The Semantics of Conditional Modality*

Richmond H. Thomason
Philosophy Department
University of Michigan

rthomaso@umich.edu
http://www.eecs.umich.edu/~rthomaso/

May 9, 2012

> ***Warning:*** This is a draft of an in-progress paper. It is likely to contain misprints, and certainly there are places in need of clarification and reformulation. Please send comments to the author.

Abstract: This paper proposes a new semantics for the conditional, which combines elements of Stalnaker's theory and the rival approach due to Lewis and Kratzer. It is claimed that this conditional enables a simple compositional treatment of combinations of conditionals with modalities.

*I owe thanks to Craige Roberts to several conversations about this topic while the ideas were in an early stage of development, and to Kai von Fintel for helpful discussions concerning Conditional Excluded Middle.

1. Introduction

An approach to modality, conditionals, and their interpretation due to David Lewis and Angelika Kratzer has gained widespread acceptance in linguistic circles. Although this may be due in part to the fact that few competitors have been presented in the linguistic literature, the approach is attractive in itself, incorporating some good ideas. Here are three of them.

1. The approach uses possible worlds semantics, and there are robust connections to the logical literature in modal logic.
2. Modals combine with conditionals to produce *conditional modalities*: constructions whose meanings are problematic is some ways, but which, one would hope, would somehow depend systematically and uniformly on the meaning of the conditional and the particular modality with which it combines. The Lewis-Kratzer account provides a theory of this.
3. In accounting for the meaning of both modals and conditionals it is useful to use relations of "relative nearness" or preference among worlds. The Lewis-Kratzer account uses such relations, and, while making minimal assumptions about the properties of nearness relations, provides uniform satisfaction conditions for modalities, conditionals, and combined constructions.

But the approach has some features that, to me at least, seem like flaws.

1. The theory doesn't explain some things that it has been claimed to explain.
2. The satisfaction conditions that go along with this approach enforce a logic of the conditional that is controversial and that doesn't match the linguistic evidence well.
3. Although preference certainly plays a crucial role in the semantics of modals, the theory depends on a uniform treatment of the role preference plays in satisfaction conditions for modals and related constructions. This makes it difficult to reconcile the linguistic theory with work on preferential semantics in logic, economics, and computer science. This work suggests that the relation of preference to truth conditions is less uniform and more complex.

2. The Lewis-Kratzer semantics for modals and conditionals

The problem of "counterfactual conditionals" emerged as a problem in analytic philosophy during the first half of the 20th century.[1] The development and acceptance of possible worlds semantics for modal logic allowed this problem to be reinterpreted as a logical challenge, taking the following form: provide plausible satisfaction conditions for "counterfactual" or "subjunctive" conditionals.

[1] I don't know of an extended historical survey of the literature on conditionals from this period; but see [Bennett, 2003, Chapter 20] and [Schneider, 1953]. Two important representative works are [Goodman, 1947] and [Goodman, 1955].

Two independent solutions to this logical problem appeared in the late 1960s: one due to Robert Stalnaker and the other to David Lewis.[2] Both theories used orderings over worlds and appealed to the idea that a conditional $\phi > \psi$[3] is true if and only if ψ is true in the closest worlds satisfying ψ; but they did this in very different ways.

We can ignore Stalnaker's account for the moment. In general, logical languages for modality or conditionals are interpreted with respect to a *frame* or *modal structure* $\mathcal{S}$ and a model M on $\mathcal{S}$. A *Lewis frame* is a pair $\mathcal{S} = \langle W, \prec \rangle$, where W is a nonempty set and, for each $w \in W$, $\prec_w$ is a preorder over W[4] with minimal element w.

Lewis's satisfaction condition for $>$ in [Lewis, 1973, pp. 48–49] reads as follows.

Definition 2.1. Lewis satisfaction for $>$.

