On functional duality in MIMO source and channel coding problems having one-sided collaboration

S. Sandeep Pradhan
Dept. of EECS
Univ. of Michigan, Ann Arbor, USA
e-mail: pradhanv@eecs.umich.edu

Kannan Ramchandran
Dept. of EECS
Univ. of California, Berkeley, USA
e-mail: kannanr@eecs.berkeley.edu

Abstract — **We address duality in a variety of multiple-input-multiple-output (MIMO) source and channel coding problems of interest under different scenarios of "one-sided" inter-terminal collaboration at either the transmitter or at the receiver, including certain cases of duality between (i) broadcast channel coding and distributed source coding, and (ii) multi-access channel coding and multiple-descriptions source coding. Our notion of duality in this paper is in a functional sense, where the optimal encoder mapping for a MIMO source coding problem becomes identical to the optimal decoder mapping for the dual MIMO channel coding problem, and vice versa. For ease of illustration we give the formulation only for two-input-two-output systems, which can be easily extended to the MIMO case. We present the precise mathematical conditions under which these encoder-decoder mappings are swappable in the two dual MIMO problems, identifying the key roles played by the source distortion and channel cost measures respectively in the MIMO source and channel coding problems in capturing this duality.**

I. Introduction

Since the first observation by Shannon in 1959 [1] that source and channel coding problems can be studied as information-theoretic duals of each other, a number of researchers have made significant contribution to furthering this understanding, as noted in excellent textbooks like [2, 3]. In [4], this duality has been studied in the context of quantization and modulation. As multiuser information theory has gained significant interest recently, there has been a corresponding interest in extending this duality to these scenarios. Duality between source and channel coding with side information was first reported in [5], and later studied in [6].

A mathematical formulation of the functional duality between conventional point-to-point source and channel coding problems was given in [7], where the important roles of distortion and cost measures for these two problems were highlighted, inspired by seemingly unrelated work on the optimality of uncoded transmission in [8]. This concept was then extended to the case of source and channel coding with side information. The notion of duality addressed in [7] is in a functional sense, i.e. the optimal encoder-decoder mappings for one problem become the optimal decoder-encoder mappings for the dual problem. The precise conditions under which such swappable mappings are feasible involve dual relationships between distortion and cost measures in the source and channel coding problems respectively. Study of this functional duality serves two important purposes: (i) it provides new insights into these problems from the different perspectives of source and channel coding, and allows for cross-leveraging of advances in the individual fields; (ii) more importantly, it provides a basis for sharing efficient constructions of the encoder and decoder functions in the two problems, e.g., through the use of structured algebraic codes, turbo-like codes, trellis-based codes, etc.

In this work we extend this notion of functional duality to more instances of MIMO source and channel coding problems. We study various MIMO structures admitting different scenarios of collaboration among multi-terminal inputs and/or outputs[1]. The collaboration scenarios we consider involve those where *either* the multi-terminal encoders *or* the multi-terminal decoders can collaborate, i.e. be joint, but not both[2]. Under this one-sided collaboration abstraction, we address four problems of interest in source and channel coding: 1) distributed source coding, 2) broadcast channel coding 3) multiple description source coding with no excess sum-rate, and 4) multiple access channel coding with independent message sets. In 1) and 4), the decoders collaborate, whereas in 2) and 3), the encoders collaborate. These four problems have been studied in the literature extensively. In this paper we point out that for a given distributed source coding problem, under certain cases, we can obtain a specific dual broadcast channel coding problem and vice versa. Similarly, for a given multiple description coding problem, we can find a specific dual multiple access channel coding problem and vice versa. In the next section we briefly outline the definition of these four problems.

II. MIMO source and channel coding

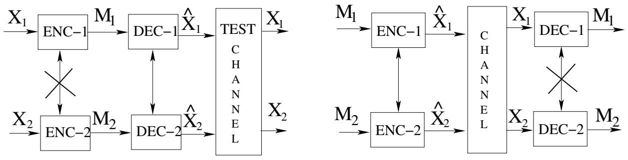
In this section we briefly define the four problems. A

[1]This work was supported in part by NSF ITR grant CCR-0219735, and DARPA Sensor Information Technology Office under the Sensorwebs project F30602-00-2-0538.

