Error Exponent Region for Gaussian Multiple Access Channels

Lihua Weng, Achilleas Anastasopoulos, and S. Sandeep Pradhan
Electrical Engineering and Computer Science Dept.
University of Michigan, Ann Arbor, MI 48109-2122
{lweng, anastas, pradhanv}@umich.edu

I. Introduction

We introduce the notion of error exponent region (EER) for a multi-user channel. We define a probability of error for each user, which, in general may be different for different users. Therefore, there are multiple error exponents, one for each user, for a given multi-user channel. The error exponent region specifies the set of error-exponent vectors, which are simultaneously achievable by all users in the multi-user channel [1]. In this work, we derive an inner bound (achievable region) and an outer bound for the error exponent region of a Gaussian multiple access channel (GMAC).

II. Formulation and Main Result

An error exponent region for a multi-user channel depends on the channel operating (rate) point. For a two-user channel, we use the notation $EER(R_1, R_2)$ to denote the EER when the channel is operated at rate pair (R_1, R_2). Consider a GMAC

$$Y = X_1 + X_2 + Z, \tag{1}$$

where X_1 and X_2 are the channel inputs for user 1 and user 2 with $E(X_1^2) = SNR_1$, $E(X_2^2) = SNR_2$, and Z is white Gaussian noise with unit variance. Denote $E(R, SNR)$ the maximum of random coding exponent and expurgated exponent of a single-user Gaussian channel. Our main result follow.

Theorem 1: For a two-user GMAC in (1), an achievable $EER(R_1, R_2)$ (inner bound) is $EER_s(R_1, R_2) \cup EER_{td}(R_1, R_2)$, where $EER_{td}(R_1, R_2)$, achievable region using time-division, and $EER_s(R_1, R_2)$, achievable region using superposition, are

$$\begin{aligned}
EER_{td}(R_1,R_2) = \{&(e_1,e_2) : 0 < \alpha < 1 \\
&e_1 \le \alpha E(\frac{R_1}{\alpha}, \frac{SNR_1}{\alpha}), e_2 \le (1-\alpha)E(\frac{R_2}{1-\alpha}, \frac{SNR_2}{1-\alpha})\},\\
EER_s(R_1,R_2) = \{&(e_1,e_2) : \\
&e_1 \le \max\{E(R_1, \frac{SNR_1}{SNR_2+1}),\\
&\quad \min\{E(R_1, SNR_1), E_{t3}(R_1+R_2, SNR_1, SNR_2)\}\},\\
&e_2 \le \max\{E(R_2, \frac{SNR_2}{SNR_1+1}),\\
&\quad \min\{E(R_2, SNR_2), E_{t3}(R_1+R_2, SNR_1, SNR_2)\}\},
\end{aligned}$$

where

$$\begin{aligned}
E_{t3}(R_3, SNR_1, SNR_2) &= \max_{\rho,\theta_1,\theta_2} \{E_{t3,0}(\rho,\theta_1,\theta_2) - \rho R_3\}\\
E_{t3,0}(\rho,\theta_1,\theta_2) &= (1+\rho)\ln\left[\frac{e\sqrt{\theta_1\theta_2}}{1+\rho}\right] - \frac{\theta_1+\theta_2}{2}\\
&\quad + \frac{\rho}{2}\ln\left[1 + \frac{SNR_1}{\theta_1} + \frac{SNR_2}{\theta_2}\right]
\end{aligned}$$

is the type-3 error exponent, and maximization is over $0 \le \rho \le 1$, and $0 < \theta_1, \theta_2 \le 1 + \rho$ [2]. □

Theorem 2: For a two-user GMAC in (1), an $EER(R_1, R_2)$ outer bound is

$$\begin{aligned}
E_1 &\le E^{su}(R_1, SNR_1)\\
E_2 &\le E^{su}(R_2, SNR_2)\\
\min\{E_1, E_2\} &\le E^{su}(R_1 + R_2, SNR_1 + SNR_2),
\end{aligned}$$

where $E^{su}(R, SNR)$ is any error exponent upper bound for a single-user Gaussian channel. □

In Fig. 1(a), the solid curve is the boundary of the EER inner bound, and the dashed curve is the boundary of the EEE outer bound. For a single-user Gaussian channel, an error exponent is known only for $R \ge R_{crit}$. Therefore, we can't expect to find EERs for all operating points in a GMAC. The EER inner bound and the EER outer bound are indeed tight for some operating points in a GMAC. This is shown in Fig. 1(b). The solid curve is the boundary of both the EER inner and outer bounds. The error exponent pair (E_1, E_2) in Fig. 1(b) achieves point-to-point single-user Gaussian error exponents for each user. For this operating point, each user seems to operate on a single-user Gaussian channel, instead of interfering with each other on a GMAC.

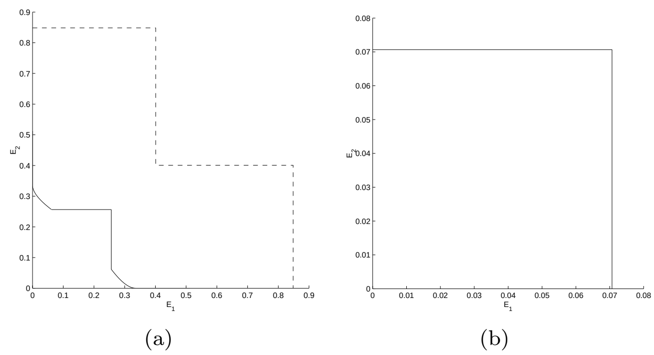

Figure 1: (a) EER inner and outer bounds ($R_1 = R_2 = 0.5$; $SNR_1 = SNR_2 = 10$). (b) EER ($R_1 = R_2 = 0.15$; $SNR_1 = SNR_2 = 1$). Inner and outer bounds are tight.

References

[1] L. Weng, S. S. Pradhan, and A. Anastasopoulos, "Error exponent region for Gaussian broadcast channels," in *Proc. Conference on Information Sciences and Systems (CISS)*, Princeton, NJ, Mar. 2004.

[2] R. G. Gallager, "A perspective on multiaccess channels," *IEEE Tran. Information Theory*, vol. 31, pp. 124-142, Mar. 1985.