Let M be a model on a Lewis frame $\langle W, \prec \rangle$ and let $w \in W$. Then:

$M, w \models \phi > \psi$ iff either $\{w \mid M, w \models \phi\} = \emptyset$ or there is a $u \in W$ such that $M, u \models \phi$ and $M, v \models \phi \rightarrow \psi$ for all $v \prec u$.

Typically, satisfaction conditions in modal logic for expressions involving some sort of necessity show up as simple universal statements. For instance the translation into a first-order theory with explicit relations over possible worlds of the satisfaction condition for $\Box\phi$ has the form $\forall x\, \phi'$.[5] Lewis has replaced this with a more complicated $\exists\forall$ first-order satisfaction condition.

The reason for the added complexity is Lewis' worry about *limits*. Intuitively, it is natural to say that the truth of $\phi > \psi$ at w has to do with the closest worlds to w that satisfy ψ, and amounts to the truth of ψ in all of these worlds. But there may be no closest worlds; there could be an infinite series of closer worlds, without a limit. Lewis cites the example 'If I were taller than I am ...', and uses this to motivate the thought that it would be wrong in general to postulate the existence of limits. Lewis' more complicated $\exists\forall$ satisfaction condition provides a logic that is very similar to the one associated with the simpler $\forall$ condition, but avoids making the limit assumption. The idea works, however, only for linear preference relations.

Kratzer rejects Lewis' linearity assumption. To obtain reasonable interpretation of modality in the presence of nonlinear orderings, she retreats to a $\forall\exists\forall$ satisfaction condition. In this context she generalizes Lewis' ideas about conditional oughts[6] to provide a general account

[2]See [Stalnaker, 1968], [Stalnaker and Thomason, 1970], and [Lewis, 1973].

[3]When I'm discussing the logic of conditionals, I'll use the notation '$>$' as a logical primitive representing a sort of generic conditional. This conditional would figure in the logical translations of many of the conditional constructions found in natural languages. In effect, I'm assuming that we can think sensibly about the interpretation of natural language conditionals, or of some common sorts of natural language conditionals, at a level of abstraction that doesn't worry about considerations having to do with mood and tense. This assumption is commonly made in the linguistic and the logical literature. Of course, there are some risks in ignoring mood and tense, but it simplifies things enormously.

[4]A preorder over W is a transitive, linear relation over W. A relation $\prec$ over W is linear if for all $u, v \in W$, $u \prec v$ or $v \prec u$ or $v = u$.

[5]See [Blackburn *et al.*, 2001, Chapter 2] for a discussion of first-order translations of modal satisfaction.

[6]In [Lewis, 1973][p. 100–104].

of conditional+modal combinations. The ideas can be found in [Kratzer, 1977], [Kratzer, 1979], [Kratzer, 1981], and [Kratzer, 1991]; they are most clearly expressed in [Kratzer, 1991].

Wishing to provide a more or less uniform semantics for modals and conditionals, and believing that linearity is inappropriate for some modalities, Kratzer gives up Lewis' linearity constraint on the ordering relation. In the case of deontic modality, for instance, she thinks that the ordering can be determined by a background set of imperatives, with $u \prec v$ iff every applicable imperative that is satisfied in v is also satisfied in u. Cases in which there are contradictions between some applicable imperatives provide counterexamples to linearity.

Kratzer's $\forall\exists\forall$ satisfaction condition for conditionals can be stated as follows.

Definition 2.2. Kratzer satisfaction for $>$.
Let M be a model on a Kratzer frame $\langle W, \prec\rangle$ and let $w \in W$. Then:
$M, w \models \phi > \psi$ iff for all $t \in W$ such that $M, t \models \phi$ there is a $u \in W$ such that $u \prec_w t$ and $M, u \models \phi$ such that for all $v \in W$ such that $v \prec_w t$, $M, v \models \phi \rightarrow \psi$.

Kratzer's satisfaction condition for deontic $\bigcirc$ is analogous.

