

Achievable rates for multiple-access channels with correlated messages

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Consider a set of transmitters wishing to accomplish reliable simultaneous communication with a single receiver using a multiple access channel (MAC) [1]. A natural extension of Shannon's point-to-point communication paradigm involving source coding and channel coding to multiple terminals was first considered by Slepian and Wolf in a subsequent work [2]. They considered a special class of such problems where the messages of the two users have a common part [3]. In [3], a joint source-channel coding scheme is considered for transmitting distributed correlated sources over MAC, where a per-letter correlation in the sources is assumed.

In this work, instead, we address this problem by using graphs as a digital interface between the above multiuser source coding and channel coding problems. In this case, the multiterminal sources can be mapped independently into a message set characterized by a graph which preserves a predetermined amount of correlation between the sources, and this message set (which we refer to as a message-graph) becomes the input to the channel encoder. In general, with a message-graph, one need not associate any stochastic characterization. Toward this goal of improving the performance of transmission of correlated sources over MAC, in this work we address the channel coding part with an associated message-graph (to be defined shortly). Note that in a system with two transmitters, many pairs of distributed sources can be mapped into a message-graph. Consider two users communicating over a MAC such that there is some kind of "correlation" between their messages (see Fig. 1). We will use bipartite graphs to

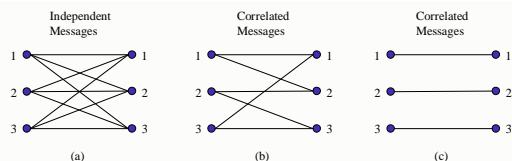


Figure 1: "Independent" and "correlated" messages.

describe "correlated" messages of the users. A word about the notation. For some finite sets \mathcal{A}, \mathcal{B} and \mathcal{C} , if $\mathcal{C} \subset \mathcal{A} \times \mathcal{B}$, then for any $a \in \mathcal{A}$, let $h_1(a, \mathcal{C})$ denote the largest subset of \mathcal{B} , where $\forall b \in h_1(a, \mathcal{C})$, we have $(a, b) \in \mathcal{C}$. Similarly define $h_2(b, \mathcal{C})$ for $\forall b \in \mathcal{B}$ to be the largest subset of \mathcal{A} such that $\forall a \in h_2(b, \mathcal{C})$ we have $(a, b) \in \mathcal{C}$. Define $h_3(\mathcal{C}) = \{a \in \mathcal{A} : h_1(a, \mathcal{C}) \neq \emptyset\}$ and $h_4(\mathcal{C}) = \{b \in \mathcal{B} : h_2(b, \mathcal{C}) \neq \emptyset\}$, where \emptyset denotes the null set.

Definition: A symmetric bipartite message-graph G , with integer parameters (θ, θ') is defined as follows. $G \subset \{1, 2, \dots, \theta\} \times \{1, 2, \dots, \theta\}$. $|h_3(G)| = |h_4(G)| = \theta$. $\forall a \in h_3(G)$ and $b \in h_4(G)$, we have $|h_1(a, G)| = |h_2(b, G)| = \theta'$. In other words, G represents just a collection of edges denoted by ordered pairs (i, j) , with $1 \leq i, j \leq \theta$, and the degree of

each vertex is θ' . The set $\{1, 2, \dots, \theta\}$ represents the message set of a transmitter.

Note that the symmetric system considered in this paper can be easily generalized to non-symmetric cases. We are given a symmetric stationary memoryless MAC with symmetric conditional distribution $p(y|x_1, x_2)$, with input alphabets being the same and given by a finite set \mathcal{X} , and a finite output alphabet \mathcal{Y} . The goal is to find the set of all symmetric message-graphs that can be transmitted reliably over a symmetric MAC, where the parameters θ and θ' are allowed to increase exponentially with the number of channel uses. In this paper we have a partial characterization of this set.

A transmission system with parameters $(n, \Delta, \Delta', \tau)$ for the given MAC with "correlated" messages would involve

- (i) A set of code-graphs \mathbb{C}^* where (a) $\mathbb{C}_i \subset \mathcal{X}^n$ and $|\mathbb{C}_i| = \Delta$ for $i = 1, 2$, (b) $\forall \mathbb{G} \in \mathbb{C}^*$, we have $\mathbb{G} \subset \mathbb{C}_1 \times \mathbb{C}_2$, (c) $|h_3(\mathbb{G})| = |h_4(\mathbb{G})| = \Delta$, (d) $|h_1(\mathbf{a}, \mathbb{G})| = |h_2(\mathbf{b}, \mathbb{G})| = \Delta'$, $\forall \mathbf{a} \in h_3(\mathbb{G})$ and $\forall \mathbf{b} \in h_4(\mathbb{G})$, (e) \mathbb{C}^* is the largest set with these properties.
- (ii) A set of decoder mappings $\{d_{\mathbb{G}} : \mathbb{G} \in \mathbb{C}^*\}$, where $d_{\mathbb{G}} : \mathcal{Y}^n \rightarrow \mathbb{G}$.
- (iii) A performance measure given by the following minimum-average probability of error criterion:

$$\tau = \min_{\mathbb{G} \in \mathbb{C}^*} \sum_{(\mathbf{a}, \mathbf{b}) \in \mathbb{G}} \frac{1}{|\mathbb{G}|} Pr[d_{\mathbb{G}}(\mathbf{Y}) \neq (\mathbf{a}, \mathbf{b}) | \mathbf{X}_1 = \mathbf{a}, \mathbf{X}_2 = \mathbf{b}].$$

The goal is to find the closure \mathcal{R}^* of the set of all achievable rate pairs (R, R') , where a rate pair (R, R') is said to be achievable for the given MAC with "correlated" message sets if $\forall \epsilon > 0$, and for sufficiently large n , there exists a transmission system as defined above with parameters $(n, \Delta, \Delta', \tau)$ with $\frac{1}{n} \log \Delta > R - \epsilon$, $\frac{1}{n} \log \Delta' > R' - \epsilon$ and the corresponding minimum-average probability of error $\tau < \epsilon$.

Theorem 1: An achievable rate region is given by the set of all (R, R') such that

$$R < \frac{1}{2} [I(X_1, X_2; Y) + I(X_1; X_2)], \quad R' < R - I(X_1; X_2)$$

for some symmetric probability distribution $p(x_1, x_2)$ defined on \mathcal{X}^2 .

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