n-Channel Multiple Descriptions: Theory and Constructions

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Abstract

We present new achievable rate regions and code constructions for the symmetric n-channel Multiple Descriptions (MD) coding problem [1] for $n > 2$. Our approach is inspired by unexplored connections between MD and the problem of distributed source coding [2, 3]. For illustrative clarity, we restrict our focus to the important special case relating to (n, k) source-channel erasure codes [4]. This involves n encodings of a source with the goal of maximizing its reconstruction fidelity with the availability of *any* k of them, while strictly improving this reconstruction fidelity with the availability of more than k descriptions. We describe the underlying information-theoretic framework, and then formulate practical constructions based on scalar quantizers and linear channel codes for the n=3 case to illustrate our concepts.

1 Introduction

The problem of multiple description (MD) coding involves generation of multiple descriptions of the source such that each description independently describes the source with a certain fidelity, and when more than one description is available, they can be synergistically combined to enhance the quality. In the interesting special case of n symmetric channels the MD problem is set up as follows: the rates on all descriptions are equal (R) and the distortions depend only on the *number* of descriptions received i.e., the set of achievable rate-distortions is represented as an $(n+1)$ tuple $(R, D_1, D_2, \ldots, D_n)$ where D_i refers to the distortion achieved when *any* i packets are received.

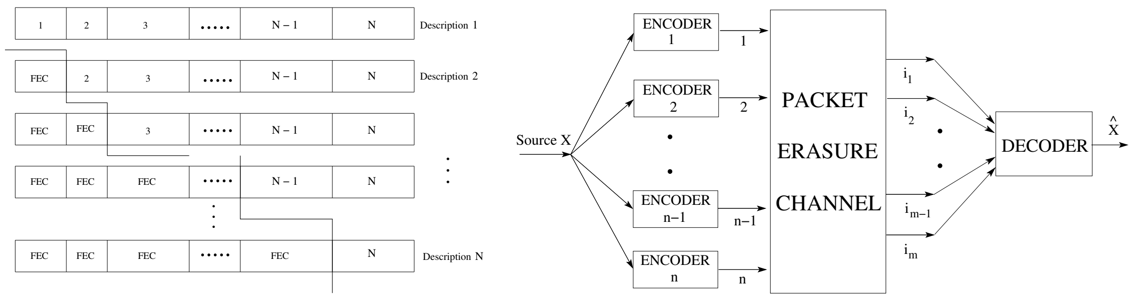
Code construction for the MD problem has been primarily focused on the n=2 case and based on approaches such as scalar quantizers, frame based expansions and transform coding. Interestingly, the most promising performance for the $n > 2$ case is given by schemes that use Maximum Distance Separable (MDS) erasure channel codes for unequal error protection of a successively refinable encoding of the source. [5, 6]. (Figure 1 (a)) reveals this approach which realizes a symmetric MD system).

We have recently studied in [4] the problem of maximum distance separable (MDS) (n, k) erasure codes in an information theoretic joint source channel coding framework where we considered the achievable rate distortion region for the problem. In this problem (see Figure 1 (b)) the channel delivers some $m \geq k$ packets from a source X. The goal is to maximize the reconstruction fidelity at the decoder with the availability of *any* k of them, while strictly improving this reconstruction fidelity with the availability of more than k descriptions. A naive approach based on the source-channel separation principle [7] would suffer from the "cliff" effect i.e., reception of fewer than k packets is not sufficient to recover any data, while the reception of *any* k packets enables the recovery of the original k source packets and further reception of packets does not aid any further recovery. In this work we highlighted the joint source-channel coding advantage by showing that, in theory, one could perform as well as the separation approach on reception of k packets while providing strictly superior quality on reception of $m > k$ packets, at least for the case of a Gaussian source.

The key observation is the fact that the channel guaranteedly delivers some k packets implies an inherent uncertainty at the source encoder as to *which* packets (side information) are received at the decoder. This connects the problem intimately to the framework of distributed source coding as considered in [3] where the statistics of the side information are available to the source encoder but the actual realization is not thus enabling the use of random binning techniques used in [3]. The concept of random binning involves the encoder sending an index for a list of codewords instead of that for a unique codeword and thus reduces the rate of transmission. The hope is that the side information can be used to disambiguate the transmitted codeword from the list.

