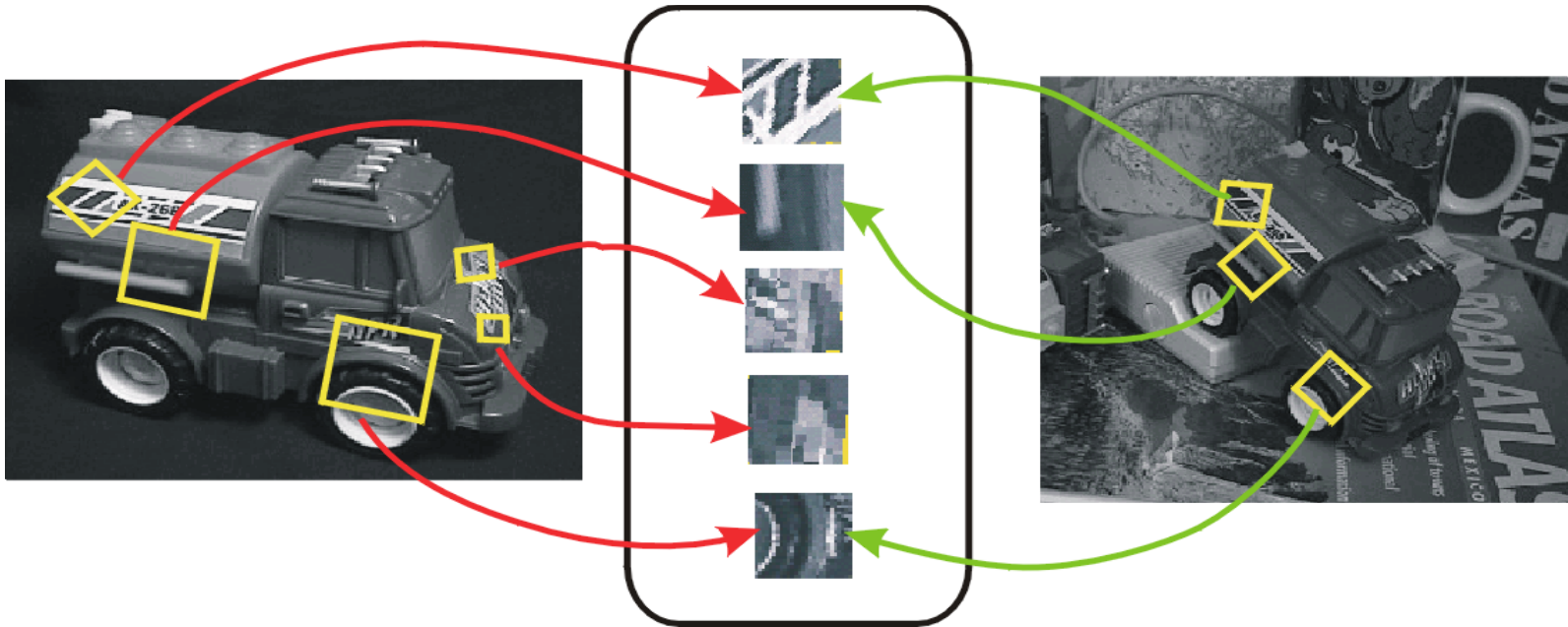
Find dominant orientation of the image patch

- This is given by  $\mathbf{x}_+$ , the eigenvector of  $\mathbf{H}$  corresponding to  $\lambda_+$ 
  - $\lambda_+$  is the *larger* eigenvalue
- Rotate the patch according to this angle



Figure by Matthew Brown

# SIFT Rotational Invariance Example



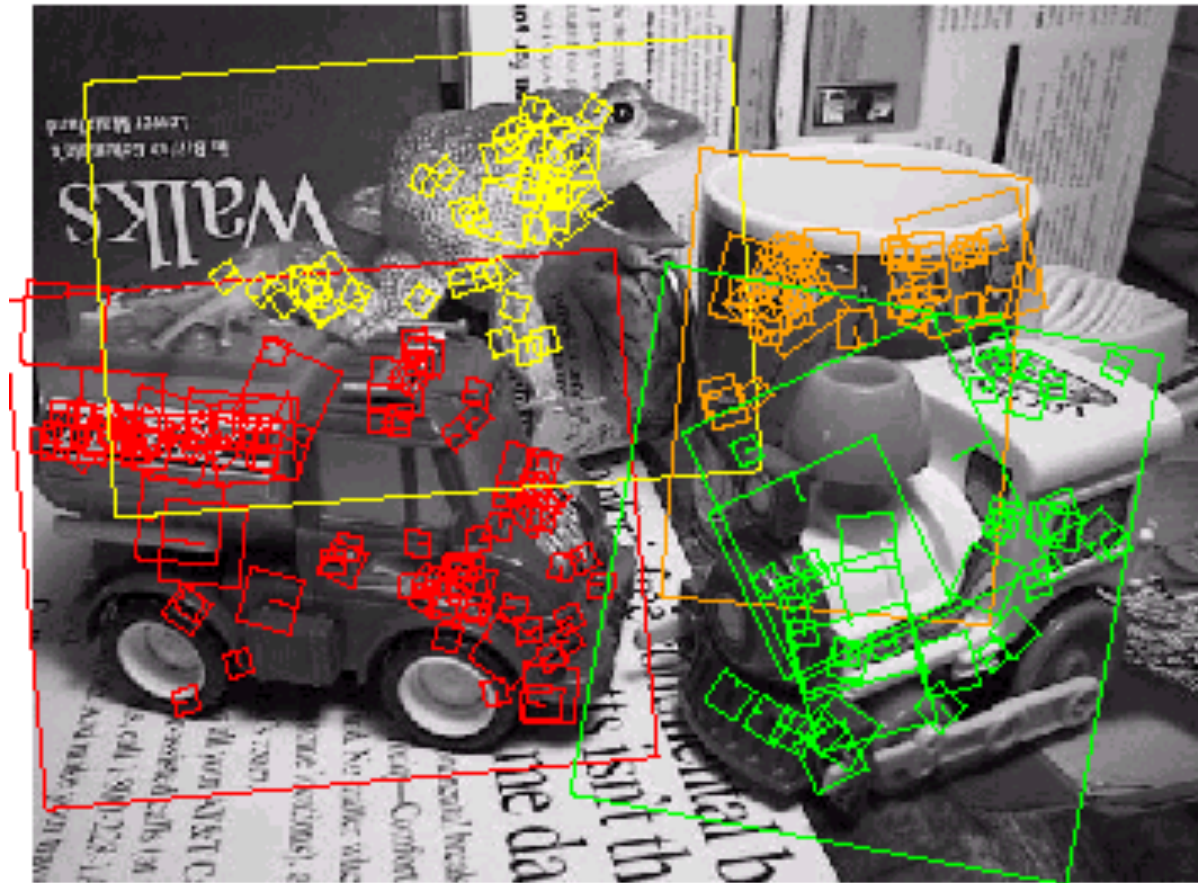
# Matching Using SIFT

David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" *IJCV* 60 (2), 04



# Matching Using SIFT

David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" *IJCV* 60 (2), 04



<b>Detector</b>	<b>Illumination</b>	<b>Pose</b>	<b>Intra-class variab.</b>
PATCH	Good	Poor	Poor
FILTERS	Good	Medium	Medium
SIFT	Good	Good	Medium

# Next Lecture: Segmentation and Clustering

- Readings: FP 6.2, 9; SZ 5.2-5.4