[1]To keep the exposition simple, in this paper we describe only two-input-two-output systems.

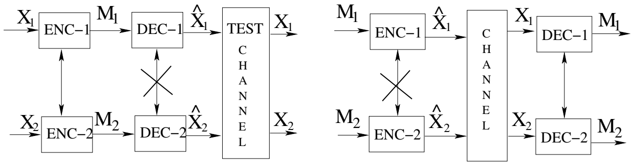
[2]The case of collaboration at both ends degenerates to point-to-point MIMO systems.

word about the notation: for the source coding problems, we denote $\mathbf{X}$ as the vector source and $\hat{\mathbf{X}}$ as the vector reconstruction, whereas in channel coding we denote $\mathbf{X}$ as the vector channel output with $\hat{\mathbf{X}}$ denoting the vector channel input. The components of $\mathbf{X}$ are X_1 and X_2, and similarly those of $\hat{\mathbf{X}}$ are $\hat{X}_1$ and $\hat{X}_2$. In the source coding problems, we interchangeably use 1) $\hat{X}_i$ for $i = 1, 2$ as different reconstructions of the pair $\hat{X}_1$ and $\hat{X}_2$; or 2) $\hat{X}_i$ as a reconstruction of X_i alone. This will be evident from the distortion measure of the particular problem. In all these problems we use a single distortion/cost measure and optimize sum-rate of representation/transmission. More general cases are studied in [9].



(a) (b)

Figure 1: (a) Distributed source coding (b) Broadcast channel coding



(a) (b)

Figure 2: (a) Multiple description source coding (b) Multiple access channel coding

1. Distributed source coding [10]: (see Fig. 1(a)) Let X_1 and X_2 be two correlated memoryless sources with a given joint distribution $\bar{p}(X_1, X_2)$, and with alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$ respectively. The encoder associated with each of these sources wishes to represent its corresponding source using an index sequence. The encoders do not communicate with each other. The joint decoder having access to these two index sequences, wishes to get a joint estimate of the sources. Let $d : \mathcal{X}_1 \times \mathcal{X}_2 \times \hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$ denote the joint distortion measure, where $\hat{\mathcal{X}}_1$ and $\hat{\mathcal{X}}_2$ denote the reconstruction alphabets. The encoders are given by: $\mathcal{X}_i^L \to \{1, 2, \ldots, 2^{R_i L}\}$ for $i = 1, 2$, and the decoder is given by $\{1, 2, \ldots, 2^{R_1 L}\} \times \{1, 2, \ldots, 2^{R_2 L}\} \to \hat{\mathcal{X}}_1^L \times \hat{\mathcal{X}}_2^L$. The achievable [10] sum rate $R_1 + R_2$ for a distortion constraint D, for this problem is given by

$$R_{DS}(D) = \min_{p(u_1,u_2|x_1,x_2)} \min_{p(\hat{x}_1,\hat{x}_2|u_1,u_2)} [I(X_1;U_1)+$$

$$I(X_2;U_2) - I(U_1;U_2)] \qquad (1)$$

such that $U_2 \to X_2 \to X_1 \to U_1$, and $(X_1, X_2) \to (U_1, U_2) \to (\hat{X}_1, \hat{X}_2)$ form a Markov chain, and $Ed(X_1, X_2, \hat{X}_1, \hat{X}_2) \leq D$, where U_1 and U_2 are auxiliary random variables with alphabets $\mathcal{U}_1$ and $\mathcal{U}_2$ respectively. *Examples of the distortion function*: 1) reconstruction of the sum of X_1 and X_2, $(x_1 + x_2 - \hat{x}_1 - \hat{x}_2)^2$; 2) sum distortion of X_1 and X_2, $(x_1 - \hat{x}_1)^2 + (x_2 - \hat{x}_2)^2$.