Definition 2.3. Kratzer satisfaction for $\bigcirc$.
Let M be a model on a Kratzer frame $\langle W, \prec\rangle$ and let $w \in W$. Then:
$M, w \models \bigcirc\phi$ iff for all $t \in W$ such that $M, t \models \phi$ there is a $u \in W$ such that $u \prec_w t$ and $M, u \models \phi$ such that for all $v \in W$ such that $v \prec_w t$, $M, v \models \phi$.

3. Lewis-Kratzer conditional modality

The combination of 'if' and 'ought' will serve to illustrate the ideas. Take a simple example like

(3.1) *If it's raining, you should take an umbrella.*

According to Lewis, this sentence is true in a world w iff either there are no worlds in which it's raining or there is a world u in which it's raining such that in every world v where it's raining that's at least as good as u, you take an umbrella.

Let P formalize 'It's raining' and Q formalize 'You take an umbrella'. On Kratzer's theory, the logical representation of (3.1) would have the following satisfaction conditions at world w, where $\prec^O_u$ is the ordering relation associated with $\bigcirc$ in world u.

(3.2) For all $t \in W$ such that $M, t \models P$ there is a $u \in W$ such that $u \prec^O_w t$ and $M, u \models P$ such that for all $v \in W$ such that $v \prec^O_w t$, $M, v \models Q$.

The idea is that the contribution of the conditional clause to the conditional modality is restriction of the quantifiers in the satisfaction condition to worlds that satisfy the content of the clause. The contribution of the modal 'should' is a particular ordering relation. This idea can be generalized to provide a uniform account of how the conditional interacts with modals.

4. Some problems with the Lewis-Kratzer approach

I'll concentrate here on what I believe to be the most worrisome flaws in this theory of conditional modality. The problems with modality have to do with the way in which preference is linked to truth conditions; the problems with the conditional have more to do with linguistic evidence.

4.1. Modality

Kratzer connects the interpretation of the modal 'should', and of 'ought', to an associated ordering source. At each world this source delivers an ordering relation comparing worlds with respect to the contextually relevant values.

But the truth conditions of 'ought' sentences aren't always related to preferences over worlds in the way the theory requires. Suppose, for instance, that you're offered an even bet on whether the next card that is turned up from a deck will be red. Whether you ought to accept the bet will depend on whether it is more likely than not that the card will be red. Now, Kratzer includes likelihoods among the modal family she wishes to explain with preference relations over worlds.[7]

But likelihoods can't be explained in this way, even in very simple situations. Suppose, for instance, that there are only three worlds, w_1, w_2, and w_3, all of them equally likely. Then a proposition true at two of these worlds is more likely than a proposition true at just one of them. None of these worlds is preferable to another. There is no way that conditions over preferences of the sort that Kratzer entertains can deliver the right results for a case like this.

More complex cases illustrate the problem more dramatically. Lebesgue measure is a technique that is often used to model probability in continuous cases. For measurable sets over the closed interval $[0, 1]$, for instance, a Lebesgue measure can be found that assigns each subinterval $[r, r']$ a probability equal to its length. Here, many propositions have nontrivial likelihoods, but each world (each point in the interval $[0, 1]$ gets measure 0—in a preference ordering account, every world would have to be maximally bad.

If likelihoods are to be included in the modal family, then we need interpretational mechanisms that are more general and flexible than first-order conditions on relations.

Moreover, in the logical literature there are many interpretations of broadly modal expressions that do not match either the Lewis' or Kratzer's formulas.