In this work we apply the results presented in [4] to the symmetric n-channel MD problem using the code concatenation framework proposed in [5]. The MD region so obtained achieves new rate-distortion tuples previously unknown in literature. We also propose practical code constructions based on scalar quantizers and linear codes for the component codes that shed insight into new constructive ways of doing MD source coding. Although, we present constructions for the $n = 3$, $k = 2$ case, using relatively simple quantizers and codes, the simulation results presented here point out the promise of the proposed framework.

2 (n, k) MDS Source Channel Erasure Codes



(a) (b)

Figure 1: (a) N channel MD coding: The N source layers of decreasing importance are protected by N channel codes of decreasing strength. (b) MDS (n, k) source-channel erasure code: A source is encoded into n packets and transmitted over n erasure channels. Reconstruction is enabled at the decoder with the availability of any $m \geq k$ packets.

We briefly state the main information theoretic result presented in [4] for the MDS (n, k) erasure code problem. We introduce some notation first. Consider a source $X \sim q(x)$ whose alphabet takes values in $\mathcal{X}$. We are given a reconstruction alphabet $\hat{\mathcal{X}}$, with a *bounded* distortion measure d : $\mathcal{X} \times \hat{\mathcal{X}} \rightarrow \mathcal{R}^+$ such that $d(\cdot,\cdot) \leq d_{max}$. Further, let $\mathcal{I}_n = \{1, 2, \ldots, n\}$ and $M = \{J : J \subseteq \mathcal{I}_n, |J| \geq k\}$. For any subset S of $\mathcal{I}_n$, let $\mathbf{x}_S = \{\mathbf{x}_{i_1}, \mathbf{x}_{i_2}, \ldots, \mathbf{x}_{i_{|S|}}\}$.

Let $Y_1, Y_2, \ldots, Y_n$ be n symmetrically correlated [1] random variables where Y_i has some alphabet $\mathcal{Y}_i$. Y_i corresponds to quantization random variables for X that is transmitted over the ith channel. Let $p(x, y_1, y_2, \ldots, y_n) = q(x) \cdot p(y_1, y_2, \ldots, y_n|x)$ denote their joint p.m.f with X.

Theorem: *An achievable rate region for a given distortion tuple* $(D_k, D_{k+1}, \ldots, D_n)$ *is given by*

$$R > \frac{1}{k} H(Y_1, Y_2, \ldots, Y_k) - \frac{1}{n} H(Y_1, Y_2, \ldots, Y_n|X). \tag{1}$$

for some probability mass function $p(x, y_1, y_2, \ldots, y_n) = q(x) \cdot p(y_1, y_2, \ldots, y_n|x)$, *and a set of decoding functions* $g_J : \bigotimes_{i \in J} \mathcal{Y}_i \longrightarrow \hat{\mathcal{X}}$, $\forall J \in M$, *such that* $\forall h \in \{k, k+1, \ldots, n\}$,

$$E\left[d\left(X, g_J(Y_{i_1}, Y_{i_2}, \ldots, Y_{i_h})\right)\right] \leq D_h, \quad J = (i_1, \ldots, i_h). \tag{2}$$

The main steps to attain the above region essentially comprise of first choosing random codebooks containing codewords of a large blocklength l for random variables Y_is using their marginal densities. The codebooks are sufficiently big so that an n-tuple of codewords (one from each codebook) can be found that is jointly typical with each X of blocklength l that occurs. Then exploiting the fact that there is "help" available at the decoder in the form of at least another $k-1$ packets, the rate of transmission is reduced by randomly binning codewords in each of the codebooks. The above theorem implies that if R is as given by (1) then with the reception of *any* $h \geq k$ packets the corresponding random variables Y_i can be received, and distortion D_h is achieved. We note that with the reception of $m > k$ packets, $D_m \leq D_k$ is achieved since a decoding function operating on a set of random variables can do no worse than one operating on its subset. Thus the estimation gain due to multiple "new" looks at the source translates to the reception of extra parity bits serving to enhance the received quality!

