Qualitative Simulation Using Time-Scale Abstraction*

Benjamin Kuipers
Department of Computer Sciences
University of Texas at Austin
Austin, Texas 78712.

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Abstract.

Qualitative simulation faces an intrinsic problem of scale: the number of limit hypotheses grows exponentially with the number of parameters approaching limits. We present a method called *Time-Scale Abstraction* for structuring a complex system as a hierarchy of smaller, interacting equilibrium mechanisms. Within this hierarchy, a given mechanism views a slower one as being constant, and a faster one as being instantaneous. A perturbation to a fast mechanism may be seen by a slower mechanism as a displacement of a monotonic function constraint. We demonstrate the time-scale abstraction hierarchy using the interaction between the water and sodium balance mechanisms in medical physiology, an example drawn from a larger, fully implemented, program. Where the structure of a large system permits decomposition by time-scale, this abstraction method permits qualitative simulation of otherwise intractibly complex systems.

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1 The Problem of Scale

Qualitative simulation is a promising method for reasoning with incomplete knowledge about the structure and behavior of physical systems [de Kleer and Brown, 1984; Forbus, 1984; Kuipers, 1984, 1985, 1986]. The structure of a system is described in terms of a collection of continuous parameters and constraints among them. Behavior is described in terms of changes to position and direction in qualitative *quantity spaces*. Such a constraint model may be derived from a component-connection description [deKleer and Brown, 1984], from a process-view description [Forbus, 1984], or be given as part of the problem-solver's model of the domain [Kuipers, 1984; Kuipers and Kassirer, 1984]. The advantage of these qualitative reasoning methods is their ability to express and reason with incomplete knowledge of functional relationships. For example, one may say that wind resistance increases monotonically with velocity, without needing to know or assume their exact relationship: $resistance = M^{+}(velocity)$.

A fundamental operation in qualitative simulation is *limit analysis*: when several variables are changing, and moving toward limiting values, the constraints among them are analyzed to determine which limits may be reached, and hence which qualitative states may come next. For the small to moderate-sized systems examined thus far in the literature, the natural constraint model is often sufficiently powerful to limit the possibilities to a reasonable set.

Unfortunately, there is an intrinsic problem of scale. When dealing with a large system, the number of changing variables moving toward limits may be very large. The set of global limit hypotheses grows exponentially with the number of variables. During a period when two variables in the system do not interact, the temporal reasoning methods of Williams [1986] can isolate them. However, we are frequently faced with large systems consisting of variables that *do* interact, which appear intractible to current qualitative reasoning methods.

Numerous examples throughout AI and computer science demonstrate that a powerful method for handling a complex problem is to impose a modular, hierarchical structure that allows it to be solved in pieces of a manageable size. In order to apply this method, we need to define a valid hierarchical structure that breaks a complex system into a collection of tractible mechanisms. The structure must also support a discipline for moving the focus of attention among the individual mechanisms in the hierarchy, and a mapping relation for communicating information meaningfully among the mechanisms. This paper presents one such structure.

We have encountered this problem of scale in our studies of the expert physician's knowledge of human physiology, especially the systems whereby the body regulates its sodium and water balances [Kuipers and Kassirer, 1984; Kuipers, 1985]. The examples presented in this paper will draw on our models of these physiological mechanisms, but the techniques have more general applicability to qualitative modeling and simulation of large-scale systems.