4.2. The conditional

Commitment to a view of the conditional as *variably strict* is an essential part of Kratzer's treatment of modal+conditional combinations. This corresponds to a long-standing issue in

[7] In [Kratzer, 1991], what she says suggests that ϕ is more likely than ψ iff for every world satisfying ψ there is a world at least as good satisfying ϕ, but not vice versa.

the logic of conditionals. Lewis' theory of the conditional in [Lewis, 1973] is variably strict, treating the conditional as a sort of necessity operator over possibilities that depend on the antecedent. Stalnaker's theory in [Stalnaker, 1968] is *variably material*, selecting contingent truths in a world that depends on the antecedent. The difference between the two approaches centers on the validity of *Conditional Excluded Middle*, (CEXM) $(\phi > \psi) \vee (\phi > \neg\psi)$. To put it another way, the issue is whether the negation of 'If P (then) Q' amounts to 'If P, then not Q'.

Up to logical equivalence, ψ is a negation of ϕ iff: (i) ϕ and ψ are contradictory (never true at once) and (ii) ϕ and ψ are exhaustive (always one of them is true). For purposes of debating CEXM, only (ii) is actually at issue.

The linguistic evidence for Conditional Excluded Middle is very (I would say overwhelmingly) strong. Here is a sample.

1. More or less direct evidence

1.1.a. A: If she drinks the pink bottle, she'll grow taller.
B: If she drinks the pink bottle, she'll grow shorter.
C: Neither: if she drinks the pink bottle, she'll stay the same.

1.1.b. A: If she drinks the pink bottle, she'll grow taller.
B: If she drinks the pink bottle, she'll grow shorter.
C: #? Neither. If she drinks the pink bottle, she might stay the same.

1.1.c. A: If she drinks the pink bottle, she'll grow taller.
B: If she drinks the pink bottle, she won't grow taller.
C: #? Neither.
C': #? Neither. If she drinks the pink bottle, she'll stay the same.

Comments: These examples are dialogues between three parties, A, B, and C. 1.1.a is weak evidence, since an exclusivity implicature may not be present here. 1.1.b is convincing evidence that 'If P then might-not Q' is not the denial of 'If P then Q, as Lewis and Kratzer claim. 1.1.c is direct evidence for exclusivity.

2. Indirect evidence about denials of conditionals

2.1. #?? I don't think it will break if you sit on it, but I'm not sure about whether it won't break if you sit on it.

2.2. #?? I know whether he walked to work if his bus was late' but 'I don't know whether he didn't walk to work if his bus was late.

Comments: 2.1 is inconsistent if CEXM is adopted. If not, it is consistent.[8]As for 2.2., on a variably strict theory of the conditional, 'I don't know whether P if Q' amounts to 'I don't know that 'If P then must Q and I don't know that 'If P then might-not Q.' and 'I don't know whether 'P if not Q' amounts to 'I don't know that 'If P then must-not Q and I don't know that 'If P then might Q.' So 2.2 would be consistent.

[8]Kai von Fintel points out that this depends on the assumption that conditionals have their regular interpretation when embedded under propositional attitudes.

3. Further points about 'might' conditionals

4.1. Purported evidence for the idea that 'If P then might-not Q' is the denial of 'If P then Q' can be converted into similar "evidence" for the same conclusion about nonconditionals. For instance, suppose someone cites

> A: If you go outside, you'll get wet.
> B: No, if I go outside, I might not get wet.

as evidence for this account of the denial of a conditional. This example can be converted an unconditional one as follows.

> A: You'll get wet.
> B: No, I might not get wet.

4.2. 'It might not rain, but I'm still inclined to think that it will rain' is a way of expressing hedged belief. On the other hand, 'It might not rain, but it will rain' is Moore-paradoxical and, in particular, is not inconsistent.

Similarly, 'If it rains then it might not flood, but I'm inclined to think that if it rains, then it will flood' is not Moore-paradoxical, but is a hedged belief, as in the unconditional case. And 'If it rains then it might not flood, but if it rains, it will flood' is Moore paradoxical and not inconsistent.

These facts, I think, build up a very strong case and make it very desirable to find a treatment of conditional+modal combinations that makes room for other theories of modality and the conditional.

