Distributed source coding: Symmetric rates and applications to sensor networks. †

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Abstract

We address the problem of distributed source coding using a practical and constructive approach [1], referred to as Distributed source coding using syndromes (DISCUS), with applications to sensor networks. We propose low complexity encoding and decoding methods based on linear codes, to achieve all points in the achievable rate region of Slepian-Wolf [2] problem. The extension of these concepts to the construction of Euclidean-space codes is also studied and analyzed for the case of trellis and lattice codes. The performance of these symmetric methods for encoding with a fidelity criterion is shown to be the same as that of asymmetric encoding. Simulations are presented to corroborate these results.

1 Introduction

Many phenomena of interest can be described as events occurring between some intervals in space and time. Since most sensing systems are typically spatially localized, it is desirable to have a multitude of such sensors which are distributed spatially, to capture these events. These sensors are typically constructed to operate under stringent constraints on system resources like bandwidth and power. They therefore need to encode their signals with maximal efficiency for transmission to a remote central decoding unit. The signals captured by these sensors are inherently redundant as the events typically have high spatio-temporal correlations.

This justifies the need for an inter-sensor communication network aimed at removing this redundancy in order to have an efficient joint description of the data of all sensors. Such a network itself might be a heavy burden on the system resources. This highlights the need for distributed non-collaborative compression techniques that exploit the inter-sensor correlation while obviating the need for (or at least lightening the load on) an elaborate inter-sensor network.

Consider the sensor communication system shown in Fig. 1. Here, a number of sensors observe an event, which is characterized by the signal X. The sensors observe a noisy version of this event[1], represented by the signal set $\{Y_i\}$ for i sensors. The sensors transmit their signals efficiently to a joint decoder, which forms an optimal reconstruction of X based on the information received from all the sensors. The optimal fusion of information at the decoder involves both coding and estimation. This problem was first analyzed by Flynn and Gray in [3], where an achievable rate region for an optimal encoding system was obtained. This concept of distributed source coding was first introduced in [2], where it was shown that if X and Y are two correlated memoryless discrete sources, they can be separately compressed at a rate approaching their joint entropy [4]. This was extended in [5] to the

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[1]In this paper we restrict ourselves to this setup, though more complex models can also be treated.

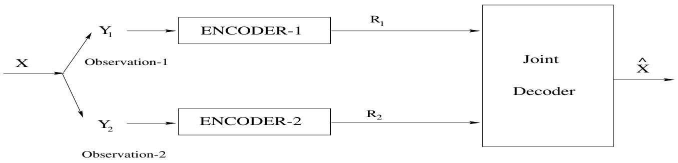


Figure 1: Sensor network communication system: Encoders observe corrupted version of the source X, and transmit their information to the decoder whose task is to get the best estimate, $\hat{X}$, of X. The encoders do not communicate with each other.

case of encoding of a source in the presence of a correlated source (side information) at the decoder.

A practical design of encoders for a class of such problems was considered in [1], where the potential to approach the gains promised by theory was exposed. This approach is referred to as Distributed Source Coding Using Syndromes (DISCUS). In [1], an asymmetric version of the problem was considered, where one of the sensors is considered as a "primary source", and the other sensor is considered as a "secondary" source[2].

In practice, it may be desirable to have more flexibility in transmission rates and include the case of symmetric encoding, where all sensors send only partial information to the decoder, without compromising the reconstructed signal quality the decoder. One solution to this is to do time-division-multiplexing between the sensors so that at any time, one of the sensors will be acting as a primary source. This requires synchronism between the sensors and the encoders need to switch between these operating modes. In this work, we provide a practical and constructive framework for encoding of correlated observations in both asymmetric and symmetric encoder settings. This framework can easily draw from the large class of coding strategies involving block-channel codes on GF(q), trellis codes, lattice codes and multi-level codes etc.

The paper is organized as follows. In section 2, we introduce the concept using discrete binary sources. In Section 3 and 4, we extend these concepts to Euclidean space codes. Section 5 concludes the paper with simulation results and discussions.