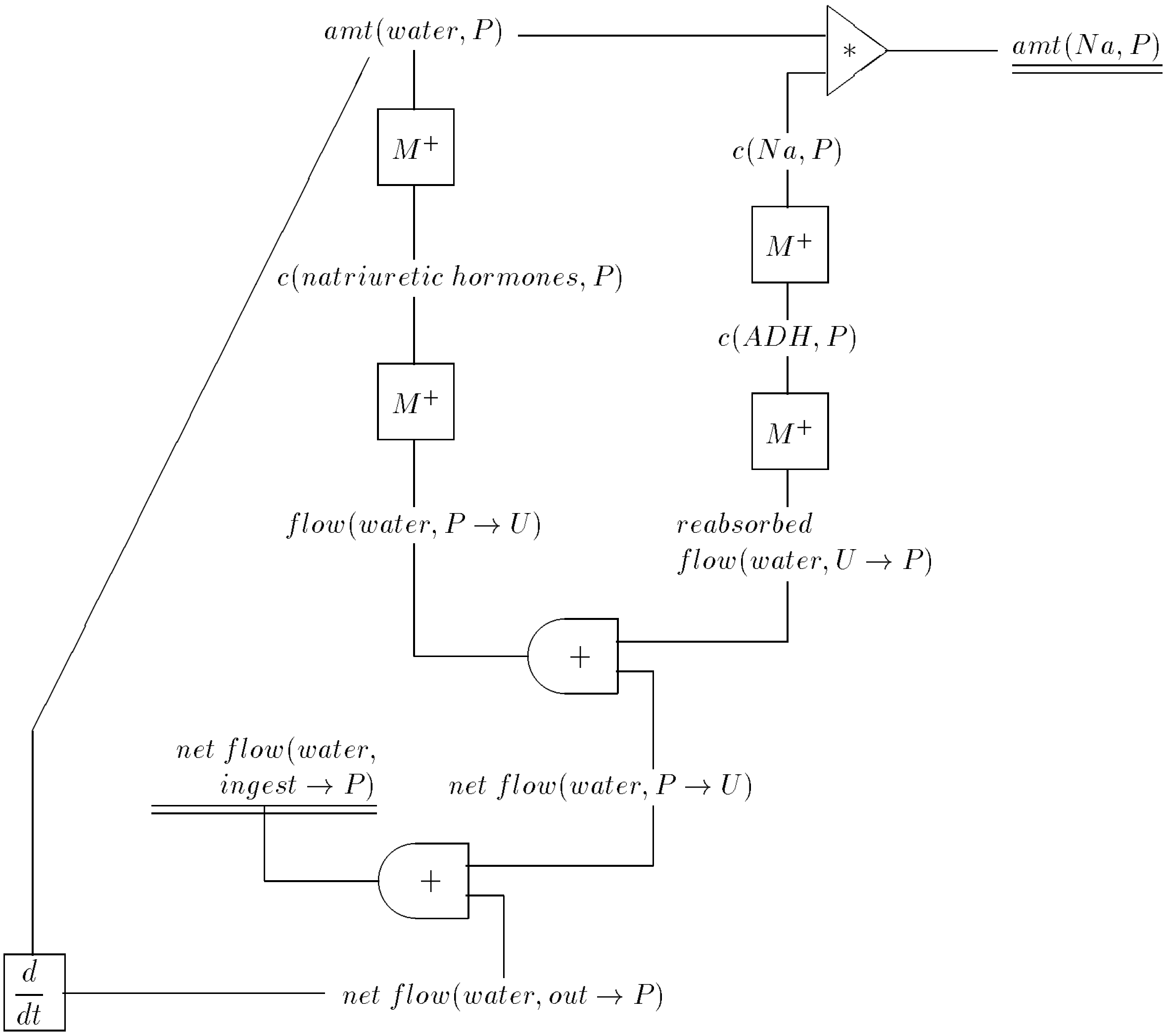


Figure 1: Constraint model for the Water Balance mechanism (fast).

2 Time-Scale Abstraction

Looking at expert physicians for our inspiration, we observe that although the human regulatory systems are immensely complicated, the experts reason effectively about them by focusing on one aspect at a time. One important method for distinguishing closely related mechanisms within the same large system is the time-scale at which they operate.

For example, two closely related mechanism in the kidney help regulate the body's sodium and water balances [Valtin, 1973].

- The water balance mechanism responds to changes in plasma water volume by adjusting water excretion, through a hormone called *antidiuretic hormone* or ADH. Water volume is not sensed directly, but through its effect on sodium concentration. The water balance mechanism responds to changes within a period of minutes. (Figure 1)

- The sodium balance mechanism responds to changes in the amount of sodium in the plasma by adjusting sodium excretion through a hormone called *aldosterone*. The amount of sodium is not sensed directly, but through its effects on water volume. The sodium balance mechanism responds to changes over a period of hours to days. (Figure 2)

Figures 1 and 2 give a graphical representation of the QSIM constraint models of the water balance and sodium balance mechanisms, respectively.

The separation in time-scales of these two mechanisms allows physicians to reason about them separately. For example, in discussing the related but distinct problem of blood pressure regulation, Guyton [1981] presents graphs of the responses of eight different mechanisms, with time-scales ranging from seconds to days (Figure 3).

These observations lead us to define the concept of *time-scale abstraction* applied to a complex system made up of interacting equilibrium mechanisms:

> *If a complex system can be decomposed into equilibrium mechanisms that operate at widely separated time-scales, then a particular mechanism can view a faster one as being instantaneous, and a slower one as being constant.*

When a faster mechanism in a time-scale hierarchy views a slower one as constant, the slower one can simply be treated as a source of values for certain parameters. When a slower mechanism views a faster one as instantaneous, a relation among shared variables may be treated by the fast mechanism as the result of a process over time, and by the slow mechanism as a functional relationship.