For the case of a Gaussian source X distributed as $\mathcal{N}(0, 1)$, the rate of transmission being R bits/sample/packet, one set of achievable distortions with the availability of any $(k+r)$ packets of information can be shown to be:

$$D_{k+r} = \frac{k}{2^{2kR}(k+r) - r}, \tag{3}$$

for $0 \leq r \leq (n-k)$. For the case when r equals 0 i.e., the total rate received is kR, the observed distortion is 2^{-2kR} which is *on* the rate-distortion curve for a unit variance Gaussian source. Further the reconstruction distortion monotonically decreases as a function of the number of received packets.

[1] The joint probability mass function (p.m.f) of any subset of them with a particular cardinality is the same and this is true for all cardinalities.

3 Achievable performance for MD

Using results from Section 2, we now present the rate region for the MD problem the details of which have been discussed in [1]. The techniques used here derive their inspiration from those used in the Wyner-Ziv theorem [3] and are based on the "source coding with side information" philosophy in contrast with the approaches pursued in [8, 9] which rely on a conventional "successive refinement" framework.

3.1 Basic Concepts

Let us consider, without loss of generality, the case of symmetric 3-channel MD coding. The system has to use a $(3,1)$ code (base layer) to satisfy the single channel distortion constraints. Let $I = \{1,2\}$ and $J = \{1,2,3\}$. Let Y_{1j} be the random variable corresponding to the j^{th} packet in this code (see Figure 2). If the 2-channel distortion (which is the same for the reception of any two packets) given by such a code does not satisfy the desired 2-channel distortion constraints, then we have to add refinement layers.

Instead of a conventional successive refinement strategy, let us consider the transmission of the second layer using a distributed paradigm [2]. In the given scenario, with the availability of the first layer in any two descriptions, the pairs (Y_{11}, Y_{12}), (Y_{12}, Y_{13}) and (Y_{13}, Y_{11}) which have the same joint statistics with X can each be treated as side information about the source X. Using the source coding with side information approach, a single refinement layer that will work for all of these can thus be transmitted. This information which is meant to be decoded with the reception of any 2 packets, can be packetized into 2 packets and then encoded using a conventional $(3,2)$ parity-check code in the second layer. However, we can use $(3,2)$ source-channel erasure codes proposed in [4] to get the source-channel coding advantage over a conventional $(3,2)$ parity-check code. In short, we notice that every portion of the bitstream serves to enhance the received quality.

3.2 Rate region for MD

For paucity of space and for notational convenience, we present the results for $n = 3$ case. We borrow some of the notation from Section 2. Let $Y_{11}, Y_{12}, Y_{13}, Y_{21}, Y_{22}, Y_{23}, Y_3$ be random variables as shown in Figure 2. Let $\mathcal{Y}_3$ denote the alphabet for Y_3 and $\mathcal{Y}_{ij}$ denote alphabet for Y_{ij} where $i \in I$, and $j \in J$. As in Section 2 we further assume symmetry between the random variables in any given layer. The total rate of transmission per packet R is partitioned as $R_1 + R_2 + R_3$. Rate R_1 per packet is used to send Y_{11}, Y_{12}, Y_{13} as a $(3,1)$ code. Rate R_2 is used to send the $(3,2)$ code. The final

[2]The concept of source coding with side information involves encoding of a source in the presence of another correlated source (also referred to as side information) at the decoder [3] . The encoder does not have access to the realization of the correlated source but has knowledge of the joint statistics. Such a system has the advantage that a given encoding of the the source is useful to any decoder that has access to side information having the same joint statistics regardless of the actual realization

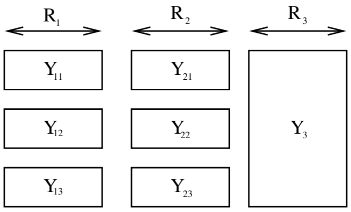


Figure 2: 3 channel MD coding using concatenation of $(3, 1), (3, 2), (3, 3)$ Codes.