2 Preliminaries and Illustration

Let us consider the following key illustrative example as motivation, whose generalization will be considered later. Let X and Y be two discrete memoryless uniformly distributed 7-bit binary random variables which are correlated in the following way: $d_H(X,Y) \leq 1$, where $d_H(.,.)$ is the Hamming distance. The entropies of X and Y and their joint entropies are given by $H(X) = H(Y) = 7$ bits, and $H(X,Y) = H(Y) + H(X|Y) = 10$ bits. According to Slepian and Wolf [2], X and Y can be separately compressed at rates R_1 and R_2 respectively such that $R_1 \geq 3, R_2 \geq 3, R_1 + R_2 \geq 10$. The achievable rate region is given in Fig. 2. Consider the point A as shown in Fig. 2. This vertex point (corresponding to $R_1 = 3$ and $R_2 = 7$ bits) can be achieved by using the $(7,4,3)$ binary Hamming code, denoted by $\mathcal{C}$, with

[2] In this paper we consider the network with two sensors, for ease of illustration of the concepts. Extension to multiple sensors is straightforward.

$\mathbf{H}$ as its parity check matrix. Y can send its complete data $\mathbf{y}$ using 7 bits of information. X can send the syndrome given by $\mathbf{s} = \mathbf{Hx}$, where $\mathbf{x}$ is the outcome of X, thus needing 3 bits. Clearly with the availability of $\mathbf{y}$ and $\mathbf{s}$ at the decoder, one can recover the value of X since we have $d_H(X, Y) \leq 1$. Similarly the point B can be achieved by reversing the roles of X and Y. Any point on the line AB can be achieved by suitably time-division multiplexing between these two modes of operation. In [6], a source-splitting approach has been taken to achieve these non-vertex points. This approach is asymptotic and non-constructive and involves the splitting of the sources into sub-sources having a lower entropy. We now examine whether other rate pairs, (R_1, R_2) are attainable. Consider the point C in Fig. 2

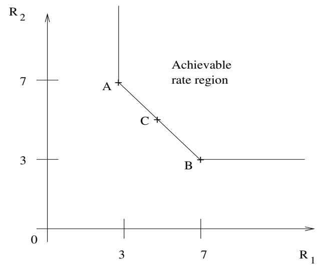


Figure 2: Achievable rate region of two correlated sources for the "Hamming code" example: any rate pair (R_1, R_2) can be achieved as long as $R_1, R_2 \geq 3$ and $R_1 + R_2 \geq 10$. The sources considered here are $X, Y \in \{0, 1\}^7$ and $d_H(X, Y) \leq 1$.

corresponding to $R_1 = R_2 = 5$ bits. We need to build an encoder-decoder pair such that X and Y individually transmit 5 bits of information, and the decoder, having received these 10 bits, should be able to recover X and Y errorlessly. We use linear codes for this system, similar to the ones used in [1]. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two linear binary codes with $\mathbf{H_1}$ and $\mathbf{H_2}$ as their parity check matrices respectively, which are associated with X and Y respectively.

Proposition: There exists a linear code pair $\mathcal{C}_1$ and $\mathcal{C}_2$ and an encoder-decoder pair based on linear codes such that the decoder can recover X and Y perfectly. The encoder is given by: $\mathbf{s_1} = \mathbf{H_1 x}$ and $\mathbf{s_2} = \mathbf{H_2 y}$ where $\mathbf{x}$ and $\mathbf{y}$ are the outcomes of X and Y respectively.

Proof: Let us consider the necessary conditions for the decoder to make an error. $\mathbf{s_1}$=$\mathbf{H_1 x_1}$ and $\mathbf{s_2}$=$\mathbf{H_2 y_1}$ are the syndromes of the cosets of $\mathcal{C}_1$ and $\mathcal{C}_2$ containing $\mathbf{x_1}$ and $\mathbf{y_1}$ respectively, where $\mathbf{x_1}$ and $\mathbf{y_1}$ are the outcomes of X and Y respectively. Suppose there exists another pair $\mathbf{x_2}$ and $\mathbf{y_2}$ with the same syndromes as those of $\mathbf{x_1}$ and $\mathbf{y_1}$ respectively, such that $d_H(\mathbf{x_2}, \mathbf{y_2}) \leq 1$; i.e., $\mathbf{H_1 x_1} = \mathbf{H_1 x_2}$ and $\mathbf{H_2 y_1} = \mathbf{H_2 y_2}$.