Consider the relationship between the (slow) sodium and (fast) water balance mechanisms:

- The water balance mechanism (Figure 1) includes the following parameters. (P stands for the plasma compartment of the body fluids, and the N in ANP stands for sodium (Na).)

AWP	$amt(water, P)$	dependent
ANP	$amt(sodium, P)$	independent
NFWIP	$net\ flow(water, ingest \rightarrow P)$	independent

ANP and NFWIP are independent, or "context", parameters of the water balance mechanism. The parameters AWP and ANP are shared with the sodium balance mechanism (Figure 2), where they are both dependent variables.

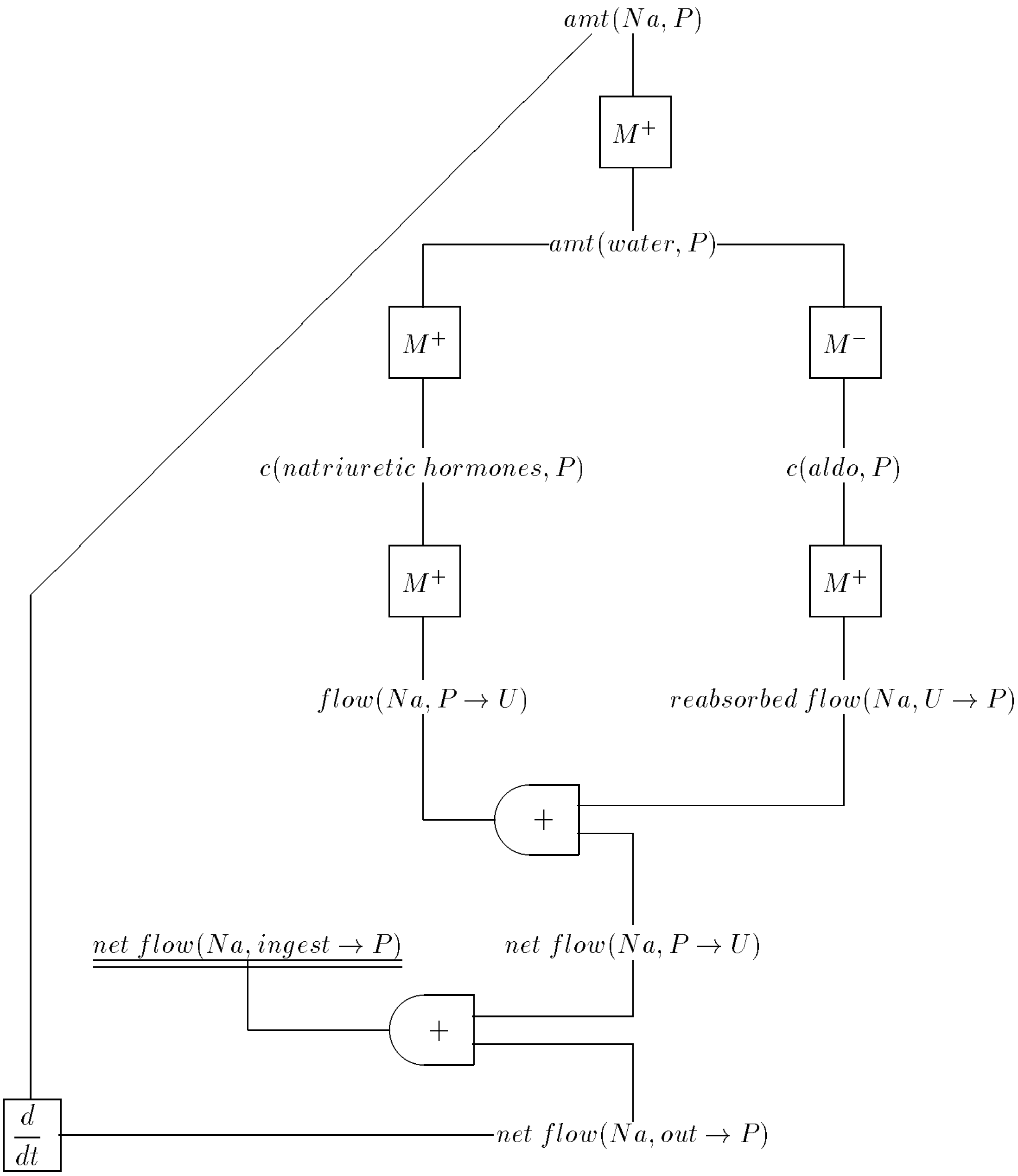


Figure 2: Constraint model for the Sodium Balance mechanism (slow).

