EECS 487 Fall 2008 Homework - 3

Due 03 Nov 2008

1 OpenGL Lighting Model (25 pts)

You are an intelligence agent who has to communicate a secret code to another agent on the field. You decide to do this in an abstract art museum by scribbling a code on a painting and making it selectively visible to the field-agent[1]. The painting you choose is a 3-ft-by-6-ft square painted uniformly with only one color and the ink conforms to an OpenGL material with diffuse color $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$ and is titled *CitiZen*[2]. It is lit with a directional light of OpenGL color $(1, 1, 1)$ at an angle 30^0 from the vertical, from the ceiling. During exhibit hours, the museum runs a white fog in the room (for surrealism) such that a person at a distance d feet from the center of the painting sees only a fraction $\frac{1}{1+\frac{1}{2}d}$ of the color from the painting and the rest, white from the ambient fog. To take advantage of the white fog you decide that the code should appear in white to the field-agent so that others not actively looking for it may dismiss it as an illusion from the fog should they see it by accident. You sneak up the previous night and scribble the code on the painting at a height of 2.5 ft knowing that the field-agent is 5-ft. tall. You write the code with "magic" ink which conforms to an OpenGL material with diffuse color $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$ and specular color $\left(\frac{3}{4}, \frac{7}{8}, \frac{3}{4}\right)$. You have a full-feature flashlight from OpenGL Spy-Supplies Inc. which behaves like a spot-light with a narrow cone. Your aim is to shine your flashlight on the painting when the field-agent is in the critical position so that he sees the code from his position but others continue to see the painting. All lighting parameters available for any OpenGL light are controllable on the flashlight. The attenuation of the light from the flashlight due to fog and distance has a constant coefficient of 1, a linear coefficient of $\frac{8}{143}\text{ft}^{-1}$ and a quadratic coefficient of 0. To make out the code on the painting from the background there must be a minimum difference of $\frac{1}{10}$ in each of the R,G,B components of the colors reaching the agent's eye from both materials.

1. What is the expression for the total color observed by the field-agent at a distance d from a point of actual color (r_p, g_p, b_p) (after lighting calculations etc) ? (3 pts)

2. What general settings would you choose for your flashlight? (2 pts)

3. What is the maximum distance from the painting that the code will be visible at all? (10 pts)

4. Suppose the agent has parked himself at a total distance of 6 ft. from the painting and is 3 ft. to the left of the center, where should you position yourself so that you can shine your flashlight? What exact settings would you make on your flashlight? (10 pts)

Perform all calculations using rational numbers. All colors components are in the range $[0, 1]$.

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[1] Do not try this "at home"!

[2] The author's blurb by its side says that it is a full-length mirror and that a zen monk would be able to see his silhouette in it in the same color as the background

2 Splines (20 pts)

2.1 Quartic spline (10 pts)

Determine the expression for the basis and constraint matrices a quartic spline curve that of the form,

$$\mathbf{f}(u) = \sum_{i=0}^{4} \mathbf{a}_i u^i, \tag{1}$$

that interpolates/approximates 5 control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ and $\mathbf{p}_4$ according to the following specifications:

1. The curve must interpolate $\mathbf{p}_0$, $\mathbf{p}_2$ and $\mathbf{p}_4$,
2. The slope at $\mathbf{p}_0$ must be twice the displacement from $\mathbf{p}_0$ to $\mathbf{p}_1$,
3. The slope at $\mathbf{p}_4$ must be thrice the displacement from $\mathbf{p}_3$ to $\mathbf{p}_4$.
4. The basis matrix $\mathbf{B}$ should be of the form,

$$\begin{bmatrix} \mathbf{p}_2 \\ \mathbf{p}_0 \\ \mathbf{p}_3 \\ \mathbf{p}_1 \\ \mathbf{p}_4 \end{bmatrix} = \mathbf{B} \begin{bmatrix} \mathbf{a}_4 \\ \mathbf{a}_2 \\ \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_3 \end{bmatrix}. \tag{2}$$

Show the development of all expressions leading upto Eqn. (2).

2.2 Spline interconversion (8 pts)

OpenGL supports interpolation and approximation through *evaluators*. An evaluator is an engine which is initialized with 4 control points and it returns points on the Bezier curve formed by these four points. Because all cubic splines can be interconverted, evaluators support only Bezier curves and leave the task of expressing custom cubic splines in terms of a related Bezier curve to the programmer. Given the requirement of generating a Catmull-Rom spline with control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$, how would you go about instructing OpenGL to implement this using an evaluator?

2.3 Lagrange polynomial interpolation (2 pts)

The Lagrange-polynomial method of interpolation is way of interpolating an arbitrary set of points. What are its advantages and disadvantages?

3 Texture Cube-Mapping (15 pts)

On page 254 of the textbook, the expression is derived for the cube map mapping from (x, y, z) direction vector to (u, v) coordinates of one of the cube faces for which the condition is $x > |y|, x > |z|$. Derive similar expressions for the other five faces of the cube.

For the faces that are parallel to the z-axis assume that the v coordinate goes in the direction of the z-axis, and u-coordinate is oriented so that the coordinate system (u, v, n)(where n is the *outward* normal direction) is right-handed. For the top and bottom faces orthogonal to z-axis, orient u and v in the same way as x and y are oriented. Note that on each face u and v should vary from 0 to 1. In your answer, write down the condition for each face followed by the expression converting (x, y, z) to u and v.