5. Cumulative variably material conditionals

A prominent virtue of Stalnaker's theory is the materiality of the conditional: conditionals don't sum up what is common to many worlds, but report on what is true in a single world. A prominent virtue of the Lewis-Kratzer theory is the idea that conditionals act as restrictors, confining the interpretation of the consequent to worlds where the antecedent is true. I propose to combine these two virtues; this idea leads to a *cumulative* variably material conditional.

To carry out this idea, we incorporate Kratzer's "modal base" as a parameter in the satisfaction conditions for formulas. Satisfaction will now be a relation $M, s, X, w \models \phi$ between a model M on a Stalnaker frame, a selection function s, a set X of worlds of the model, a world w of the model, and a formula ϕ. If we don't mention the model, we have a relation $s, X, w \models \phi$.

A *Stalnaker frame* for pure conditional logic is a pair $\mathcal{S} = \langle W, s \rangle$, where W is a nonempty set—the set of worlds—and s is a partial function from subsets of W and members of W to subsets of W: for each $Y \subseteq W$ and $w \in W$, $s(Y, w) \subseteq W$. Intuitively, s is supposed to choose the conditionally closest world to w in Y; the values of s are sets merely to accommodate the case in which $Y = \emptyset$.

The selection function s satisfies the following conditions.

1. If $Y \neq \emptyset$ then for some $u \in W$, $s(Y, w) = \{u\}$. Otherwise, $s(Y, w) = \emptyset$.
2. $s(Y, w) \in Y$.
3. If $w \in Y$ then $s(Y, w) = \{w\}$.
4. If $s(Y, w) \subseteq Z$ and $s(Z, w) \subseteq Y$ then $s(Y, w) = s(Z, w)$.

The satisfaction condition is like Stalnaker's, except that it restricts the context to worlds satisfying the antecedent.

Definition 5.4. Cumulative Stalnaker satisfaction for the conditional.
Let M be a model on a Stalnaker frame $\langle W, s\rangle$, $w \in W$, and $X \subseteq W$. Then:
$s, X, w \models \phi > \psi$ iff $s, X \cap V^X(\phi), s(X \cap V^X(\phi), w) \models \psi$.[9]

The resulting conditional validates Conditional Excluded Middle, but differs in some important respects from Stalnaker's conditional. (1) It validates cumulativity—that is, it validates $\phi > (\psi > \phi)$. (2) It validates "importation-exportation"—that is, it renders $\phi > (\psi > \chi)$ and $(\phi \wedge \psi) > \chi$ logically equivalent. (3) It doesn't validate *modus ponens*.[10]

The invalidity of *modus ponens* may seem strange, but in fact the validity of this principle for nested conditionals has been challenged,[11] the semantic interpretation provides a plausible explanation of why *modus ponens* is invalid in certain cases, and $(\phi > \psi) \rightarrow (\phi \rightarrow \psi)$ is valid as long as ψ doesn't contain a conditional or modal operator. I will not elaborate on the subject of *modus ponens* here.

Importation-exportation has been a problem for conditional logic since logics in the Stalnaker-Lewis' families were first presented. These logics invalidate exportation—that is, they invalidate $((\phi \wedge \psi) > \chi) \rightarrow (\phi > (\psi > \chi))$—but this is puzzling because informal examples of the principle seem intuitively to be valid. Cumulativity is similar. A conditional like 'If it will rain today then if I take an umbrella it will rain today' sounds peculiar, but this seems to a pragmatic effect.

On the other hand, the Stalnaker-Lewis logics make it difficult to explain why assumptions introduced in the antecedent of a conditional persist. Suppose, for instance, that my belt and my belt buckle are normally attached. Any world in which they aren't attached is pretty strange; to get to such a world we need an antecedent that postulates sumething like an action of detaching the buckle from the belt. Now, consider the conditional 'If my belt is in the closet then if my belt buckle is in the dresser drawer the buckle is detached'. This seems true, while on the other hand 'If my belt is in the closet then if my belt buckle is in the dresser drawer the belt isn't in the closet' is very peculiar. I have no difficulty thinking it's false.