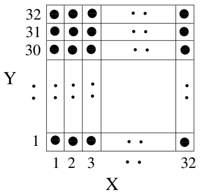
$$\mathbf{y_1} = \mathbf{x_1} + \mathbf{e_1} \text{ and } \mathbf{y_2} = \mathbf{x_2} + \mathbf{e_2}, \text{ where } \mathbf{e_1} \text{ and } \mathbf{e_2} \text{ are such that } w(\mathbf{e_1}), w(\mathbf{e_2}) \leq 1 \quad (1)$$

where $w(.)$ represents the Hamming weight. Thus, we have from (1):

$$\mathbf{x_1} + \mathbf{x_2} \in \mathcal{C}_1 \text{ and } \mathbf{x_1} + \mathbf{x_2} + \mathbf{e_3} \in \mathcal{C}_2 \quad (2)$$

where $\mathbf{e_3} = \mathbf{e_1} + \mathbf{e_2}$ and $w(\mathbf{e_3}) \leq 2$. Now if we ensure that one of the conditions in (2) is not satisfied then there will be no error. Consider the (4×7) generator matrix $\mathbf{G}$ of the binary Hamming code. Let $\mathbf{G_1}$ be formed by taking any two rows from $\mathbf{G}$ and let $\mathbf{G_2}$ be formed using the remaining two rows of $\mathbf{G}$. $\mathcal{C}_1$ and $\mathcal{C}_2$ are the subcodes of the Hamming code which has a minimum distance of 3. Thus the condition (2) cannot be satisfied, implying that the decoder can recover X and Y errorlessly. Q.E.D.

This concept is illustrated in Fig. 3. Each subcode has 32 cosets. Thus we need to distribute the all possible 2^{10} outcomes of (X, Y) in the coset matrix of the subcodes as shown. A given pair of cosets of these subcodes should determine uniquely the outcome of (X, Y).



(a)

$$\mathbf{G} = \left[\begin{array}{c} 1\,0\,1\,1\,0\,1\,0 \\ 0\,1\,0\,0\,1\,0\,1 \\ \hline 0\,1\,1\,0\,0\,1\,0 \\ 1\,1\,1\,0\,0\,0\,1 \end{array}\right] \begin{array}{c} G_1 \\ \\ G_2 \end{array}$$

(b)

Figure 3: (a) Distribution of 2^{10} outcomes of (X, Y) in the coset matrix of the subcodes. Each subcode has 32 cosets (b) Hamming Example: The generator matrix $\mathbf{G}$ is partitioned into two sub-matrices which are assigned to two encoders.

Decoder with a fast decoding algorithm: We need to search for a pair of codewords, one from each coset of $\mathcal{C}_1$ and $\mathcal{C}_2$, such that their distance is at most 1. Without loss of generality, let $\mathbf{H_1}$ and $\mathbf{H_2}$ (of dimension 5×7) be in their systematic form: $\mathbf{H_1} = [\mathbf{A_1}|\mathbf{I}]$ and $\mathbf{H_2} = [\mathbf{A_2}|\mathbf{I}]$. Thus we have $\mathbf{x} = \mathbf{u_1 G_1} + [\mathbf{0}|\mathbf{s_1}]$, $\mathbf{y} = \mathbf{u_2 G_2} + [\mathbf{0}|\mathbf{s_2}]$ and $\mathbf{y} = \mathbf{x} + \mathbf{e}$, where $w(\mathbf{e}) \leq 1$, $[\mathbf{0}|\mathbf{s_1}]$ and $[\mathbf{0}|\mathbf{s_2}]$ are the representatives of the cosets of $\mathcal{C}_1$ and $\mathcal{C}_2$ with syndromes $\mathbf{s_1}$ and $\mathbf{s_2}$ respectively and $\mathbf{u_1 G_1}$ and $\mathbf{u_2 G_2}$ represent arbitrary linear combinations of the rows of $\mathbf{G_1}$ and $\mathbf{G_2}$ respectively. We have $\mathbf{e} = \mathbf{uG} + [\mathbf{0}|\mathbf{s}]$ where $\mathbf{u} = [\mathbf{u_1}|\mathbf{u_2}]$ and $[\mathbf{0}|\mathbf{s}] = [\mathbf{0}|\mathbf{s_1}] + [\mathbf{0}|\mathbf{s_2}]$. As the minimum distance of the code is 3, the decoding strategy is to find the codeword nearest to $[\mathbf{0}|\mathbf{s}]$ in $\mathcal{C}$. Thus we can decode X and Y perfectly.

