Toward a Reformalization of QSIM

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Abstract

A more detailed formalization of part of the QSIM framework has been found to be useful. That formalization is presented here in brief. The new formalization makes it easier to make and prove some useful statements. The formalization of temporal reasoning about continuous systems [7] and the precise definition of completeness are examples of the usefulness of this formalization.

1 Introduction

In many applications in which ordinary differential equations (ODEs) are used, complete information about initial conditions or the specific relationship between a pair of quantities is not completely known. In some cases constants are known to lie in a certain range or the relationship between quantities is only known to be monotonic. Qualitative reasoning allows this information to be used to generate descriptions of solutions to any ODE which abstracts to the known information. We call such an abstract ODE a qualitative differential equation (QDE). In many such applications we want to draw conclusions about the solution to *any* ODE consistent with the limited information we have about a system. That is exactly where the QSIM framework helps.

The purpose of this report is to reformalize part of the framework of the Guaranteed Coverage Theorem for QSIM. The intention is not to reformalize every detail of the proof but merely to mention some areas whose reformalization lends deeper understanding and clarity.

Most of the paper consists of definitions which are all used in the statement of the Guaranteed Coverage Theorem. Therefore, we will not be able to state the theorem carefully until the end of the report. However, in order to motivate the definitions, it will be useful if the reader has an idea about what the theorem says. Therefore, we will use loose statements of the theorem throughout the report and these statements should not be considered to have formal meaning but only to be lending understanding. Here is our first loose statement of the theorem: given an ODE, F, which abstracts to a QDE, C, if QSIM produces the tree M from C, then there is a path in M which describes the solution to F. This theorem is important because it guarantees that the tree produced by QSIM does indeed describe every solution to any ODE which abstracts to the QDE which produced the tree.

First we describe the relationship between an ODE, F, and a set, F', of simultaneous equations and how to pass from one to the other. Given an ODE, there is a unique set of simultaneous equations associated with it modulo renaming variables. Conversely, given a set of simultaneous equations there is a unique ODE associated with it modulo renaming variables.

Any set of simultaneous equations obtained in this way can be "abstracted" to a QDE. We call the QDE an "abstraction" of the associated ODE. The relationship between ODEs and QDEs is not a bijection. Generally, there are a vast number of ODEs which abstract to a given QDE. In applications, due to incomplete knowledge of the system, an ODE cannot be constructed. However, there is enough information to construct a QDE which is an abstraction of the ODE.

The QSIM algorithm takes a QDE and a qualitative initial state (i.e. a specification of the values of the variables) and produces a tree-like structure of qualitative states. Details of the QSIM algorithm itself will not be discussed in this paper.

If there are quantitative information or functional envelopes given in the QDE or initial state, then the quantitative extensions to QSIM are applied to add this information to the states and branches of the tree.

Therefore, the QSIM user can deal with QDEs and gain information about the solution to any ODE which abstracts to the given QDE. This has great advantages in applications in which complete quantitative and functional information cannot be gathered.

An appendix defines notions of completeness for QSIM.

2 The QSIM Framework

2.1 A Refinement to QSIM

The structure constructed by the QSIM algorithm is guaranteed to represent any "reasonable" solution to ODEs which abstract to the given QDE.

Here we give a refinement of the definition of a reasonable function. This refinement does not change any of the soundness results in the QSIM literature. On the contrary, use of this new definition simplifies the QSIM algorithm slightly and can allow the detection of more spurious behaviors.

Here R denotes the reals and R^* denotes the extended reals.

Definition 1 (Reasonable Function) *Where* $[a, b] \subset R^*$, *the function* $f : [a, b] \rightarrow R^*$ *is a* reasonable function over $[a, b]$ *if*

1. f *is continuous on* $[a, b]$,
2. f *is continuously differentiable on* (a, b) *with derivative* f',
3. f *has only finitely many critical points in any bounded interval of* R^*, *and*
4. *the one-sided limits* $\lim_{t \rightarrow a^+} f'(t)$ *and* $\lim_{t \rightarrow b^-} f'(t)$ *exist. Furthermore,* $\lim_{t \rightarrow a^+} f'(t) \in R$ *and if* $b \in R$ *then* $\lim_{t \rightarrow b^-} f'(t) \in R$. *Define* $f'(a)$ *and* $f'(b)$ *to be equal to these limits.*

Readers familiar with the QSIM literature will notice that the only difference between this definition and the traditional one is that we require that the derivatives at the endpoints be finite with the exception that if $b = \infty$ then $f'(b)$ may be infinite.