[9] $V^X(\phi)$ is the set $\{w \mid s, X, w \models \phi\}$. Strictly speaking, $s, X, w \models \phi$ and $V^W(\phi)$ would have to be defined by a simultaneous induction.

[10] In fact, in view of a theorem of Allen Gibbard's, it *can't* validate *modus ponens*. [Gibbard, 1981] contains a proof that any logic of conditionals (i) validating exportation-importation, (ii) validating $\phi > \psi$ if $\phi \rightarrow \psi$ is a logical validity, and (iii) validating *modus ponens*, collapses to the material conditional. The cumulative Stalnaker conditional satisfies (i) and (ii), but not (iii).

[11] See [McGee, 1985].

But on the Stalnaker and Lewis theories, the antecedent clause 'My belt is in the closet' should take us to a world or worlds where the belt is in the closet. Once it is taken there it is forgotten, and new conditionals are evaluated afresh in these worlds. But in a world where the belt is in the closet, the conditional 'If the buckle is in the dresser drawer then the belt is in the dresser drawer' is true, because nothing in the antececdent requires the belt to be taken apart.[12]

On the cumulative theory, the antecedent 'My belt is in the closet' is added to the background assumptions in force when the second conditional, 'If the buckle is in the dresser drawer, then . . .' is evaluated. These assumptions, together with the fact that dresser drawer is not in the closet, force the belt to come apart.

This behavior of the conditional is wuite robust, providing a a case for cumulativity that, on the whole, is very convincing.

6. Conditional oughts

This approach to conditionals provides a simple and direct account of conditional modality—modal conditionals are conditionals with modal consequents. For instance, a conditional ought would have the form $\phi > \bigcirc\psi$.

Simple compositional accounts of conditional obligation have often been rejected in the literature, but when this happens it is usually a material conditional of the form $\phi \to \bigcirc\psi$ that is in question, and the problems with these formalizations of conditional modality can be attributed to the shortcomings of this theory of the conditional.

It is harder to find flaws when the conditional is more sophisticated. But $\phi > \bigcirc\psi$ does have one glaring problem as a formalization of conditional oughts when $>$ is the Stalnaker conditional. Take a case like 'If you don't return a library book you should pay a fine'. Formalizing this as $P > \bigcirc Q$, this is true in case 'You should pay a fine' is true in the closest world in which you don't return the book. But in fact, there's no reason to expect this to be true. The closest world where the book is not returned is very similar to the actual one, and in the actual one you ought to return the book and so ought not to pay the fine. There is no principled way to ensure that things are any different in the closest world where the book is not returned.

As we'll see, a cumulative semantics for the conditional solves this problem.

6.1. Satisfaction conditions for conditionals and modality

We will use Lewis' preference-based semantics for $\bigcirc$. This works perfectly well for some cases, although (as we pointed out in Section 4.1), it doesn't work for all of them. Then a Stalnaker frame for conditional+deontic logic is a triple $\mathcal{S} = \langle W, s, \prec\rangle$, where W is a nonempty set, s is a partial function as before, and for each $w \in W$, $\prec_w$ is a transitive,

[12] I first heard examples of this sort from Allen Gibbard. My memory is hazy about where and when that happened.

linear ordering over W. (So $\prec$ itself is a function taking members of W into orderings over W.)[13]

The satisfaction condition for $>$ as in Definition 5.4, and the satisfaction condition for $\bigcirc$ is as follows.

Definition 6.5. Satisfaction for $\bigcirc$.
Let M be a model on a Stalnaker frame $\langle W, s, \prec\rangle$ for conditional+deontic logic. Let $w \in W$, and $X \subseteq W$. Then:
$s, X, w \models \bigcirc\phi$ iff there is a $u \in X$ such that for all $v \in X$ such that $v \preceq_w u$, $s, X, v \models \phi$.