Thus using a linear code, we can achieve all the integral rate pairs on the line AB in Fig. 2 using the method described above. Thus for any rate pair: $R_1, R_2 \geq 3$ and $R_1 + R_2 = 10$ (let $R_1 \leq R_2$ without loss of generality), $\mathbf{G_1} = (7 - R_1) \times 7$ and $\mathbf{G_2} = (7 - R_2) \times 7$, where the rows of $\mathbf{G_1}$ and $\mathbf{G_2}$ are obtained from the disjoint partition of the rows of $\mathbf{G}$. The concepts developed here can be generalized to convolutional codes and codes on any $GF(q)$.

Theorem: Any pair of correlated uniformly distributed sources, $X, Y \in \{0,1\}^n$, with $d_H(X, Y) \leq t$, can be compressed separately at a rate pair (R_1, R_2) such that $R_1, R_2 \geq n - k$, $R_1 + R_2 \geq 2n - k$, and R_1 and R_2 are integers , where $k \leq n - log\left\{\sum_{i=0}^{t} \binom{n}{i}\right\}$ (sphere packing bound [7]). This can be achieved using an (n,k,2t+1) binary linear code.

3 Encoding with a fidelity criterion

Large classes of Euclidean space codes such as trellis codes, lattice codes [8] and multilevel codes [9] have a group structure [10]. Such codes are built using some generation process having a group structure such as generator polynomials in the case of trellis codes, generator vectors in the case of lattices and generator matrices in the case of multilevel codes built on block codes. Let us consider the problem of encoding of distributed sources with a *fidelity* *criterion*[3].

In this paper we consider the encoding of correlated observations as shown in Fig. 1. Let X be a random process observed by the sensors. Let Y_1, Y_2 be the random processes received by the sensors. We consider a specific model for the correlation between Y_1 and Y_2. Let X be characterized by discrete memoryless *i.i.d.* sequences $\{X_i\}_{i=1}^{\infty}$ and the observations are given by the *i.i.d.* sequences $\{Y_{1i}\}_{i=1}^{\infty}$ and $\{Y_{2i}\}_{i=1}^{\infty}$ which are given by $Y_{1i} = X_i + N_{1i}$ and $Y_{2i} = X_i + N_{2i}$ where X_i and N_{1i}, N_{2i} are zero-mean independent Gaussian random variables with variances given by 1 and σ^2 respectively. Y_1 and Y_2 are remotely compressed and transmitted to the joint decoder which forms an optimal estimate of X. We encode these observations in blocks of length n while minimizing the average mean squared error.

In [3], a "solely" source coding technique based on the generalized Lloyd-algorithm, dubbed Quantization Based on Index Reuse has been presented. In stark contrast to this previous approach, our technique involves careful design of "channel" subcodes having "good" distance properties.

3.1 Key ideas

The observations $\{Y_i\}$ are first quantized using a root quantizer (which may be a scalar, trellis or lattice quantizer) designed for their marginal distribution without considering their correlations. We then take a subset (having a good group structure like trellises or lattices or multilevel codes) of the n-product root quantizer space to form the main code. Two "linearly independent" subcodes are obtained by partitioning of the generation process of the group structure of the main code. These subcodes are assigned to the encoders. The n-product root quantizer space can be partitioned in two different ways into cosets of these subcodes. Each encoder (remotely) sends the index of the coset of its corresponding subcode containing the quantized codeword. At the decoder, we need to find the most-likely codeword pairs from the cosets of the subcode whose indices are received from the encoders. Then the decoder gets an optimal reconstruction of X using all the received information. This problem uses the concepts of source coding, channel coding and estimation theory, all interwoven in an intricate group structure. In [11, 10] a group-theoretic formulation of generalized coset-codes was presented and it was shown that trellis codes and lattice codes are special cases of generalized coset codes. A formulation of generalized coset codes for distributed source coding and their distance properties are analyzed in [12]. In this section we give results for the special case of trellises and lattices.