2. Broadcast channel coding [11]: (see Fig. 1(b)) Consider a general broadcast channel with a given conditional distribution $\bar{p}(X_1, X_2|\hat{X}_1, \hat{X}_2)$, with $\hat{\mathcal{X}}_i$ as input alphabet and $\mathcal{X}_i$ as output alphabet for $i = 1, 2$. The joint encoder wishes to simultaneously communicate with two receivers with rates R_1 and R_2. There is a joint cost measure associated with this channel: $w : \hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$. The encoder is a mapping $\{1, 2, \ldots, 2^{R_1 L}\} \times \{1, 2, \ldots, 2^{R_2 L}\} \to \hat{\mathcal{X}}_1^L \times \hat{\mathcal{X}}_2^L$. The decoders are given by: $\mathcal{X}_i^L \to \{1, 2, \ldots, 2^{R_i L}\}$ for $i = 1, 2$. The achievable [11] sum rate $R_1 + R_2$ for a cost constraint of W is given by

$$C_{BC}(W) = \max_{p(u_1,u_2)} \max_{p(\hat{x}_1,\hat{x}_2|u_1,u_2)} [I(X_1;U_1)+$$

$$I(X_2;U_2) - I(U_1;U_2)] \qquad (2)$$

such that $(U_1, U_2) \to (\hat{X}_1, \hat{X}_2) \to (X_1, X_2)$ and $(X_1, X_2) \to (U_1, U_2) \to (\hat{X}_1, \hat{X}_2)$ form a Markov chain, and $Ew(\hat{X}_1, \hat{X}_2) \leq W$, where U_1 and U_2 are auxiliary random variables with alphabets $\mathcal{U}_1$ and $\mathcal{U}_2$ respectively. *Examples of the cost function*: 1) sum power, $(\hat{x}_1^2 + \hat{x}_2^2)$; 2) power of sum of X_1 and X_2, $(\hat{x}_1 + \hat{x}_2)^2$.

3. Multiple Description source Coding with no excess sum-rate [12, 13]: (see Fig. 2(a)) Here we have two correlated sources X_1 and X_2 with a given joint distribution $\bar{p}(X_1, X_2)$ with alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$. The joint encoder wishes to represent these sources using two descriptions to be sent to two receivers. The receivers do not communicate with each other. The encoder is a mapping: $\mathcal{X}_1 \times \mathcal{X}_2 \to \{1, 2, \ldots, 2^{R_1 L}\} \times \{1, 2, \ldots, 2^{R_2 L}\}$. R_i denotes the rate of the ith description. The ith decoder is given by the mapping: $\{1, 2, \ldots, 2^{R_i L}\} \to \hat{\mathcal{X}}_i^L$, and let $d : \mathcal{X}_1 \times \mathcal{X}_2 \times \hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$ denote the joint distortion measure[3], where $\hat{\mathcal{X}}_1$ and $\hat{\mathcal{X}}_2$ denote the reconstruction alphabets. The sum-rate-distortion function [13] for a distortion constraint D for this problem is given by

$$R_{MD}(D) = \min_{p(\hat{x}_1,\hat{x}_2|x_1,x_2)} I(X_1, X_2; \hat{X}_1, \hat{X}_2) \qquad (3)$$

such that $\hat{X}_1, \hat{X}_2$ are independent and $Ed(X_1, X_2, \hat{X}_1, \hat{X}_2) \leq D$.

4. Multiple access channel coding with independent message sets [14, 15]: (see Fig. 2(b)) Consider a multiple access channel with a given conditional distribution $\bar{p}(X_1, X_2, |\hat{X}_1, \hat{X}_2)$. Two non-collaborating encoders wish to transmit messages to a joint decoder

[3]In this paper for a precise notion of duality, we consider a slightly different version (with a single distortion measure) of the multiple description coding than that studied in the literature (with three distortion measure). In the latter, three distortion measures are associated with different subsets of descriptions received. The former is a subset of the latter in the following sense: the reconstruction $\hat{X}_0$ for the reception of all descriptions is a function of the reconstructions $\hat{X}_1, \hat{X}_2$ corresponding to the reception of individual descriptions. The single distortion measure $d(x_1, x_2, \hat{x}_1, \hat{x}_2)$ is able to capture this function.

with rates R_1, R_2. The ith encoder is given by the mapping: $\{1, 2, \ldots, 2^{R_i L}\} \to \hat{\mathcal{X}}_i^L$. The joint decoder is given by $\mathcal{X}_1 \times \mathcal{X}_2 \to \{1, 2, \ldots, 2^{R_1 L}\} \times \{1, 2, \ldots, 2^{R_2 L}\}$. There is a joint cost function associated with this channel: $w : \hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$. The sum capacity-cost function [14] is given by

$$C_{MA}(W) = \max_{p(\hat{x}_1, \hat{x}_2)} I(X_1, X_2; \hat{X}_1, \hat{X}_2) \tag{4}$$

such that $\hat{X}_1, \hat{X}_2$ are independent and $Ew \leq W$.