According to this theory, s, X, and w satisfy $P > \bigcirc Q$ iff the closest world to w in $V^X(P) \cap X$ satisfies $\bigcirc Q$ when contextually restricted to $V^X(P) \cap X$. This means that, where $u = s(V^X(P) \cap X, w)$, there is an x such that for all y such that $y \preceq_u x$ and $y \in V^X(P) \cap X$, $s, V^X(P) \cap X, u \models Q$.

Crucially, this means that in the consequent of a conditional with antecedent P, the worlds used to evaluate $\bigcirc Q$ will be restricted to ones that satisfy P. This solves the problem concerning reparational obligation that was raised in Section 6, because the conditional not only shifts the world to one where the book is not returned, but restricts the worlds used to interpret modals and conditionals in the consequent to these worlds.

6.2. Testing this theory of conditional ought

When things get complicated enough, I believe that one or two hand-picked examples don't give very reliable results about the performance of a theory of truth conditions. In this case, it is better to pick a domain where intuitions about both 'if' and 'ought' are relatively clear, supporting a large sample of statements of the relative sort which can then be tested. I try to do something of the sort in [Thomason, 2012]. I believe those results are promising.

Here, I will do something else, arguing that given a plausible condition on models, this delivers a theory of conditional oughts that is equivalent to Lewis'.

The satisfaction condition for conditional 'ought' that Lewis gives in [Lewis, 1974][p. 4] is as follows. (This is equivalent to the condition in terms of "nesting" that he gives in [Lewis, 1973].)

A Lewis deontic frame is a pair $\langle W, \prec\rangle$, where W is a nonempty set and $\prec$ is a function from W to transitive, linear orderings over W.

Definition 6.6. Lewis satisfaction for $\bigcirc\!\!\rightarrow$.

[13] I don't think that Kratzer's case for nonlinear precedence orderings is entirely convincing. For one thing, in most cases it doesn't make sense to derive preferences over outcomes from a set of propositions serving as "laws." For another, even though there may be cases where worlds are intuitively incomparable, a "supervaluatonal" approach is possible for these, where one takes what is common to all the linearizations of a nonlinear order.

Let M be a model on a frame $\langle W, \prec \rangle$ and let $w \in W$. Then:

$M, w \models \phi \,\Box\!\!\!\!\!\bigcirc\!\!\rightarrow \psi$ iff either $V(\phi) = \emptyset$ or there is a $u \in V(\phi)$ such that $M \models \phi \rightarrow \psi$ for all $v \in v(\phi)$ such that $v \prec_w u$.

How does the interpretation of $\phi > \bigcirc\psi$ compare with this in the cumulative logic? Well, it amounts to this.

$s, \emptyset, w \models \phi > \bigcirc\psi$ iff either $V(\phi) = \emptyset$ or. there is a $u \in V(\phi)$ such that $M \models \phi \rightarrow \psi$ for all $v \in v(\phi)$ such that $v \prec_{s(V(\phi),w)} u$.

The only difference in the satisfaction conditions is that, where Lewis evaluates with respect to deontic preference in the base world w, the condition I propose evaluates with respect to deontic preference in a world close to w.

But factually close worlds should not differ in the preferences that prevail. In considering what we ought to do under various circumstances, we are in general changing the set of worlds we are taking into account, not the preferences.

So-called "anankastic conditionals," like 'If you want to get to work on time you should leave now', provide a class of cases where preservation of preferences through conditional transitions is violated. I don't claim to have a theory of such conditionals, and it would be premature at this point to make comparative claims based on these examples. But it may be a little easier to accommodate anankastic conditionals on the cumulative approach.

7. Conclusion

The account that I've proposed of conditional modality is, I think, well motivated on logical grounds.

On the linguistic side, it postulates the simplest sort of compositional structure for conditional+modal combinations. These are simple syntactic combinations of the two constructions, with the modality taking narrow scope. In general, I believe that wide scope combinations of conditional+modal constructions are not easily available, and in any case would not have truth conditions that are very sensible.