layer which is desired to be decodable only with the reception of all the packets can be sent at rate R_3 per packet using the conventional successive refinement approach.
Theorem: An achievable rate region for a given distortion tuple (D_1, D_2, D_3) is given by $R > R_1 + R_2 + R_3$, where,

$$R_1 > H(Y_{11}) - \frac{1}{3}H(Y_{1J}|X),\ R_2 > \frac{1}{2}H(Y_{21}, Y_{22}|Y_{11}, Y_{12}) - \frac{1}{3}H(Y_{2J}|X),\ R_3 > \frac{1}{3}I(X; Y_3|Y_{IJ}) \tag{4}$$

for some p.m.f $p(x, y_{IJ}, y_3) = q(x)p(y_{1J}|x)p(y_{2J}|x)p(y_3|x, y_{IJ})$, where $Y_{1J} \to X \to Y_{2J}$ is a Markov chain and a set of decoding functions

$$g_i^1 : \mathcal{Y}_{1i} \longrightarrow \hat{\mathcal{X}}, i \in J,\ \ g_S^2 : \bigotimes_{k \in I, i \in S} \mathcal{Y}_{ki} \longrightarrow \hat{\mathcal{X}}\ \ \forall S \subset J, |S| = 2,\ \ g^3 : \mathcal{Y}_3 \times \bigotimes_{k \in I, i \in J} \mathcal{Y}_{kj} \longrightarrow \hat{\mathcal{X}}. \tag{5}$$

such that $\forall i \in J$,

$$E[d(X, g_i^1(Y_{1i}))] \leq D_1;\ \ \forall S \subset J, |S| = 2, E[d(X, g_S^2(Y_{I,S}))] \leq D_2;\ \ E[d(X, g^3(Y_{I,J}, Y_3))] \leq D_3 \tag{6}$$

4 Constructive Approach

4.1 Example for illustration

Let us consider the key concepts involved in the efficient practical construction of (n, k) source-channel erasure codes. Let us consider two correlated memoryless binary sources, i.e., $Y_1, Y_2 \in \{0, 1\}$. Let the correlation between them be characterized as follows: $Y_i = X \oplus e_i$ with $P(X = 1) = 0.5$ and $P(e_i = 1) = a$ for some $0 \leq a \leq 1$, and X, e_1 and e_2 are independent, where $\oplus$ denotes modulo-2 addition. Note that X in this example is the hidden variable capturing the correlation structure between Y_1 and Y_2. It can be shown that $H(Y_1) = H(Y_2) = 1$ bit/sample and $H(Y_1, Y_2) = 1 + h(p)$ where $p = 2a(1 - a)$ and $h(p)$ is the binary entropy function [7]. Note that Y_1 and Y_2 are related in such a way that one of them can be thought of as the output of a binary symmetric channel (BSC) with the other as the input, and the probability of error of this channel is p. If the encoders of these sources, without direct communication between them, wish to transmit the corresponding sources to a joint remote decoder, they have to adopt a distributed source coding strategy as proposed in [2]. Consider a (n, k) capacity-achieving [10, 11] binary linear code, denoted by $\mathcal{C}$ with $k \approx n(1 - h(p))$ (capacity of the BSC is $1 - h(p)$) and the generator matrix given by $\mathbf{G}$. Consider encoding using block-length of n. As given by [3], one of the encoders sends its full information using n bits and the other encoder sends the index of the coset of the code $\mathcal{C}$ containing its outcome, enabling the joint decoder to recover both sources Y_1 and Y_2, thus approaching the joint entropy bound of $1 + h(p)$ bits/sample for the rates of the sum rate of transmission.

A symmetric solution to this problem was proposed in [12] where a subcode construction of the linear code $\mathcal{C}$ (and in general any algebraic code under consideration) was considered. The k rows of $\mathbf{G}$ are partitioned into two sets denoted by $\mathbf{G_1}$ and $\mathbf{G_2}$, containing $k/2$ rows each. Linear subcodes (given by $\mathcal{C}_1$ and $\mathcal{C}_2$) are constructed using each of these sets, and these are assigned to the encoders. The encoders send the index of the coset of their corresponding subcodes containing their respective outcomes. Using the decoding algorithm proposed in [12], the decoder can recover both the sources without any error. This is briefly outlined below:

Let $\mathcal{C}_i$ be in their systematic form. Let $\mathbf{H_i}$ denote the parity check matrix of $\mathcal{C}_i$. The messages sent by the encoder i is given by $\mathbf{s_i} = \mathbf{H_i y_i}$ where $\mathbf{y_i}$ is the outcome of $\mathbf{Y_i}$. The decoder after receiving $\mathbf{s_1}$ and $\mathbf{s_2}$, has to recover $\mathbf{Y_1}$ and $\mathbf{Y_2}$ by noting that $\mathbf{y_i} = \mathbf{u_i G_i} \oplus [\mathbf{0}|\mathbf{s_i}]$ for $i = 1, 2$. Now $\mathbf{s} = \mathbf{uG} \oplus \mathbf{e}$ where $\mathbf{e} = \mathbf{y_1} \oplus \mathbf{y_2}$, $\mathbf{s} = [\mathbf{0}|\mathbf{s_1}] \oplus [\mathbf{0}|\mathbf{s_2}]$ and $\mathbf{u} = [\mathbf{u_1}|\mathbf{u_2}]$ which implies that we can recover $\mathbf{u}$ as $\mathbf{e}$ corresponds to the noise of the BSC, the capacity of which is achieved by $\mathcal{C}$. This guarantees the recovery of $\mathbf{Y_1}$ and $\mathbf{Y_2}$ with vanishing probability of error. The rates of transmission (R_1, R_2) given by: $R_i \geq h(p)$ and $R_1 + R_2 \geq 1 + h(p)$ can be achieved using this method.

In this case we have compressed these sources at a sum rate equal to their joint entropy. But if we leave some redundancy in the representation of these sources, we can make the system robust against link failures as discussed below: suppose that in addition to the two sources considered above, let there be a third source having the same alphabet, $Y_3 \in \{0, 1\}$ and further $Y_3 = X \oplus e_3$ with $e_3 \in \{0, 1\}$, $P(e_3 = 1) = a$ and independent of X, e_1 and e_2. Clearly $(1/3)H(Y_1, Y_2, Y_3) < (1/2)H(Y_i, Y_j)$ for $i, j = 1, 2, 3$ and $i \neq j$. Consider the transmission of these 3 sources in a distributed manner to the joint decoder. Let us also suppose that one of the transmission links breaks down during this communication. We would like to have a compression strategy which is robust to such link failures. In other words, the decoder with the reception of information bits from any two active channels wishes to reconstruct the corresponding sources with arbitrarily low probability of error, while keeping the redundancy to a minimum. We propose a scheme where the rates of transmission $\{R_i\}$ such that $R_i = H(Y_1, Y_2)/2$ are achievable.

Let $\mathbf{G_3} = \mathbf{G_1} \oplus \mathbf{G_2}$ where $\oplus$ here denotes term by term modulo-2 addition of the elements of the matrix. Let $\mathcal{C}_3$ denotes the set of codewords generated from this matrix $\mathbf{G_3}$. Clearly $\mathcal{C}_3$ is a subcode of $\mathcal{C}$. Let $\mathbf{H_3}$ denotes the parity check matrix of $\mathcal{C}_3$. Note that $\{\mathbf{G_i}\}$ are pairwise linearly independent. We observe that the subcodes $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ partition in three different ways, the space of all n-length binary codewords $\{0, 1\}^n$ into $2^{n-k/2}$ cosets. The message sent by the third encoder is $\mathbf{s_3} = \mathbf{H_3 y_3}$ where $\mathbf{y_3}$ denotes the outcome of $\mathbf{Y_3}$, with the messages sent by the other encoders remaining as before.

Now with the channels associated with any two sources Y_i and Y_j active, the decoding operation is identical to that of above because the reception of syndromes with respect to any 2 of $\mathbf{G_1}$, $\mathbf{G_2}$ and $\mathbf{G_3}$ still amounts to decoding of a codeword in $\mathbf{G}$ that is nearest to $\mathbf{s} = [\mathbf{0}|\mathbf{s_i}] \oplus [\mathbf{0}|\mathbf{s_j}]$. Thus the pairwise linear independence of $\mathbf{G_1}$, $\mathbf{G_2}$ and $\mathbf{G_3}$ which ensures that any 2 of them are equivalent to $\mathbf{G}$ enables successful recovery of the corresponding $\mathbf{y_i}$ and $\mathbf{y_j}$. Thus the rate tuple: $R_i \geq H(Y_1, Y_2)/2$ for all $i = 1, 2, 3$ are achievable. From the converse to the Shannon source coding theorem, even if the encoders were to communicate, the rate tuple must satisfy the constraint given by (note that $H(Y_i, Y_j) = 1 + h(p)$ for all $i, j = 1, 2, 3$ and $i \neq j$)

$$R_i + R_j \geq H(Y_1, Y_2) = 1 + h(p) \forall i, j = 1, 2, 3, i \neq j$$

Thus the key concept to note here is that we need three different pairwise linearly independent partitions of the space of outcomes of the corresponding random variables to make the system robust against 1 link failure. This concept can be generalized to Euclidean space codes using the concept of generalized coset codes [13] which includes trellis codes and lattice codes as examples.