III. Main results

Now we give a formulation of duality between these MIMO source and channel coding problems with limited collaboration. Note that although the rate region for general multiple description coding and general multiple access channel coding have been shown to be tight, similar converses for the general case for distributed source coding and broadcast channel coding are still not available. This duality deals only with the available single-letter characterization of rate regions. In the sequel we show that for a given distributed source coding problem, under certain conditions, we can associate a dual broadcast channel coding problem where both problems induce the same joint distribution of variables with suitable correspondence.

Similarly, for a given broadcast channel coding problem, under certain conditions, we can associate a specific dual distributed source coding problem. In a pair of dual problems, the encoder of the one is functionally identical to the decoder of the latter and vice versa. Similarly, we can obtain a duality formulation between multiple description coding with no excess sum-rate and multiple access channel with independent message sets. These results are stated as the following theorems.

Theorem 1a: For a given distributed source coding problem with a given source $\bar{p}(X_1, X_2)$, and alphabets $\mathcal{X}_i$ and reconstruction alphabets $\hat{\mathcal{X}}_i$ for $i = 1, 2$, a distortion measure $d : \mathcal{X}_1 \times\ \mathcal{X}_2 \times\ \hat{\mathcal{X}}_1 \times\ \hat{\mathcal{X}}_2 \to \mathbb{R}^+$, a distortion constraint D, let the conditional distributions satisfying the optimality condition of (1) be given by $p^*(u_1, u_2, |x_1, x_2)$ and $p^*(\hat{x}_1, \hat{x}_2 | u_1, u_2)$, inducing the joint distribution $p^*(x_1, x_2, u_1, u_2, \hat{x}_1, \hat{x}_2) = \bar{p}(x_1, x_2)$ $p^*(u_1, u_2 | x_1, x_2)$ $p^*(\hat{x}_1, \hat{x}_2 | u_1, u_2)$. Let

$$p^*(u_1, u_2) = \sum_{x_1, x_2, \hat{x}_1, \hat{x}_2} p^*(x_1, x_2, u_1, u_2, \hat{x}_1, \hat{x}_2),$$

$$p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2) = \frac{\sum_{u_1, u_2} p^*(x_1, x_2, u_1, u_2, \hat{x}_1, \hat{x}_2)}{\sum_{u_1, u_2, x_1, x_2} p^*(x_1, x_2, u_1, u_2, \hat{x}_1, \hat{x}_2)}.$$

If $p^*(x_1, x_2, u_1, u_2, \hat{x}_1, \hat{x}_2)$ satisfies $(U_1, U_2) \to (\hat{X}_1, \hat{X}_2) \to (X_1, X_2)$, then $\exists$ a dual broadcast channel coding problem for the channel $p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2)$, with input alphabets $\hat{\mathcal{X}}_i$ and output alphabets $\mathcal{X}_i$, a cost measure $w : \hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$, and a cost constraint W such that

- achievable rate bound $R_{DS}(D) = C_{BC}(W)$, i.e.,

$$\min \quad I(X_1; U_1) + I(X_2; U_2) - I(U_1; U_2) =$$

$$\max \quad I(X_1; U_1) + I(X_2; U_2) - I(U_1; U_2) \tag{5}$$

  where the minimization is carried out over $p(u_1, u_2 | x_1, x_2)$ and $p(\hat{x}_1, \hat{x}_2 | u_1, u_2)$ with source distribution $\bar{p}(x_1, x_2)$ held fixed such that $Ed \leq D$, $U_2 \to X_2 \to X_1 \to U_1$ and $(X_1, X_2) \to (U_1, U_2) \to (\hat{X}_1, \hat{X}_2)$; and the maximization is carried out over $p(u_1, u_2)$ and $p(\hat{x}_1, \hat{x}_2 | u_1, u_2)$ with channel conditional distribution $p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2)$ held fixed such that $Ew \leq W$, $(U_1, U_2) \to (\hat{X}_1, \hat{X}_2) \to (X_1, X_2)$, and $(X_1, X_2) \to (U_1, U_2) \to (\hat{X}_1, \hat{X}_2)$,