Unlike the Lewis-Kratzer theory, this approach allows conditionals to combine with arbitrary modalities and near-modalities, with the interpretation of the conditional remaining fixed, and the contribution of the modality depending on whatever interpretation seems appropriate. In complex cases, such as the probabilistic ones, this interpretation may not even involve a modal satisfaction condition, but could use a probability measure or some other interpretive device. This makes the approach much more flexible, and provides a more promising way to reconcile the two desiderata of linguistic plausibility and sensitivity to the variety of logical analyses of broadly modal constructions.

In other, unpublished work I have proved a completeness theorem for the conditional logic described here, and have investigated the consequences of this approach for some of the traditional problems of deontic logic.

Bibliography

[Bennett, 2003] Jonathan Bennett. *A Philosophical Guide to Conditionals*. Oxford University Press, Oxford, 2003.

[Blackburn *et al.*, 2001] Patrick Blackburn, Maarten de Rijke, and Yde Venema. *Modal Logic*. Cambridge University Press, Cambridge, England, 2001.

[Gibbard, 1981] Allan F. Gibbard. Two recent theories of conditionals. In William L. Harper, Robert C. Stalnaker, and G. Pearce, editors, *Ifs: Conditionals, Beliefs, Decision, Chance, Time*, pages 211–247. D. Reidel Publishing Co., Dordrecht, 1981.

[Goodman, 1947] Nelson Goodman. The problem of counterfactual conditionals. *The Journal of Philosophy*, 44:113–118, 1947.

[Goodman, 1955] Nelson Goodman. *Fact, Fiction and Forecast*. Harvard University Press, 1955.

[Kratzer, 1977] Angelika Kratzer. What 'must' and 'can' must and can mean. *Linguistics and Philosophy*, 1(3):337–356, 1977.

[Kratzer, 1979] Angelika Kratzer. Conditional necessity and possibility. In Rainer Bäuerle, Urs Egli, and Arnim von Stechow, editors, *Semantics from Different Points of View*, pages 117–147. Springer-Verlag, Berlin, 1979.

[Kratzer, 1981] Angelika Kratzer. The notional category of modality. In Hans-Jürgen Eikmeyer and Hannes Rieser, editors, *Words, Worlds and Contexts*, pages 38–74. Walter de Gruyter, Berlin, 1981.

[Kratzer, 1991] Angelika Kratzer. Modality. In A. von Stechow and D. Wunderlich, editors, *Semantics: An International Handbook of Contemporary Research*, pages 639–650. de Gruyter, Berlin, 1991.

[Lewis, 1973] David K. Lewis. *Counterfactuals*. Harvard University Press, Cambridge, Massachusetts, 1973.

[Lewis, 1974] David K. Lewis. Semantic analyses for dyadic deontic logic. In Sören Stedlund, editor, *Logical Theory and Semantic Analysis: Essays Dedicated to Stig Kanger on His Fiftieth Birthday*, pages 1–14. D. Reidel Publishing Co., Dordrecht, 1974.

[McGee, 1985] Vann McGee. A counterexample to modus ponens. *Journal of Philosophy*, 82:462–471, 1985.

[Schneider, 1953] Erna Schneider. Recent discussions on subjunctive conditionals. *Review of Metaphysics*, 6:623–649, 1953.

[Stalnaker and Thomason, 1970] Robert C. Stalnaker and Richmond H. Thomason. A semantic analysis of conditional logic. *Theoria*, 36:23–42, 1970.

[Stalnaker, 1968] Robert C. Stalnaker. A theory of conditionals. In Nicholas Rescher, editor, *Studies in Logical Theory*, pages 98–112. Basil Blackwell Publishers, Oxford, 1968.

[Thomason, 2012] Richmond H. Thomason. Formalizing conditional oughts. Unpublished manuscript, University of Michigan, 2012.