3.2 Trellis codes for encoding

3.2.1 Construction of sub-codes

As an illustrative framework, we consider with no loss of generality, trellis codes based on a scalar quantizer (extensions to other cases are straightforward). Let us consider a simple fixed-length, 8-level, uniform scalar quantizer with step size $\Delta = 1.0$. Suppose $R_1 = R_2 = 2$ bits/source sample. Let $\nabla = \{r_0, r_1, \ldots, r_7\}$ be the set of reconstruction levels of the scalar quantizer. Consider the Ungerboeck trellis [13, 14, 11] built on ∇. We have a finite state machine operating on a bit-stream at the rate of 2 bits/unit time. It has a rate 2/3

[3]It can be noted that this problem is not the same as the quantization followed by coding in the binary space.

convolutional encoder, set partitioning rules and a mapping function: $Q : \{0,1\}^3 \longrightarrow \nabla$. Consider the generator matrix polynomial $\mathbf{G(t)}$ which is given by:

$$\mathbf{G(t)} = \begin{bmatrix} g_{11}(t) & g_{12}(t) & g_{13}(t) \\ g_{21}(t) & g_{22}(t) & g_{23}(t) \end{bmatrix} \tag{3}$$

where $g_{ij}(t)$ is a rational polynomial for $1 \leq i \leq 2$ and $1 \leq j \leq 3$ and let $\mathcal{C}$ denote the code associated with this generator matrix polynomial. The two rows of $\mathbf{G(t)}$ are linearly independent. We partition $\mathbf{G(t)}$ into $\mathbf{G_1(t)}$ and $\mathbf{G_2(t)}$ which are given by:

$$\mathbf{G_1(t)} = [\ g_{11}(t) \quad g_{12}(t) \quad g_{13}(t)\ ] \text{ and } \mathbf{G_2(t)} = [\ g_{21}(t) \quad g_{22}(t) \quad g_{23}(t)\ ] \tag{4}$$

The encoders-1 and -2 are associated with the set of codewords generated by $\mathbf{G_1(t)}$ and $\mathbf{G_2(t)}$ respectively. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be the codes associated with $\mathbf{G_1(t)}$ and $\mathbf{G_2(t)}$ respectively. $\mathcal{C}_1$ and $\mathcal{C}_2$ have 2^{2n} cosets and partition ∇^n in two different ways.

Each of the encoders sends the index of the coset containing the observed quantized output, thus spending 2 bits/source sample. For example, Fig. 4 shows the trellises associated with $\mathcal{C}_1$ and $\mathcal{C}_2$ obtained from a 4-state Ungerboeck's trellis. We define the cosets $\mathcal{C}_1$ and $\mathcal{C}_2$ as given in [1]. The coset information transmitted by the encoders are given by the result of the following syndrome computation. Let $\hat{\mathbf{y}}_1$ and $\hat{\mathbf{y}}_2$ be the observed scalar-quantized output (called the *active codeword pair*). The syndromes are given by: $\mathbf{s_1(t)} = \mathbf{H_1(t)}[Q^{-1}(\hat{\mathbf{y}}_1\mathbf{(t)})]$ and $\mathbf{s_2(t)} = \mathbf{H_2(t)}[Q^{-1}(\hat{\mathbf{y}}_2\mathbf{(t)})]$, where $\mathbf{H_1(t)}$ and $\mathbf{H_2(t)}$ are the parity check matrix polynomials associated with $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively.

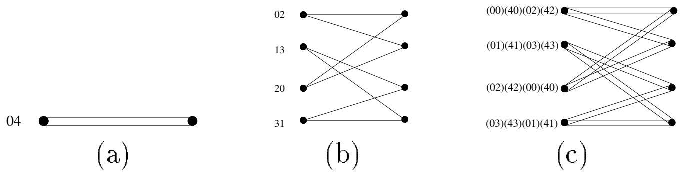


Figure 4: Trellises: (a) First constituent trellis with generator matrix polynomial given by $\mathbf{G_1(t)} = [1\ 0\ 0]$. (b) Second constituent trellis with the generator matrix polynomial given by $\mathbf{G_2(t)} = [0\ 1\ \frac{t}{1+t^2}]$. They are obtained by partitioning Ungerboeck's 4-state trellis [13]. (c) Tensor Product trellis: This is formed using the constituent trellises. Each path has two labels taking one from each of the constituent trellises.

Decoding algorithm: The joint decoder needs to recover $\hat{\mathbf{y}}_1$ and $\hat{\mathbf{y}}_2$ before doing further processing on them. To obtain a fast decoding algorithm, let us consider the tensor product trellis of any coset of $\mathcal{C}_1$ and $\mathcal{C}_2$ as shown Fig. 4(c) (consider the principal trellises [1] for illustration). Each path in the trellis has two labels associated with the constituent trellises. In an n-stage trellis, each section is a tensor product of the trellises of any coset of $\mathcal{C}_1$ and $\mathcal{C}_2$ as directed by the encoders. We obtain the most probable sequence-pair in this tensor product trellis using the Viterbi algorithm.