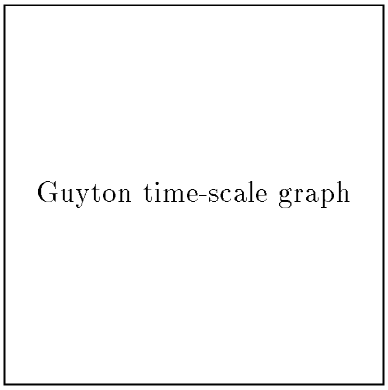


Figure 3: Time-scales of physiological processes, from [Guyton, 1981].

- From the point of view of the water balance mechanism, an externally given increase in sodium (ANP) results in the water balance moving, over some period of time, to a new equilibrium where the amount of water (AWP) is also increased (Figure 4a).
- From the point of view of the sodium balance mechanism, the relationship between ANP and AWP is seen as instantaneous, and is expressed by the monotonic function constraint, $AWP = M^{+}(ANP)$ (Figure 4b).

Thus, different levels of the time-scale hierarchy view the relation between two parameters in quite different ways. A structural constraint at one level is the result of an embedded process at a faster level.

3 Communicating Across Time-Scales

In order to use a hierarchical model linked by time-scale abstraction for qualitative simulation of a complex system, information must be transmitted through shared variables among mechanisms operating at different time-scales.

3.1 The Pattern of Shifting Focus

We need a discipline for shifting the focus of attention among different time-scales and for making valid use of previously derived information in subsequent computations. The two directions of shift in focus from a given mechanism require different methods.

- **Faster to Slower**. Given an initial perturbation to its environment, qualitative simulation predicts the resulting equilibrium state of the fast mechanism, and shifts attention to the next slower one. The final values of parameters that are shared with the slower mechanism can be treated as part of the initial state of the slower mechanism. There are also effects on the constraints which will be treated in the next section.

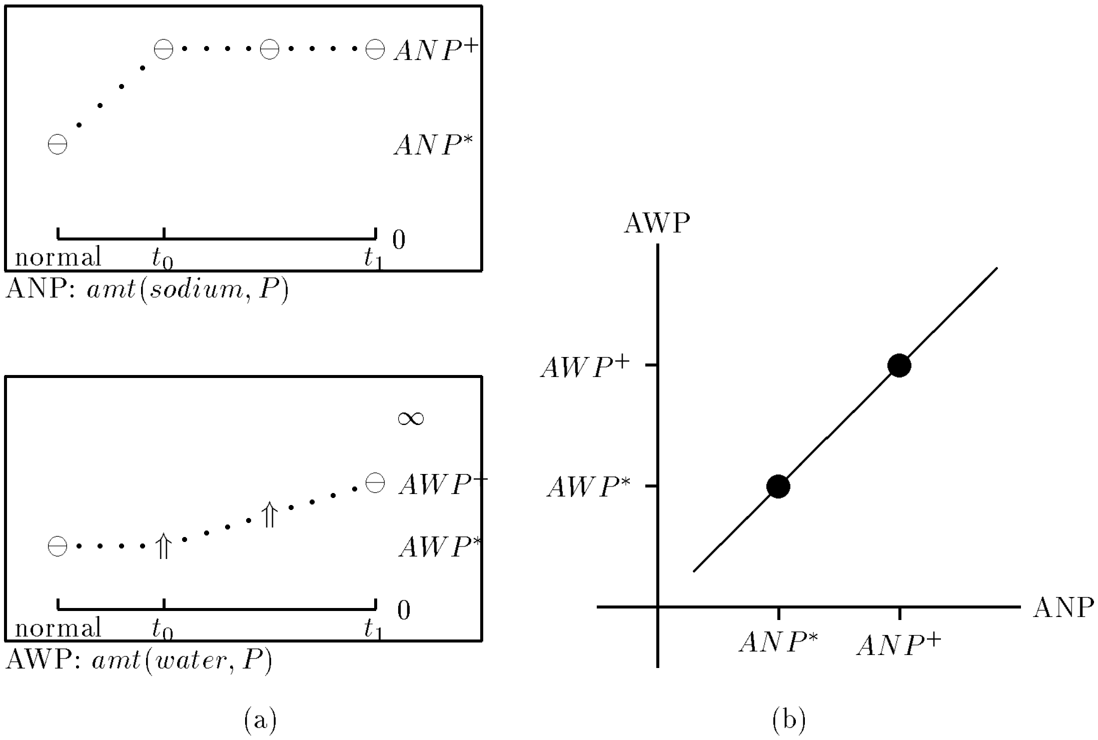


Figure 4: The relationship between ANP and AWP

- (a) From the point of view of the Water Balance mechanism (Figure 1), a change to ANP *causes* a subsequent change to AWP.
- (b) From the point of view of the Sodium Balance mechanism (Figure 2), the monotonic function constraint $AWP = M^+(ANP)$ requires the two parameters to change together.

