ISIT 2002, Lausanne, Switzerland, June 30 – July 5, 2002

n-Channel Symmetric Multiple Descriptions: New Rate Regions

Rohit Puri
EECS Dept.,
University of California, Berkeley
e-mail: rpuri@eecs.berkeley.edu

S. Sandeep Pradhan
EECS Dept.,
University of Michigan, Ann Arbor
e-mail: pradhanv@eecs.umich.edu

Kannan Ramchandran
EECS Dept.,
University of California, Berkeley
e-mail: kannanr@eecs.berkeley.edu

Abstract — **A new achievable region for the symmetric n-channel multiple descriptions (MD) problem ($n > 2$) is presented. This is derived from our recent result for the problem of information theoretic characterization of maximum distance separable erasure codes [1] in a source-channel coding framework. The rate region presented here is a generalization of the ElGamal and Cover region [2] for the case of $n = 2$ descriptions and achieves points that have not been known in the literature previously.**

I. Introduction

The problem of maximum distance separable (n, k) erasure codes was recently studied [1] in a joint source channel coding framework. In this problem, the channel guarantees the delivery of $m \geq k$ packets but there is uncertainty at the encoder as to *which* packets are received at the decoder. This attribute connects the problem intimately to the framework of distributed source coding as considered in [3]. Random binning ideas used in [3] were applied in [1] to exploit correlation with side information by reducing the transmission rate.

This work highlighted the theoretical advantage of doing joint source-channel coding over the traditional separation principle. While an approach based on the latter would suffer from the "cliff" [1] effect, a source channel coding based approach, at least for the case of a Gaussian source, could perform as well as the separation approach on reception of k packets while providing strictly superior quality on reception of $m > k$ packets.

In this work, we present a new rate region for the symmetric n-channel MD problem obtained by concatenation of $\{(n, 1), \ldots (n, n)\}$ source channel erasure codes as in Figure 1.

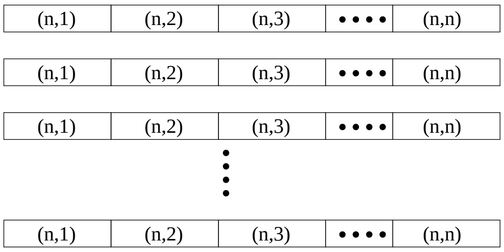

Figure 1: n-channel MD using source-channel erasure codes.

II. Rate Region for MD

For paucity of space and for notational convenience, we illustrate the approach for the $n = 3$ case. The results can be generalized to the case $n > 3$. Let X denote the source with p.m.f. $q(x)$. Let $Y_{11}, Y_{12}, Y_{13}, Y_{21}, Y_{22}, Y_{23}, Y_3$ be symmetrically distributed random variables as shown in Figure 2. Let $\mathcal{Y}_3$ denote the alphabet for Y_3 and $\mathcal{Y}_{ij}$ denote alphabet for Y_{ij} where $i \in I, I = \{1, 2\}$ and $j \in J, J = \{1, 2, 3\}$.

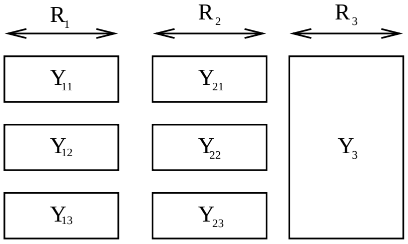

Figure 2: 3 channel MD coding using concatenation of $(3, 1), (3, 2), (3, 3)$ Erasure Channel Codes.

The total rate of transmission per packet R is partitioned as $R_1 + R_2 + R_3$. Rate R_1 per packet is used to send Y_{11}, Y_{12}, Y_{13} as a $(3, 1)$ code. The $(3, 2)$ code (that uses rate R_2) used here is encoded as a universal refinement [3] as opposed to a conventional successive refinement. That is, from the point of view of random variable Y_{21} the side information consists of one of Y_{22} and Y_{23} and also one of Y_{12} and Y_{13} besides Y_{11}. The final layer which is desired to be decodable only with the reception of all the packets can be sent at rate R_3 per packet using the conventional successive refinement approach.

Theorem: *An achievable rate region for a given distortion tuple (D_1, D_2, D_3) is given by $R > R_1 + R_2 + R_3$, where,*

$$R_1 > H(Y_{11}) - \frac{1}{3}H(Y_{1J}|X), \quad R_2 > \frac{1}{2}H(Y_{2I}|Y_{1I}) - \frac{1}{3}H(Y_{2J}|X)$$

$$R_3 > \frac{1}{3}I(X; Y_3|Y_{IJ}) \quad \textit{for some p.m.f } p(x, y_{IJ}, y_3)$$

$= q(x)p(y_{1J}|x)p(y_{2J}|x)p(y_3|x, y_{IJ})$, where $Y_{1J} \to X \to Y_{2J}$ is a Markov chain and a set of decoding functions

$$g_i^1 : \mathcal{Y}_{1i} \longrightarrow \hat{\mathcal{X}}, i \in J, \quad g_S^2 : \bigotimes_{k \in I, i \in S} \mathcal{Y}_{ki} \longrightarrow \hat{\mathcal{X}} \quad \forall S \subset J, |S| = 2$$

$$g^3 : \mathcal{Y}_3 \times \bigotimes_{k \in I, i \in J} \mathcal{Y}_{kj} \longrightarrow \hat{\mathcal{X}} \;\; \textit{such that} \;\; E[d(X, g_i^1(Y_{1i}))] \leq D_1 \; \forall i \in J$$

$$E[d(X, g_S^2(Y_{I,S}))] \leq D_2 \; \forall S \subset J, |S| = 2 \;\; E[d(X, g^3(Y_{I,J}, Y_3))] \leq D_3$$

[1] Reception of fewer than k packets is not sufficient to recover any data, while the reception of *any* k packets enables the recovery of the original k source packets and further reception of packets do not aid any further recovery.

References

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