- the distributions $p^*(u_1, u_2)$ and $p^*(\hat{x}_1, \hat{x}_2 | u_1, u_2)$ induced from the first optimization achieve this maximization,

provided the cost measure and the cost constraint are chosen such that

$$w(\hat{x}_1, \hat{x}_2) = c_1 D(p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2) || \bar{p}(x_1, x_2)) + \theta, \tag{6}$$

and $W = E_{p^*(\hat{x}_1, \hat{x}_2)}[w(\hat{x}_1, \hat{x}_2)]$, for arbitrary $c_1 > 0$ and θ.

Similarly, for a given broadcast channel coding problem, if the optimizing distribution satisfies the Markov chain necessary for the distributed source coding problem, then we can find a dual distributed source coding problem. Due to lack of space, we illustrate this with the following example. Note that the encoder of the first problem is functionally identical to that of the decoder of the latter and vice versa. Further, the optimal random codebooks are constructed using the distributions $p^*(u_1)$ and $p^*(u_2)$.

Example 1: Consider a two-input-two-output broadcast channel $\mathbf{X} = H\hat{\mathbf{X}} + \mathbf{N}$, where $\mathbf{N}$ is a zero-mean Gaussian vector with identity matrix I as the covariance matrix, where H is a matrix. The cost measure is sum-power: $w(\hat{x}_1, \hat{x}_2) = (\hat{x}_1^2 + \hat{x}_2^2)$, and $W = 2P$ for some $P > 0$. It was shown in [16] that the sum-rate as given by (2) for this channel is tight using Sato's bound, which says that the capacity of a broadcast channel depends only on the marginals $\bar{p}(x_i | \hat{x}_1, \hat{x}_2)$ for $i = 1, 2$. It was shown that by carefully varying the covariance matrix of the noise in the channel while keeping the marginals (ones on the diagonals) the same, the capacity of this channel if the receivers were to cooperate can be made exactly equal to that given by (2). Let Σ_n be this least favorable noise covariance matrix (see [16, 17]). Then it can be shown that the induced joint distribution does indeed satisfy $U_1 \to X_1 \to X_2 \to U_2$. The covariance matrix of the channel output is $(H\Sigma_{\hat{x}}H^T + \Sigma_n)$, where $\Sigma_{\hat{x}}$ is the optimum input distribution. In other words, if the noise covariance matrix of the given broadcast channel is changed to Σ_n, then we can define its dual distributed source coding problem with jointly Gaussian source with zero-mean and covariance matrix $(H\Sigma_{\hat{x}}H^T + \Sigma_n)$, and a quadratic distortion measure of the form $(\mathbf{x} - H\hat{\mathbf{x}})^T \Sigma_n^{-1} (\mathbf{x} - H\hat{\mathbf{x}})$.

Theorem 2a: Given a multiple description coding problem with no excess sum-rate, with a given source $\bar{p}(X_1, X_2)$, with alphabets $\mathcal{X}_i$ and reconstruction alphabets $\hat{\mathcal{X}}_i$ for $i = 1, 2$, a distortion measure $d : \mathcal{X}_1 \times\ \mathcal{X}_2 \times$

$\hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$, a distortion constraint D, let the conditional distributions satisfying the optimality condition of (3) be given by $p^*(\hat{x}_1, \hat{x}_2 | x_1, x_2)$, inducing the joint distribution $p^*(x_1, x_2, \hat{x}_1, \hat{x}_2) = \bar{p}(x_1, x_2) p^*(\hat{x}_1, \hat{x}_2 | x_1, x_2)$. Let

$$p^*(\hat{x}_1, \hat{x}_2) = \sum_{x_1, x_2} p^*(x_1, x_2, \hat{x}_1, \hat{x}_2),$$

$$p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2) = \frac{p^*(x_1, x_2, \hat{x}_1, \hat{x}_2)}{p^*(\hat{x}_1, \hat{x}_2)}.$$