4.2 Generalization to (n, k) codes

Till now we have considered transmission of 3 sources with 1 link failure. This corresponds to the $(3, 2)$ source-channel erasure codes. We would like to point out that the choice of the generator matrices for the 3 encoders was motivated by the observation that a $(3, 2)$ MDS block code (a parity check code) has the property that any two columns of its generator matrix [3] are linearly independent. Thus each of the three columns can be used to weight and add the 2 parts of $\mathbf{G}$ giving rise to the three generator matrices $\mathbf{G_1}$, $\mathbf{G_2}$ and $\mathbf{G_3}$. The linear independence property of *any* two columns ensures that the reception from any two encoders is equivalent to that from any other two encoders.

This concept can be easily generalized to the construction of such partitions for system with n sources with $(n - k)$ link failures. The ingredients required are an MDS (n, k) block code that gives us columns which have the property that *any* k of them are linearly independent and a linear code generator matrix which can be split into k parts of equal rate and which is strong enough to combat the correlation noise between the observations. Note that in order to use the columns of the MDS code to weight and combine the parts of the generator matrix we require that both come from the same finite field.

In the rest of the paper, however, for illustrative clarity, we will restrict our attention to the case of $n = 3$ and $k = 2$. In general, in source-channel erasure codes, the random variables Y_1, Y_2 and Y_3 correspond to quantized representations of a given source, say X. There are two issues to be addressed here: 1)quantization and 2) partition of the codeword space.

[3]The generator matrix is given by $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

4.3 Code Construction for $(3,2)$ code

4.3.1 Quantization

The first part of the encoder involves quantizing the source X to get some representations: Y_1, Y_2 and Y_3. It was shown in [4] that for the case when a $(3,2)$ code is desired to give rate-distortion performance for reception of *any* 2 packets and further improvement on reception when 3 packets arrive, the quantization noises are independent. The constructive strategy presented here focuses on approaching this performance point. We use the staggered quantizer configuration proposed in [14] to have approximately independent quantization noises. We thus take a 22-level quantizer and partition it into 3 staggered quantizers with 8 reconstruction levels each (corresponding to a source coding rate of 3 bits/sample). Let $\{\Delta_i\}$ denote the alphabets of these quantizers.

4.3.2 Trellis partition

The next part of the encoder consists of obtaining an efficient way of partitioning the space of quantizer codewords into cosets. We need three different ways of partition of the three different quantizer codebooks. The encoder sends the index of the coset containing the quantized outcome in the i^{th} quantizer on the i^{th} channel. Since the nature guarantees that at least any two channels are active (say i and j), the decoder wishes to recover the corresponding quantized outcomes characterized by Y_i and Y_j from the coset indices received. Finally, if all the channels are active, the decoder recovers all the quantized outcomes. In both the above cases, finally, the decoder gets a best representation of the source X based on all the information recovered. In the sequel, we propose a structured code construction based on trellis codes. This construction can be formulated in a very general framework of generalized coset codes which includes trellis as well as lattice codes. This generalization is not presented here for lack of space.

Let $R_i = 2$ bits/sample for $i = 1,2,3$ be the transmission rate which is 1 bit less than the source coding rate. Consider a rate 2/3 trellis code with 2 bits/sample as input and which maps (one-one) the set of all $2n$-bit binary sequences into Δ_i^n (without loss of generality assume $i = 1$). Let the generator matrix polynomial of this code be $\mathbf{G}(\mathbf{t})$ with dimension 2×3 (see an example below for a 2-state trellis).

$$\mathbf{G}(\mathbf{t}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & t \end{bmatrix} \tag{7}$$

Let $\mathbf{G_1}(\mathbf{t}), \mathbf{G_2}(\mathbf{t})$ denote $[1\ 0\ 0]$ and $[0\ 1\ t]$ respectively and let $\mathbf{G_3}(\mathbf{t}) = \mathbf{G_1}(\mathbf{t}) \oplus \mathbf{G_2}(\mathbf{t})$. Now consider the i^{th} trellis code (for $i = 1,2,3$) with the trellis as shown in Figure 3, with the generator matrix polynomial $\mathbf{G_i}(\mathbf{t})$ and built on the alphabet Δ_i. With a block-length of n, these trellis codes consists of 2^n sequences. We can partition the space of quantizer outcomes given by Δ_i^n into 2^{2n} cosets each containing 2^n sequences, using the i^{th} trellis code similar to the trellis-based coset partition proposed in [12].