One consequence of the use of this definition is that a non-quiescent behavior can only become quiescent at infinite time. This result agrees with the uniqueness theorem for differential equations.

2.2 The Abstraction of an ODE

Given an ODE, there is a unique set of simultaneous equations associated with it modulo renaming variables. Conversely, given a set of simultaneous equations there is a unique ODE associated with it modulo renaming variables.

The advantage of referring to the set of simultaneous equations associated with an ODE is that a set of simultaneous equations can be easily abstracted to a QDE. We call this QDE the abstraction of the ODE. We say that the ODE *abstracts to* the QDE.

ODE and Simultaneous Equations There is an algorithm which takes any ODE, F, and translates it into a set, F', of simultaneous equations. (See Section 3.3.1 of Kuipers' *Qualitative Reasoning* [6].) This set, F', of equations can likewise be translated back to F. Given a solution, U, of F', a similar procedure translates U into a solution to F.

The variables which are introduced in F' stand for functions of time. A solution, U, to the set of simultaneous equations consists of a set containing specific functions of time for each variable in F' such that if the function is substituted for the variable, the set of equations is true. By applying the inverse of the process which constructed F' from F, the set U becomes a function which is the solution to F.

The Abstraction Mapping The set of simultaneous equations is useful because it is easy to translate it into a QDE by abstraction. The transformation of the set of simultaneous equations into a QDE provides a bijection between the variables in the set of simultaneous equations and the set of variables in the QDE. The QSIM algorithm takes a QDE as input and produces a behavior tree M. The Guaranteed Coverage Theorem says that the solution, U, to F' is described in M. We let $\psi_{U,M}$ denote the bijection from the set U to the set of variables in M. Recall that we use U to refer to the solution to F', thus U is in correspondence with the set of variables in F'. Since there are many ODEs which may abstract to a single QDE, we can only mention the mapping $\psi_{U,M}$ when we have a particular ODE or set of simultaneous equations in mind. Conversely, for any QDE, C, if F is an ODE such that F' abstracts to C, then we will use $\psi_{U,M}$ again to map the variables in the set of simultaneous equations to the set of variables in C.

The algorithm which produces F' from F is such that the top-level operator in each equation in F' is associated with some QSIM constraint. Thus F' is translated into a QDE. (See Section 3.3.1 of Kuipers' *Qualitative Reasoning* [6].)

There are constraints which provide for the translation of multivariate functions so that complex ODEs can be handled. Additional information can be encoded by the user into the QDE and initial state. More flexible and powerful constraints are still being added to the QSIM framework. As this work continues, the class of ODE which can be abstracted into QDEs is widened. See Kuipers' *Qualitative Reasoning* for a more complete description of the various constraints. Whether an ODE is in the domain of the QSIM framework can be ascertained by considering whether the operators in the ODE can be abstracted to QSIM constraints.

What the user generally wants to do is find a QDE which contains all of the information he has so that the solution to every ODE which abstracts to the QDE will be interesting to him.

The set of simultaneous equations is thus abstracted into a QDE. We call this QDE an abstraction of the original ODE, F. This abstraction provides a bijection between the set of simultaneous equations and the set of constraints in the QDE and a bijection, $\psi_{U,M}$ between the variables in the set of simultaneous equations and the variables in the QDE.

2.3 Semantics of QSIM Trees

QDE to QSIM tree A *qualitative differential equation* (QDE) consists of a finite set of *variables* (each of which is associated with a *quantity space* which is a totally-ordered set of *landmarks*), and a set of constraints on the values of the variables. A QDE is a structural abstraction of a class of ordinary differential equations (ODEs). The QDE codifies the QSIM user's incomplete knowledge of a physical system.

The QSIM algorithm takes a QDE, C, and an initial state, I, and produces a tree, M, of qualitative states. We call I the root state of the tree. Each state assigns a value to each variable in the QDE.