3.2.2 Distance properties

Here we analyze the distance properties of trellis-based distributed coding. Let us denote the i^{th} coset of $\mathcal{C}_1$ and $\mathcal{C}_2$ as $\mathcal{C}_1(i)$ and $\mathcal{C}_2(i)$ respectively, for $i \in \{1, 2, \ldots, 2^{2n}\}$. For any coset pair (i, j): define $\alpha(i, j)$= minimum of distances $d_e(\mathbf{c_1}, \mathbf{c_2})$ between any two codewords

$\mathbf{c_1} \in \mathcal{C}_1(i)$ and $\mathbf{c_2} \in \mathcal{C}_2(j)$ such that there exists at least one pair $(\mathbf{c_3}, \mathbf{c_4}) \neq (\mathbf{c_1}, \mathbf{c_2})$ such that $d_e(\mathbf{c_1}, \mathbf{c_2}) \geq d_e(\mathbf{c_3}, \mathbf{c_4})$, $\mathbf{c_3} \in \mathcal{C}_1(i)$ and $\mathbf{c_4} \in \mathcal{C}_2(j)$ (i.e., $(\mathbf{c_1}, \mathbf{c_2})$ is the codeword pair having second smallest distance). We define the correlation distance, d_c as follows:

$$d_c = \begin{array}{c}\text{minimum} \\ i, j \in \{1, 2, \ldots, 2^{2n}\}\end{array} \{\alpha(i, j)\} \tag{5}$$

Thus for the decoder to make an error, *the distance between any two active codevector pair* $\mathbf{y_1}$ *and* $\mathbf{y_2}$ *must be at least* d_c. We have the following theorem which guarantees that d_c is at least half the minimum distance (d_{min}) of $\mathcal{C}$.

Theorem: For trellis codes, using the partition of the generator polynomial matrix as given above, $d_c \geq \frac{d_{min}}{2}$.

Proof: Omitted due to lack of space. (see [12])

3.3 Lattice codes for encoding

A lattice $\mathbf{\Lambda}$ [8] in an N-dimensional Euclidean space is a discrete additive sub-group [15] of $\mathcal{R}^N$. An N-dimensional lattice will have N basis vectors $\mathbf{w_1}, \mathbf{w_2}, \ldots, \mathbf{w_N}$ such that

$$\mathbf{\Lambda} = \left\{ \sum_{i=1}^{N} \theta_i \mathbf{w_i} | \theta_i \text{ is an integer for } i = 1, 2, \ldots, N \right\}$$

In asymmetric encoding, the observation signals are first quantized using a fine lattice. One of the encoders sends its full information about the quantized codeword. The other encoder sends the index of the coset of a lattice denoted by $\mathbf{\Lambda}$ (with "good" distance properties and which is a sub-lattice of the fine lattice used for quantization) containing the quantized codeword.

Similar to the construction of subcodes in trellis codes, we assign $N/2$ basis vectors to one of the users and the remaining $N/2$ vectors to the other user (where the lattice $\mathbf{\Lambda}$ has dimension N and has N basis vectors). The $N/2$ basis vectors assigned to each of the encoders form a sub-lattice with $N/2$ dimension and the sub-lattices corresponding to the two encoders are linearly independent. The encoders send the index of the coset of the sub-lattices containing the codeword in the fine lattice (a more general formulation is considered in [12]). At the decoder, we need to find a codeword pair, one from each of the cosets of the corresponding sub-lattice, such that the distance between them is minimum over all the other pairs. An example of a lattice $\mathbb{Z}^2$ is shown in Fig. 5. The sublattices associated with the encoders are given by $(\mathbb{Z}, \mathbf{0})$ and $(\mathbf{0}, \mathbb{Z})$.