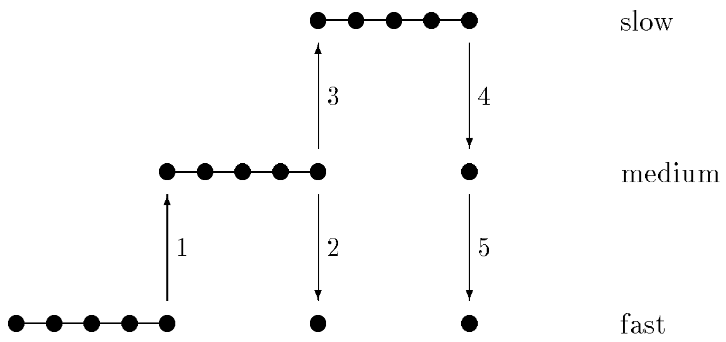


Figure 5: Control of focus of attention.
Each bead represents a qualitative state, so simulation produces a string of beads, and propagation of an equilibrium state produces a single bead. Changes in focus of attention take place in the sequence shown. (1) The equilibrium state of the fastest mechanism provides values for initializing a simulation of the next slower mechanism. (2) The final state of the second simulation is first used to propagate a new equilibrium state for the fastest mechanism. (3) Then values from both faster mechanisms are available to initialize the slowest mechanism. And so on.

- **Slower to Faster**. After a slower mechanism has reached equilibrium, the environment it provides for a faster mechanism may have changed. However, the faster mechanism, by definition, must have tracked the slower mechanism on its way to equilibrium. Thus, the fast mechanism is already in equilibrium, and simulation is not necessary. By combining the values of shared variables, the fact that the mechanism is in equilibrium, and other context information, a complete description of the equilibrium state of the fast mechanism can be derived by propagation and/or constraint satisfaction.

Figure 5 shows the pattern of control for a three-level time-scale hierarchy, deriving the effect of an initial perturbation throughout the system. Upward arrows initiate simulation to a new equilibrium, and downward arrows initiate propagation to a complete description of an existing equilibrium state. The algorithm is as follows. After simulating a mechanism, QSIM identifies the faster mechanisms which share parameters with the current mechanism, and propagates the shared values to determine the equilibrium state of the faster mechanism. Once this is done, the slower mechanisms sharing parameters are identified. The current values of parameters shared with this mechanism are used to define the initial state from which it is simulated. The process repeats recursively.

In order for the abstraction hierarchy to support correct simulation, control of the focus of attention must be combined with an appropriate interpretation of information from one level of the hierarchy, as viewed from another. In particular, if some change causes a fast mechanism to behave abnormally, this is viewed from the slower mechanism as a displacement of a monotonic function.

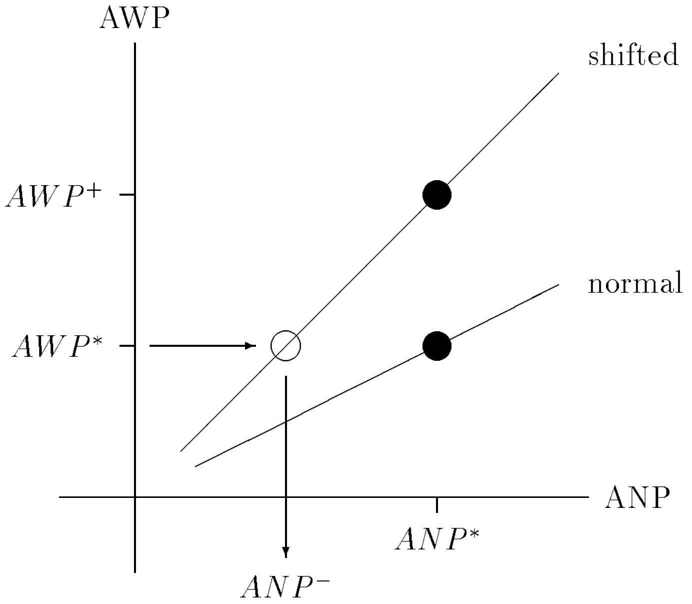


Figure 6: Normal and shifted monotonic function constraints.
The sodium balance mechanism (Figure 2) moves to bring AWP back to its normal value AWP^*. If the relation $AWP = M^+(ANP)$ is shifted upward, ANP will reach equilibrium at a value lower than normal, $ANP^- < ANP^*$. The shifted constraint is expressed by replacing the normal corresponding values (ANP^*, AWP^*) with (ANP^*, AWP^+).