Then $\exists$ a dual multiple access channel coding problem with independent message sets for the channel $p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2)$, with input and output alphabets $\hat{\mathcal{X}}_i$ and $\mathcal{X}_i$ for $i = 1, 2$ respectively, a cost measure $w : \hat{\mathcal{X}}_1 \times \hat{\mathcal{X}}_2 \to \mathbb{R}^+$, a cost constraint W, such that

- the sum rate-distortion bound $R_{MD}(D)$ is equal to the sum capacity-cost bound $C_{MA}(W)$, i.e.,

  $$\min\ I(X_1, X_2; \hat{X}_1, \hat{X}_2) = \max\ I(X_1, X_2; \hat{X}_1, \hat{X}_2),$$

  where the minimization is carried out over $p(\hat{x}_1, \hat{x}_2 | x_1, x_2)$ such that $Ed \leq D$, and $\hat{X}_1$ and $\hat{X}_2$ are independent, with source distribution $\bar{p}(x_1, x_2)$ held fixed; and the maximization is carried out over $p(\hat{x}_1, \hat{x}_2)$ such that $Ew \leq W$, and $\hat{X}_1$ and $\hat{X}_2$ independent, with channel conditional distribution $p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2)$ held fixed.

- the distribution $p^*(\hat{x}_1, \hat{x}_2)$ induced from the first optimization achieves this maximization,

provided the cost measure and cost constraint for this problem are chosen such that

$$w(\hat{x}_1, \hat{x}_2) = c_1 D(p^*(x_1, x_2 | \hat{x}_1, \hat{x}_2) || \bar{p}(x_1, x_2)) + \theta, \quad (7)$$

and $W = E_{p^*(\hat{x}_1, \hat{x}_2)}[w(\hat{x}_1, \hat{x}_2)]$, for arbitrary $c_1 > 0$ and θ.

As in Theorem 1, here too for a pair of dual problems, the encoder of one is functionally identical to the decoder of the other and vice versa. Further, for a given multiple access channel coding problem with independent messages we can associate a specific dual multiple description problem with no excess sum rate. This is illustrated by the following example. The optimal random codebooks are constructed from the distribution $p^*(u_1)$ and $p^*(u_2)$.

Example 2: Given a two-input-two-output multiple access channel coding problem with channel given by: $\mathbf{X} = H\hat{\mathbf{X}} + \mathbf{N}$, where $\mathbf{N}$ is a zero-mean Gaussian vector with identity matrix I as the covariance matrix, where H is a matrix. The cost measure is sum-power: $w(\hat{x}_1, \hat{x}_2) = (\hat{x}_1^2 + \hat{x}_2^2)$, and $W = 2P$ for some P. For simplicity let the rows of H be $[1, \alpha]$ and $[\alpha, 1]$ for some $\alpha > 0$. The optimization of the sum rate yields $C_{MA}(2P) = \frac{1}{2}\log\left[(P(1+\alpha^2)+1)^2 - 4\alpha^2 P^2\right]$. This results in the covariance matrix of the channel output being $P(HH^T) + I$. Each encoder transmits at power P. Its dual multiple description coding problem will have a Gaussian vector source with zero mean and covariance matrix given by $P(HH^T) + I$, which is the same as that of the channel output of the given multiple access problem. The appropriate distortion measure is quadratic and is given by $d(\mathbf{x}, \hat{\mathbf{x}}) = (\mathbf{x} - H\hat{\mathbf{x}})^T (\mathbf{x} - H\hat{\mathbf{x}})$.