Starting with a QDE, C, and an initial state, I, qualitative simulation with QSIM produces a finite tree, M, of *qualitative states*, linked by the QSIM *successor* relation, R. The finite tree which it produces is called a *QSIM behavior tree* in the literature. We let $M = \langle S, B \rangle$ represent the behavior tree where S is the set of states, B is the set of fullpaths (represented as finite, totally-ordered sets). We will say that this QSIM behavior tree was *generated by* the pair $\langle C, I \rangle$ of the QDE and the initial state. A *QSIM behavior* is a path in the behavior tree, starting at the root and terminating at a leaf of the tree, i.e. B is the set of QSIM behaviors in M. Each state describes the *qualitative value* of each *variable* appearing in the QDE model. Each variable will represent a function of time. The qualitative value of a variable v over a state s is of the form $\langle qmag, qdir \rangle$, where *qmag* describes the magnitude of v as equal to a landmark or in an open interval defined by two landmarks, and *qdir* is the sign of the derivative of v. By considering the qualitative values of the variables at a state, and the constraints in the QDE, QSIM is able to derive a number of properties of the states and behaviors, including quiescence, stability and cycles.

A QSIM state is called a *transition state* if it has no R-successors due to the fact that the value of one of its variables crosses a boundary of the QDE description. QSIM allows the user to produce *transition relations* between a transition state in one tree and the root of a tree generated by another QDE. This allows the user to produce a tree which has different models for its behavior in different ranges. The theorems here could be extended to take transition relations into account, however, the extension is tedious and unenlightening so, in the theorems in this paper, we assume that transition states have no successors.

Closed Trees Ideally, given a QDE, the QSIM algorithm will terminate, not because it runs out of memory or other resources, but because it has finished simulating all possible behaviors. When the QSIM algorithm terminates in this "natural" way, we call the tree it produces *closed.* In this case, every behavior in the behavior tree returned by QSIM terminates with a state which is a transition state, a cycle state or a quiescent state. There are cases, however, in which QSIM does not return a closed tree regardless of how long it is allowed to run. In cases where QSIM returns a tree which is not closed, the hypotheses of the Guaranteed Coverage Theorem do not hold. If the behavior tree M is not closed then it is possible that an actual behavior of the system is not represented by any rooted fullpath in the represented structure $\widehat{M}$.

The normal QSIM simulation style creates new landmarks for critical values, applies a strong cycle-match criterion (all variables must have identical landmark values), and does not allow *cross-edge* cycles (i.e. considers cycle matches only within the same QSIM behavior.) Under this simulation style, certain systems such as the damped spring never close. However, by applying the *envisionment* simulation style (no new landmarks, weak cycle-match criterion, and cycle matches anywhere in the behavior tree), *every* qualitative model has a finite closed behavior tree. (See Chapter 5 of Kuipers' *Qualitative Reasoning* [6].)

The Represented QSIM Structure Due to the fact that some QSIM behaviors terminate at cycle states, we do not want to claim that it is QSIM behaviors (i.e. elements of B) which describe the solutions to ODE, rather we want to reason about paths which consider cycle states to have successors. Therefore, we define the structure, $\widehat{M} = \langle S, X\rangle$, *represented by* the QSIM tree, $M = \langle S, B\rangle$, to take cycles into account.

QSIM may use various matching criteria when it detects cycles. The *strong-match* criterion requires that the value of each variable in the states to be matched is a landmark (rather than an interval) and that those landmarks match the values of the variables in the previously-existing state (the qualitative derivatives have to match regardless of the match criterion). The *weak-match* criterion allows a match when the values are intervals or landmarks. The QSIM user may also dictate whether cycles are detected across QSIM behaviors (cross-edge cycles) or only on the same QSIM behavior. The type of cycle detection chosen makes a difference in the structure represented by the tree.

If we are using strong-match cycle detection and no cross-edge cycles, then when a cycle is detected, the system has returned to a state where it has been before; by the uniqueness theorem for differential equations, it must continue on the exact path which it followed previously. Therefore, when using strong-match and no cross-edge cycle detection a cycle behavior represents a single fullpath.

We begin by constructing a set, X^r, which is called the set of *rooted fullpaths*. X will be the suffix closure of X^r. The paths in X^r will all begin at the root of the QSIM tree but will not terminate at cycle states. The resulting X may not be fusion closed. A set X is fusion closed if, whenever $x_1sy_1, x_2sy_2 \in X$, then $x_1sy_2 \in X$. See Emerson's survey article for a review of these issues [2]. A behavior $b \in B$ is in X^r if b terminates in a state which is not a cycle state. If the tree M was constructed applying the strong-match criterion with no cross-edge matches, then for each behavior $b = \langle s_0, s_1, \ldots, s_i\rangle$ in B such that s_i is a cycle state which matches some state s', there is a state s_j which is the successor of s' lying in b. In this case, the path $\langle s_0, \ldots, s_i, s_j, \ldots, s_i, s_j, \ldots\rangle$ is added to the set X^r. If weak-cycle detection was used or cross-edge cycles were allowed, then X^r is constructed by including every path, y, such that for some $b \in B, b$ is a prefix of y and for every pair of consecutive states $\langle s, s'\rangle$ in y, s and s' are related by the QSIM successor relation or s is a cycle state and s' is a successor to the state which s matches. The reason for including only this small set of rooted fullpaths in strong-match structures is the uniqueness theorem for differential equations as mentioned previously.