3.2 Changing the Monotonic Function Constraints

As we have discussed, the slower sodium balance mechanism (Figure 2) includes the monotonic function constraint, $AWP = M^+(ANP)$. In addition to the monotonically increasing direction of the relationship, the constraint specifies *corresponding values*. In the normal situation, each parameter has a normal value — called AWP^* and ANP^*, respectively — and the monotonic function includes the point (ANP^*, AWP^*). Figure 4b shows this relationship.

During the response of the sodium balance mechanism to different initial conditions, the values of ANP and AWP move along this curve. These corresponding values, and those on the other constraints, provide critical information about the possible transient and equilibrium states of the sodium balance mechanism. The faster water balance mechanism acts to move the values back to this curve if they are displaced from it.

Notice, however, that the abstracted monotonic function constraint, $AWP = M^+(ANP)$, and especially its corresponding values, also depend on the value of the context parameter NFWIP, representing the rate of water intake, which appears only in the water balance mechanism. If NFWIP is shifted to a value higher than normal, then the monotonicity of the relationship $AWP = M^+(ANP)$ is preserved, but the corresponding values are changed to (ANP^*, AWP^+), where $AWP^+ > AWP^*$. Figure 6 shows how this change implies that the relationship has been shifted upward.

In the water and sodium balance systems, we can see how a change can propagate within the hierarchy.

1. An externally imposed change affects the fast mechanism, say an increase to the rate of water intake, NFWIP.

2. The external change is not visible to the slower mechanism, which has abstracted away the changed variable, NFWIP. However, QSIM determines that the change to the water balance mechanism results in a shift of the monotonic function constraint, $AWP = M^+(ANP)$.

3. The slower mechanism adjusts to the shifted monotonic function constraint by finding a new equilibrium point. In this case, the sodium balance mechanism excretes sodium to bring the water volume, AWP, down to its normal level, AWP^*, even at the cost of reducing the amount of sodium, ANP, below normal, to AWP^-. (See figure 6)

A "flat" model combining the constraints of the sodium and water balance mechanisms produces an intractibly branching set of predicted behaviors. Using the time-scale abstraction hierarchy, we derive a single qualitative prediction for the behavior resulting from increased water intake: water volume rises quickly, followed by a slower process of sodium excretion (with simultaneous water excretion) until water volume returns to normal. In this final equilibrium state, total sodium and sodium concentration are below normal. Figure 7 shows the relation between the purely ordinal QSIM output plots and qualitative sketches that illustrate the relative time-scales of the processes. Figure 8 is a screen-image, showing a slice of the hierarchical behavior: four parameters across fast and slow mechanisms.