Let us consider 2 encoders using an N-dimensional lattice, with the corresponding outcomes given by $\mathbf{y_1}$ and $\mathbf{y_2}$. These can be represented as the linear combination of basis vectors as given in the following:

$$\mathbf{y_1} = \mathbf{v_1} + \sum_{i=1}^{N/2} u_{1i}\mathbf{w_i} + \sum_{i=N/2+1}^{N} s_{1\ i-N/2}\mathbf{w_i}, \ \mathbf{y_2} = \mathbf{v_2} + \sum_{i=1}^{N/2} s_{2i}\mathbf{w_i} + \sum_{i=N/2+1}^{N} u_{2\ i-N/2}\mathbf{w_i},$$

where $\{u_{ji}, s_{ji}\}$ and $\mathbf{v_j}$ are the coefficients of the basis vectors and the coset-representatives respectively, for $1 \leq j \leq 2$ and $1 \leq i \leq N/2$; the encoders send s_{ji} and $\mathbf{v_j}$. The decoder needs to find u_{1i} and u_{2i} such that

$$d_e\left(\mathbf{w} - \left(\sum_{i=N/2+1}^{N} u_{2\ i-N/2}\mathbf{w_i} - \sum_{i=1}^{N/2} u_{1i}\mathbf{w_i}\right)\right)$$

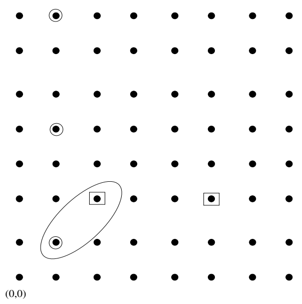


Figure 5: An example of a lattice $\mathbb{Z}^2$. The sublattices associated with the encoders are given by $(\mathbb{Z}, \mathbf{0})$ and $(\mathbf{0}, \mathbb{Z})$. The encoders send the index of the coset of these sublattices in $\mathbb{Z}^2$ containing the quantized codewords. There exists a unique pair as shown such that the distance between them is minimum among all the other pairs.

is minimized where $\mathbf{w} = \mathbf{v_1} - \mathbf{v_2} + \sum_{i=N/2+1}^{N} s_{1\ i-N/2}\mathbf{w_i} - \sum_{i=1}^{N/2} s_{2i}\mathbf{w_i}$ is known at the decoder. This decoding has the same form as that of asymmetric decoding. We can define the correlation distance d_c similar to the case of trellises. We have the following theorem about d_c:

Theorem: For Euclidean space lattices, $\mathbf{\Lambda}$, with minimum distance of d_{min}, $d_c \geq \frac{d_{min}}{2}$.
Proof: Omitted due to lack of space (see [12]).

Similarly, we can apply the construction of sub-codes for multi-level codes based on block-channel-codes and similar "good" distance properties can be guaranteed.

Thus the performance of the symmetric encoding framework is not compromised with respect to asymmetric encoding methods. Most codes having a good group structure can be used for such encoding. The computational complexity is also of the same order as that of the asymmetric case. Optimal quantizers can be designed for the marginal distribution of Y_i for the i^{th} sensor. A group structure based on block codes can also be used to partition the source codeword space, which is similar to the construction of multilevel codes which will be considered in the future work.

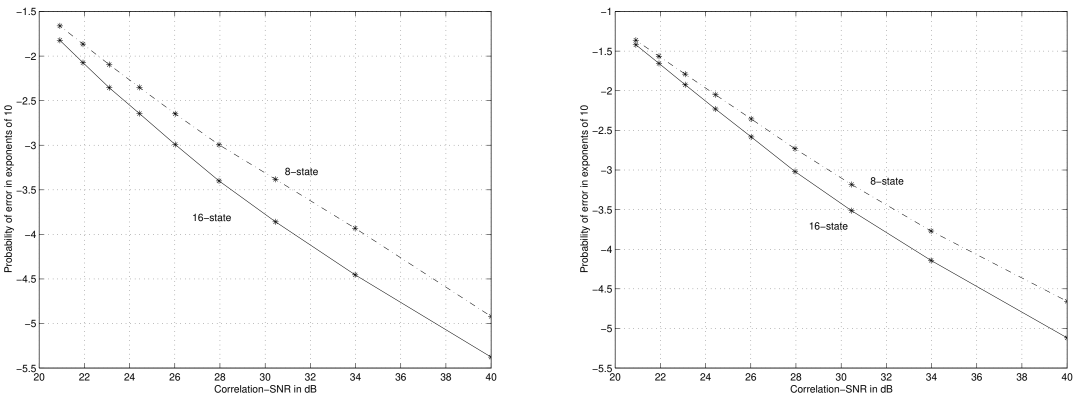
4 Simulation Results

In this section we compare the performance of symmetric encoding with that of asymmetric encoding using simulations. We first design the fixed-length scalar quantizer for quantizing the observations Y_1 and Y_2. We then apply the techniques developed in the previous sections to build codes on this scalar quantizer alphabet set.