IV. Conclusions

In this paper we have presented a formulation of functional duality in MIMO source and channel coding problems with one-sided inter-terminal collaboration. Our notion of duality is in the functional sense. We have shown that under certain conditions, for a given distributed source coding problem, we can associate a specific dual broadcast channel coding problem, and similarly vice versa. Further, we have shown that for a given multiple description coding problem with no excess sum-rate, we can associate a specific dual multiple access channel with independent message sets, and similarly vice versa. The encoders of a given problem is functionally identical to the decoders of the dual problem. The main motivation to addressing this notion of duality is to get further insight into these multi-terminal problems, and inspire constructions of structured codes for these problems within a common framework. Further, other properties of these problems along lines of [18, 19], and the topic of joint MIMO source-channel coding will be addressed as a part of the future work [9].

References

[1] C. E. Shannon, "Coding theorems for a discrete source with a fidelity criterion," *In IRE Nat. Conv. Rec.*, vol. Part 4, pp. 142–163, 1959.

[2] I. Csiszar and J. Korner, *Information Theory: Coding Theorems for Discrete memoryless sources*. Academic Press, New York, 1981.

[3] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley and Sons, 1991.

[4] M. V. Eyuboglu and G. D. Forney, "Lattice and trellis quantization with lattice and trellis-bounded codebooks- High rate theory for memoryless sources," *IEEE Trans. Inform. Theory*, vol. 39, pp. 46–59, Jan. 1993.

[5] J. Chou, S. S. Pradhan, and K. Ramchandran, "On the duality between distributed source coding and data hiding," *Proc. of 33rd Asilomar Conf. on Signals, Systems and Computers*, November 1999.

[6] M. Chiang and T. M. Cover, "Unified duality between channel capacity and rate distortion with side information," in *Proc. Int. Symp. on Info. Theory (ISIT), Washington D.C.*, June 2001.

[7] S. S. Pradhan, J. Chou, and K. Ramchandran, "Duality between source and channel coding with side information," in *UCB/ERL Technical Memorandum No. M0/34*, December 2001.

[8] M. Gastpar, B. Rimoldi, and M. Vetterli, "To code or not to code," *Proc. IEEE International Symposium on Information Theory (ISIT), Sorrento, Italy*, June 2000.

[9] S. S. Pradhan, and K. Ramchandran, "Duality in MIMO source and channel coding" *In preparation*, 2002.

[10] T. Berger, "Multiterminal Source Coding," *Information Theory Approach to Communication, (CISM Courses and Lecture Notes No. 229)*, G. Longo, Ed., Wien and New York: Springer-Verlag, 1977.

[11] K. Marton, "A coding theorem for the discrete memoryless broadcast channel," *IEEE Trans. on Inform. Theory*, vol. IT-25, pp. 306–311, May 1979.

[12] A. El Gamal and T. M. Cover, "Achievable Rates for Multiple Descriptions," *IEEE Transactions on Information Theory*, vol. 28, pp. 851–857, November 1982.

[13] R. Ahlswede, "The rate-distortion region for multiple descriptions without excess rate," *IEEE Trans. on Inform. Theory*, vol. IT-31, pp. 721–726, November 1985.

[14] R. Ahlswede, "Multiway communication channels," *Proc. 2nd Int. Symp. Inform. Theory, Tsahkadsor, Armenian S. S. R., Publishing house of Hungarian Academy of sciences*, pp. 23–52, 1971.

[15] R. Ahlswede, "An elementary proof of the strong converse theorem for the multiple-access channel," *J. Comb. Inform. Syst. Sci.*, vol. 7, pp. 216–230, 1982.

[16] G. Caire and S. Shamai, "On achievable rates in a multi-antenna Gaussian broadcast channel," in *Proc. Int. Symp. on Info. Theory (ISIT), Washington D.C.*, June 2001.

[17] W. Yu and J. Cioffi, "Sum capacity of a Gaussian vector broadcast channel," in *Proc. Int. Symp. on Info. Theory (ISIT), Lausanne, Switzerland*, July 2002.

[18] N. Jindal, amd S. Vishwanath, and A. Goldsmith, "On the duality of multiple-access and broadcast channels," *Proc. Allerton Conf. on Commn., Computing, and Control*, Oct. 2001.

[19] D. N. C. Tse, and P. Viswanath, "Downlink-Uplink duality and effective bandwidth," *Proc. Int. Symp. on Info. Theory (ISIT), Lausanne, Switzerland*, July 2002.