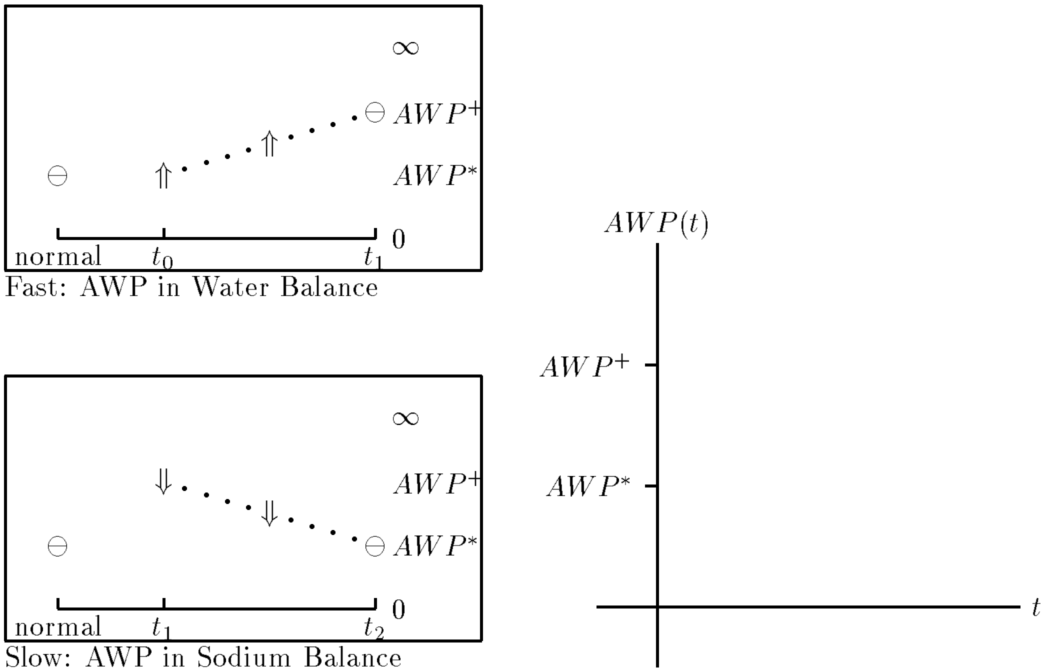
3.3 Implementation Considerations

The time-scale abstraction methods have been implemented as extensions to QSIM, developed and tested on a three-level time-scale hierarchy consisting of the water and sodium balance mechanisms and the Starling equilibrium mechanism governing the balance of water between the plasma and interstitial compartments [Kuipers and Kassirer, 1984]. A preliminary model of control of heart rate and output has also been developed in isolation [Kuipers and Kassirer, 1985] and will be incorporated into the hierarchy. In future work, we plan to extend the hierarchy to include the mechanisms referred to in Figure 3. The ultimate purpose of this physiological model is to support "deep model" reasoning and hypothesis testing in medical diagnosis.

There are several extensions to QSIM required to handle the interfaces between mechanisms acting at different time-scales.

- The time-scale orderings of the mechanisms making up a time-scale abstraction hierarchy must be given explicitly as part of the model. The values of shared variables, and shifted corresponding values, are computed automatically when information is mapped from one mechanism to another. The hierarchy may in principle be arbitrarily deep, but there is currently no method for deciding when to cut off simulation at a lowest level of detail. The hierarchy may be partially ordered, but any two mechanisms that share variables must have a known time-scale order.

- When shifting attention from one mechanism to another, the value of a shared variable must be mapped from one quantity space to another. Since the same landmarks may not appear in



(a) (b)

Figure 7: QSIM plots and qualitative sketches.

- (a) The axes of the fast and slow qualitative plots show only the ordering of the time and quantity landmarks.
- (b) A qualitative sketch (by hand) suggests the relative time-scales of the two processes.

**** Water+Sodium Behavior screen image near here. ****

Figure 8: A four-parameter slice through the fast and slow mechanisms in the sodium and water balance system, responding to increased water intake.

the two quantity spaces, a qualitative value described by a landmark in one quantity space, may be mapped into an interval in the other.

For example, suppose that X_1 and X_2 are the parameters in two different QDEs describing the same variable X, and their quantity spaces are:

$$\begin{array}{ll} X_1 & -\infty \ldots 0 \ldots A \ldots B \ldots \infty \\ X_2 & -\infty \ldots 0 \ldots C \ldots \infty \end{array}$$

Without additional knowledge, we must map $X_1(t) = A$ into $X_2(t) = (0, \infty)$, since the only landmarks with shared meaning across the two quantity spaces are $-\infty$, 0, and ∞. If, in addition, we know that landmarks B and C both represent the value of X in a particular state, such as the normal steady state of a patient, then $X_1(t) = A$ maps into $X_2(t) = (0, C)$.

- A more complex version of this mapping problem arises when shifting a monotonic function constraint in a slow mechanism that abstracts the effect of change in a faster mechanism (figure 6). The corresponding values of an abstracted constraint depend on the prevailing assumptions about the values of other independent variables in the fast mechanism. The fast mechanism must store multiple sets of corresponding values, indexed under the prevailing assumptions.

  For example, for the monotonic function constraint $AWP = M^+(AWP)$ illustrated in figure 6, which is abstracted from the water balance mechanism, the corresponding values depend on the independent variable $NFWIP$:

$$\begin{array}{lll} NFWIP = NFWIP^* & \Rightarrow & (0,0), (ANP^*, AWP^*), (\infty, \infty) \\ NFWIP = NFWIP^+ & \Rightarrow & (0,0), (ANP^*, AWP^+), (\infty, \infty) \end{array}$$

These landmarks are defined in the quantity spaces of the water balance mechanism and must be mapped, as described above, into the quantity spaces of the sodium balance mechanism.

3.4 Limitations and Extensions

There are several limitations to the current implementation of time-scale abstraction that point the way to future extensions, or to the fundamental limits of the method.