As described earlier, we take the noisy observations to be $Y_1 = X + N_1$ and $Y_2 = X + N_2$, where the actual signal of interest is X, which is *i.i.d.* Gaussian with unit variance and zero mean, N_1 and N_2 are *i.i.d.* zero-mean Gaussian with σ^2 as their variance and they are independent of each other and X. In this paper we consider a rate of transmission of 2 bits/source sample for the encoders of Y_1 and Y_2. We refer to the ratio of the variance of X and the noise as 'correlation-SNR'. Reconstruction distortion for unit variance of X is referred to as 'normalized distortion'. The trellises are built on 8-level Lloyd-Max fixed-length scalar quantizer. The trellis codes for Y_1 and Y_2 are given by the partition of the generator matrix polynomial of the Ungerboeck's trellis codes. We use 8− and 16− state trellises for the simulations. Clearly the distortion performance of the system depends only on the root scalar quantizer. We minimize the probability of decoding error by designing efficient trellis codes.

As a reference system to represent asymmetric encoding, one of the observations is quantized and transmitted with "full information" using 3 bits/source sample. The other observation is quantized and the syndrome sequence is sent using the trellises given in [1] with 1 bit/source sample thus requiring a total of 4 bits for a sample of X.

Fig. 6 shows the probability of decoding error as a function of correlation-SNR. This is the result of Monte-Carlo simulations using 10^{10} samples. Fig. 6(a) and (b) show the performance of symmetric and asymmetric encoding respectively. It can be noted that there is no loss in performance by the introduction of symmetry in encoding. The distortion performance of the system is a monotonically decreasing function of correlation-SNR's. We define



(a) (b)

Figure 6: Probability of decoding error: (a) Symmetric encoding: both the encoders send partial information. Rate of transmission is 2 bit/source sample/encoder. (b) Asymmetric encoding: one of the encoders sends conventional source-encoded bit stream with 3 bits/ source sample, and the other sends the syndrome information with 1 bit/ source sample.

the ratio of the distortion and the signal variance as normalized distortion. The normalized distortion is plotted as a function of c-SNR in Fig. 7(a). The achievable distortion [3] for a rate of transmission of 2 bits/sample/branch is shown in Fig. 7(b). This is obtained using the achievable rate region given by the information-theoretic analysis.

Conclusion: We have proposed a practical constructive framework for the symmetric distributed source coding problem, both in a discrete alphabet space and in Euclidean space. The performance of these systems matches that of the asymmetric encoding methods. Although we have treated trellis and lattice codes separately, we can obtain a general formulation using generalized coset codes in a group-theoretic setting [12]. The choice of the root quantizer for different encoders is an issue that requires more scrutiny. The intricate interplay between coding and estimation has a definite role in choosing these root quantizers. We propose to investigate these issues and apply our principles to "real world" sensor data in our future research.

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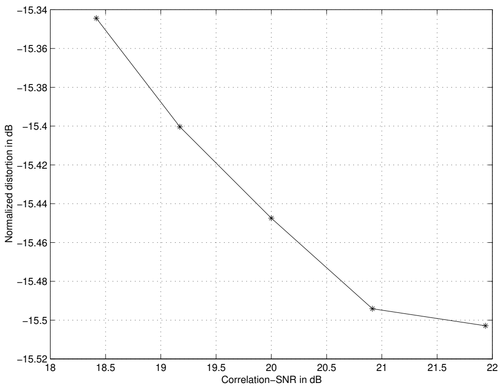
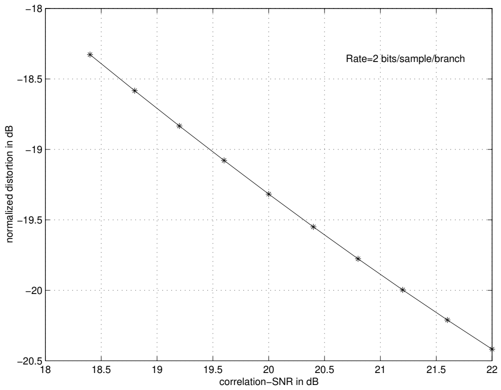


(a)



(b)

Figure 7: Distortion Performance: (a) Distortion performance for the trellis code with rate=2 bits/sample/branch using 8- and 16- trellis based on 8-level root scalar quantizer (b) theoretical achievable distortion for a rate of 2 bits/sample/branch using the information theoretic analysis of [3].

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