The set X of fullpaths is the suffix-closure of X^r. The structure $\widehat{M} = \langle S, X \rangle$ is called the structure *represented by* the QSIM behavior tree $M = \langle S, B \rangle$.

The set, X, of fullpaths is not necessarily fusion closed and hence not R-generable. (A set X is R-generable if there exists a total binary relation R on S such that a path $x = \langle s_0, s_1, s_2, \ldots \rangle \in X$ if and only if for every nonnegative integer i, $\langle s_i, s_{i+1} \rangle \in R$.) X will certainly be R-generable if weak-match of cross-edge cycle detection was used.

Notice that in the case of strong-cycle matching and no cross-edge matching, there are exactly as many members of X^r as there are of B. Furthermore, in this case, for any fullpath (i.e. an element of X) there is a unique rooted fullpath of which it is a suffix. Since a rooted fullpath in such a tree is associated with exactly one behavior in B, we define the function $z : X^r \to B$ to map a fullpath to its associated behavior. For trees with other kinds of cycle detection, the the value of the function z is unimportant. This function allows us to use the numeric information (which is really associated with a behavior) in the context of a general fullpath.

Since, in strong-match cases, any fullpath in X is a suffix of only one rooted fullpath, we can extend z so that for any $x \in X, z(x) = z(x_r)$ where x_r is the rooted fullpath of which x is a suffix. We think of z as a map from a fullpath to its associated QSIM behavior. This assignment will be useful when we prove properties of systems about which we have some quantitative information.

The Guaranteed Coverage Theorem will refer to rooted fullpaths. It is elements of X^r which we want to describe the solutions to ODEs.

Quantitative Information When strong-cycle matching is used and cross-edge cycles are not allowed, then the QDE and the initial state may be augmented with quantitative information such as numerical interval bounds on the real values denoted by landmarks and other symbolic terms in the behavior prediction [1, 3, 5, 6]. In this case, QSIM propogates this quantitative information and uses it to prune branches of the tree which are inconsistent with the information. The most important quantitative information for the purposes of this paper is the information which QSIM derives about the landmarks.

A landmark in a quantity space of a QSIM variable is intended to name some real number. The quantitative extensions to QSIM are able to restrict the possible value of a landmark on a behavior to some closed, extended-real interval. This quantitative information may be different on each QSIM behavior. That is to say that on different QSIM behaviors, a given landmark may name different real numbers. Therefore, if weak-cycle matching is used or cross-edge cycles are detected, then the numeric information loses its sense because there is no notion of the unique behavior associated with a given fullpath.

2.4 Describing Functions

For the statement of the Guaranteed Coverage Theorem we need to have a formal way of relating fullpaths in the tree with the underlying reasonable, extended real valued functions which they are meant to be describing. For this purpose, we will abstract the functions by means of a *splitting* and we will "instantiate" the qualitative fullpath by means of a *specification*. We will say that a specification of a fullpath qualitatively describes a splitting of a set of functions if the two match as outlined below.

Splitting It has already been mentioned that a fullpath in $\widehat{M}$ is meant to *describe* a set, U, of reasonable, extended real valued functions which constitutes a solution to a set of simultaneous equations which are equivalent to some ODE which abstracts to the QDE which generated M. Therefore, we will define what it means for such a set of functions such as U to be described by a fullpath. For this purpose we will define a *splitting* of a set of reasonable, extended real valued functions. Each of the functions in U is thought of as a function of time and therefore they share a domain. The splitting consists of a partition of the domain of the functions in U.