The second part of the encoder sends the index of the trellis coset containing the quantized outcome to the decoder, with the i^{th} index sent on the i^{th} channel. We can

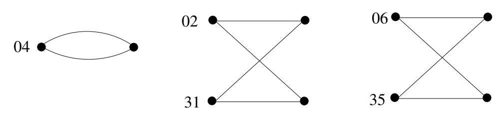


Figure 3: Trellis associated with the codes assigned to the 3 encoders.

use the parity check polynomial matrix for an efficient way of computing this coset index as given in [12]. The decoder with the reception of any two channels use tensor-product trellis [12] to obtain the actual quantized sequences using a modified Viterbi algorithm. We will not go into this computationally efficient decoding algorithm in this paper. Since the encoder only sends the coset index, there is a finite probability of decoding error. The goal is to keep this probability of error as low as possible by building efficient trellis codes with a large minimum distance.

5 Simulations Results

We now present the simulation results for the construction of a $(3, 2)$ code for a Gaussian and a uniform source. As explained in the previous section, we first partition the source space using a 22-level Lloyd-Max scalar quantizer. The 21 interval end points are then assigned alternately to each of the 3 coarse quantizers so as to give 3 8-level quantizers that are staggered with respect to each other. Each source sample that occurs is assigned a quantization index on each of the three channels corresponding to the quantizer for that channel. The sequence of quantizer indices for each channel are then sent as a syndrome sequence with respect to the subcode generator matrix assigned to that channel.

At the receiver, on the reception of any two syndrome sequences, the techniques introduced in [12] are employed to recover both the corresponding quantized sequences. To reduce the probability of decoding error, we used trellis codes [15] of increasing complexity. The table below shows the probability of decoding error (p_e) as function of complexity. Using trellises of increasing complexity [15], p_e can be made very small.

Complexity	p_e
2-state	1.575×10^{-2}
4-state	3.91×10^{-3}
8-state	7.1×10^{-4}

Gaussian		Uniform	
Desc.	Dist.(dB)	Desc.	Dist.(dB)
2	-18.53	2	-22.20
3	-22.86	3	-26.84

The adjoining table highlights the "jump" in quality obtained in going from *any* 2 to all 3 descriptions for both the Gaussian and the uniform source illustrating the joint source channel coding advantage. The reference system where the best source coded bit stream for 4 bits is passed through a $(3, 2)$ parity check code does not give any extra quality when all three descriptions are received. For the Gaussian case, with the best 16-level Lloyd-Max quantizer this value is -20.22 dB. Thus while the performance of the proposed system is inferior to that of the reference system for 2 descriptions it is strictly superior for 3 descriptions. Information theoretic arguments proposed in [4] suggest that the performance upon the reception of 2 channels matches that of the latter while for the reception of 3 channels it is strictly superior. We believe that the simplistic approach based on staggered quantizers to create the effect of independent quantization noise limits the performance of the proposed system. We are currently investigating the usage of dithering techniques to approach the effect of independent quantization noise and to try and match the performance of the reference system.

We would also like to point out that if a concatenation of $(3,2)$ and $(3,3)$ conventional channel codes were chosen as a reference system and the quality for 2 descriptions matched to -18.53 dB, then upon the reception of 3 descriptions the reference system would deliver a quality no better than -21.20 dB [16] which is about 1.6 dB below the proposed scheme. To conclude, preliminary simulation results point out that the above presented construction techniques, although relatively simple, highlight the promise of the proposed framework.

6 Conclusions and Future Work

We have presented new information theoretic techniques as well as new constructive mechanisms to approach the MD source coding problem from a distributed source coding point of view. Our ongoing work involves researching techniques such as dithered quantization for application to the source quantization part of the construction. Future work also includes building the code concatenation framework (see Figure 1 (a)) for realizing a symmetric MD system.

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