As can be seen in Figure 5, when the attention of the time-scale abstraction procedure is focused on a slower mechanism, the faster mechanisms are assumed to be following along, with no remarkable changes taking place. This is a heuristic assumption, and can prove wrong if the fast mechanism has been "dragged across" a significant landmark by the slow mechanism. For example, a slow change may bring the fast mechanism to the point where a container overflows or a cable snaps, without any qualitative change at the slow level. The current procedure needs to be augmented with methods for determining whether such changes could have taken place, and for backing up to consider them more carefully.

At the current time, the component mechanisms of the time-scale abstraction hierarchy are first-order equilibrium systems which respond to a perturbation by moving to a new steady state and becoming quiescent. As it becomes more feasible to do qualitative simulation with second-order QDEs, we need to extend our methods for abstracting fast behaviors. For example, a rapidly responding mechanism governed by an integral controller may be abstracted to a *constant* at a slower level. Similarly, a small, rapid, continuing oscillation about some value should probably be abstracted to a constant, from a slower perspective.

One of the strengths of qualitative simulation methods is its ability to reason with non-linear systems, so long as the non-linear relations are confined to monotonic function constraints (M^+ and M^-). However, non-linear systems in general may have highly complex and counterintuitive behaviors. For example, a time-scale decomposition might fail because two component mechanisms swap relative time-scales [de Kleer, personal communication]. One intuitive explanation that has been offered for such phenomena [Gleick, 1987], is that the non-linearity (more specifically, non-monotonicity) of certain relations means that transient behavior at a fast time-scale that would otherwise damp out and be encapsulated in a fast mechanism, can suddenly amplify to have significant effects on slower mechanisms. These methods for reasoning across time-scales apply only when the separation between time-scales is large and stable.

4 Related Work

It is not a new idea, of course, that mechanisms may operate on multiple time-scales, and can profitably be analyzed from that perspective.

Simon and Ando [1961] discuss methods for analyzing the relative magnitudes of coefficients in an influence matrix to determine the time-scale structure of an "almost-decomposable" system of linear equations. Chin, Hedstrom, and Howes [1985] and Kreiss [1985] discuss a variety of analytic and numerical methods applying to ordinary and partial differential equations to determine and exploit knowledge of relative time-scales. Compared with these numerical and analytical approaches, the contribution of our work is to show how time-scale relations can be exploited in the context of incomplete knowledge of a complex system, where qualitative relations are available and reliable while quantitative knowledge may be unavailable or unreliable.

De Kleer and Brown's [1984] qualitative physics with confluences is based on a quasi-equilibrium assumption: that a system can be considered to be at, or infinitely close to, equilibrium at all times. Under this assumption, true transient behavior can be ignored, and the only changes are shifts of equilibrium in response to perturbations. This is a special case of time-scale abstraction, looking at fast mechanisms from the point of view of a slower time-scale. We take their perspective on the downward sweeps (cf. figure 5) to determine the state of faster mechanisms once a slower mechanism has reached equilibrium.

Weld [1986] developed a method called *aggregation*, whereby the discrete net effect of one cycle of a repetitive process can be viewed from a different scale as an incremental change in a continuous process. For example, if a catalyst causes a reaction that consumes one molecule of a substance, and the catalyst is then freed to repeat the cycle, we can step back to view the amount of the substance as a continuously decreasing quantity, amenable to the usual limit analysis methods. Aggregation is another important type of time-scale reasoning, especially since it bridges a gap between discrete and continuous descriptions. From our current perspective, one setting where aggregation is likely to be particularly useful is qualitative reasoning about damped oscillations (e.g. integral controllers).

5 Conclusions

In the medical physiology domains we have discussed, the natural system appears to have a suitable modular structure for imposing a time-scale hierarchy. This is not necessarily always the case. Perrow [1984] argues that certain engineered systems such as nuclear power plants are simply too complex and highly interactive for human comprehension, especially under emergency circumstances. For some systems, we suspect that the modularity by time-scale necessary for this kind of hierarchical structure does not exist, and cannot validly be imposed.

In this paper, we have presented methods for qualitative simulation of complex systems that can be structured as time-scale hierarchies of interacting mechanisms. Another important application of time-scale abstraction, discussed in [Kuipers, 1987], is the use of the abstracted view of a process to determine the cause of a branching behavioral prediction, identifying a new distinction in the quantity space of some independent variable, and making the simulation deterministic.

We believe that these results, along with other recent developments in qualitative simulation (e.g. Williams [1986], Weld [1986], and Kuipers and Chiu [1987]), are significant steps towards robust qualitative reasoning methods capable of being applied to complex problems in the real world.

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