We are given a set of reasonable, extended real valued functions $U = \{u_i : 1 \leq i \leq n\}$. Since we are thinking of U as a solution to a set of simultaneous equations which was derived from an ODE, we will assume that each of the functions in U shares the domain, A, some interval in the extended reals. We define a *splitting* of U as follows. Let $\{t_i\}$ be a strictly increasing sequence of points (indexed from 0) in A satisfying the following conditions: 1) $\{t :$ for some k, t is a critical point of $u_k\} \subset \{t_i\}$, 2) $\{t_i\}$ has no finite limit point and 3) $\{t_i\}$ converges to ∞ only if $\infty \notin A$. According to the definition of a reasonable function, such a set exists, and may be infinite only if ∞ is the supremum of A and A is open on the right. We will call the pair $\langle\{t_i\}, U\rangle$ a *splitting* of U. Since the critical points of the functions all must be in $\{t_i\}$, a splitting is distinguished from another splitting by the choice of non-critical points in $\{t_i\}$. If the set $\{t_i\}$ is finite, then we will let t_J be the greatest element so that $J+1$ is the cardinality of $\{t_i\}$.

Associated with any splitting $\langle\{t_i\}, U\rangle$ there is a natural partition $\{D_k\}$ of the interval A. Each D_k is either a singleton containing one of the points in $\{t_i\}$ or an open interval whose endpoints are two consecutive points in $\{t_i\}$. The indexes on the sets in the partition follow the order of the indexes in $\{t_i\}$. That is, if A is closed on the left then $D_0 = \{t_0\}, D_1 = (t_0, t_1)$, and so on. If A is open on the left then $D_0 = (a, t_0), D_1 = \{t_0\}, D_2 = (t_0, t_1)$, and so on, where a is the infimum of A. If A is closed on the right then $D_{2J} = \{t_J\}$ (or $D_{2J+1} = \{t_J\}$ if A is open on the left). If A is open on the right, then as we have said, there may be infinitely many sets in $\{D_k\}$.

When dealing with a splitting, $\langle\{t_i\}, U\rangle$, of U, we will use the following abbreviations:

$$\begin{aligned} U|_l &= \{u_i|_{D_l} : 1 \le i \le n\} \\ U|_{l+} &= \{u_i|_{\bigcup_{l \le k}\{D_k\}} : 1 \le i \le n\} \end{aligned}$$

That is to say that $U|_l$ is the set of functions in U each restricted to the domain D_l and $U|_{l+}$ is the set of functions in U each restricted to the domain $\bigcup_{l\le k}\{D_k\}$.

Specification In order to define the notion of a splitting being *described* by a fullpath, we will need to relate the landmarks in the fullpath to specific extended real numbers. We call a mapping of landmarks to extended real numbers satisfying certain sensible conditions, a *specification* of the landmarks. We will want the extended real numbers to which the specification assigns landmarks to be consistent with the order in which the landmarks occur in the quantity spaces of the variables of which they are values. Also if quantitative information is assigned on a fullpath, we want the mapping to be consistent with that information.

A function, c, with domain, some partially ordered set, V, of landmarks and range, R^*, is called a *specification* of V if c preserves the partial order and $c(\mathtt{minf}) = -\infty, c(\mathtt{inf}) = \infty$ and $c(0) = 0$.

Given a fullpath x and a specification, c, of the landmarks of the variables in x under the partial order determined by the quantity spaces of the variables in x, we call the pair $\langle x, c\rangle$ a *specification of* x if c is also consistent with any numeric information which might be associated with $z(x)$. (Recall, $z(x)$ is the QSIM behavior associated with x when such can be determined uniquely.) In particular, if, on the behavior $z(x)$, the qualitative landmark, `X1`, has been determined, by a quantitative extension to QSIM, to refer to a number in the numeric range $[n_1, n_2]$, then $c(\mathtt{X1}) \in [n_1, n_2]$.

Definition of Qualitatively Describes Given a specification $\langle x, c\rangle$ of a fullpath $x = \langle s_0, s_1, \ldots\rangle$ in a QSIM directed graph $\widehat{M}$ and a splitting $\langle\{t_i\}, U\rangle$ of the set of reasonable, extended real valued functions $U = \{u_i : 1 \le i \le n\}$ with common domain, A, we say that $\langle x, c\rangle$ *qualitatively describes* the splitting if the specification corresponds to the splitting as described in detail in the remainder of this subsection.

The intention is that the partition, $\{D_k\}$, of the domain, A, determined by the splitting will correspond to the range of the time variable in the fullpath x in such a way that each element, D_k, of the partition will correspond to the value of the time variable in the state s_k and thus s_k will correspond to $U|_k$.

The function $\psi_{U,M}$ referred to below is the bijection which relates the variables in M with the variables in U.

The cardinality of $\{t_i\}$ must equal the number of time-point states in x. Therefore, since the states in x alternate between time-point states and time-interval states, $\langle x, c\rangle$ qualitatively describes $\langle\{t_i\}, U\rangle$ if and only if there is a correspondence between x and U in which s_k corresponds to $U|_k$ as follows:

- Given any landmark `X1` of a QSIM variable $\psi_{U,M}(u_i)$ and any nonnegative integer k, the qualitative value of $\psi_{U,M}(u_i)$ at the state s_k is `X1` if and only if $u_i|_{D_k} = c(\texttt{X1})$.
- $u_i'|_{D_k} > 0$ if and only if the qualitative derivative of the variable $\psi_{U,M}(u_i)$ in s_k is `inc`.
- $u_i'|_{D_k} = 0$ if and only if the qualitative derivative of the variable $\psi_{U,M}(u_i)$ in s_k is `std`.
- $u_i'|_{D_k} < 0$ if and only if the qualitative derivative of the variable $\psi_{U,M}(u_i)$ in s_k is `dec`.

Lemma 1 *The specification $\langle x, c\rangle$ qualitatively describes $\langle\{t_i\}, U\rangle$ if and only if $\langle x^h, c\rangle$ qualitatively describes $\langle\{t_i : h \le i\}, U|_{h+}\rangle$ for every nonnegative integer $h < \Lambda(x)$.*

Proof: The proof comes straight from the definition of what it means for a specification of a fullpath to describe qualitatively a splitting of a set of functions. This definition involves a correspondence between s_h with D_h for every nonnegative integer $h < \Lambda(x)$.

□

It follows from the definition that if $\langle x, c\rangle$ qualitatively describes a splitting $\langle\{t_i\}, U\rangle$ of a set of reasonable functions, then given any pair of landmarks, `X0` and `X1`, of a QSIM variable $v = \psi(u_i)$ and any nonnegative integer k, if qualitative value of v at the state s_k is `(X0 X1)` then $u_i(D_k) \subset (c(\texttt{X0}), c(\texttt{X1}))$. It also follows that the state s_k has been determined by QSIM to be quiescent if and only if $u_i'|_{D_k} = 0$ for each $1 \le i \le n$ and that the fullpath x is finite and QSIM has labeled the last state in x with `t=inf` if and only if $t_J = \infty$.

3 The Guaranteed Coverage Theorem

The QSIM algorithm is carefully crafted to guarantee that all reasonable functions which are solutions to an ODE which abstracts to C are described by some rooted fullpath. This is the idea behind the Guaranteed Coverage Theorem.

Theorem 1 (Guaranteed Coverage) *Let F be an ordinary differential equation with solution U, a set of reasonable, extended real valued functions. Let $\langle C, I\rangle$ be a QDE and initial state to which F abstracts. Let M be a closed QSIM behavior tree generated by the QSIM algorithm applied to $\langle C, I\rangle$. Then there is a rooted fullpath x in $\widehat{M}$ and a specification $\langle x, c\rangle$ of x such that $\langle x, c\rangle$ qualitatively describes some splitting $\langle\{t_i\}, U\rangle$ of U.*

Proof: The proof is given by Kuipers [4, 6]. Because most of the QSIM framework–the algorithm itself, for example–is beyond the scope of this report, we will not detail the proof here.

□

A Complete QSIM Trees

The notion of the completeness of a QSIM tree is useful.

Definition 2 *We call a closed QSIM behavior tree, M,* complete *if for every rooted fullpath, x in $\widehat{M}$ starting at the root, there is an ODE, $F = 0$ such that $F = 0$ is consistent with the input QDE and a splitting of the solution to $F = 0$ is qualitatively described by some specification of x.*

One way to check for the completeness of a tree is to prove, either mathematically or by numeric simulation, that there is a reasonable, extended real valued solution corresponding to each fullpath in the directed graph represented by the tree.

We could call this notion of completeness "weak completeness" and define a stronger notion of strong completeness. A tree M generated from C is *strongly complete* if for every rooted fullpath, x, and every specification, $\langle x, c\rangle$ of x, there is an ODE, F which abstracts to C such that $\langle x, c\rangle$ describes some splitting of the solution to F. It is questionable whether any but the most trivial trees satisfy this condition.

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