



ADVANCED COMPUTER NETWORKS

Leland et al., "On the Self-Similar Nature of Ethernet Traffic (Extended Version)," *IEEE/ACM Transactions on Networking*, 2(1):1–15, Feb. 1994

Self-Similarity

Viewing scale not apparent from object appearance

Object features are **statistically** similar between object parts and the overall object

For example, we always see a jagged line no matter how close we look at a coastline

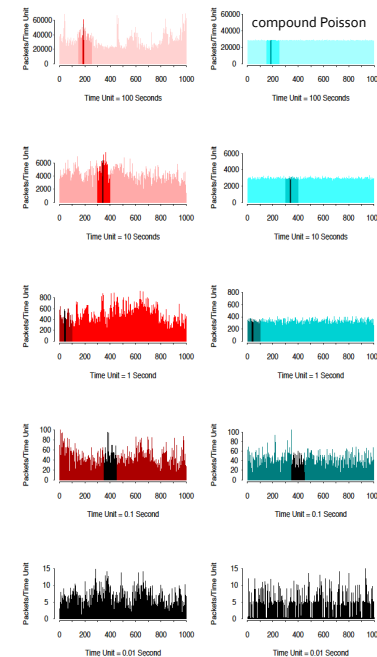
References

[ENW96] Erramilli, Narayan, and Willinger, "Experimental Queueing Analysis with Long-Range Dependent Packet Traffic," *IEEE/ACM ToN*, 4(2):209–223, Apr. 1996

[W+97] Willinger, Taqqu, Sherman, and Wilson, "Self-Similarity Through High-Variability: Statistical Analysis of Ethernet LAN Traffic at the Source Level," *IEEE/ACM ToN*, 5(1):71–86, Feb. 1997

Ethernet Traffic ...

Absence of a natural length of a "burst": at every time scale from msec to minutes and hours, bursts consist of bursty subperiods separated by *less bursty* subperiods



Comparable average packet size and arrival rate

Drastically different distribution across time scales

A Traffic Trace

Let X be a covariance stationary stochastic process

What is:

- a **stochastic process**:
a time series of a variable that changes randomly
- a **stationary process**:
a time series whose statistical properties: mean, variance, autocorrelation, stay constant over time
- **covariance**:
by how much two random variables move in tandem

By how much X_t and X_{t+h} move in tandem is not a function of time

Self-Similarity

A traffic trace is self-similar if (equivalently):

1. the variance of the sample mean remains large even as you sample at larger and larger samples (no smoothing out):

$$\text{var}(X^{(m)}) \sim a_2 m^{-\beta}, m \rightarrow \infty, 0 < \beta < 1, a_2 > 0$$

vs.

$$\text{var}(X^{(m)}) \sim a_4 m^{-1}, m \rightarrow \infty, a_4 > 0$$

Sample Mean

Let X be a covariance stationary stochastic process

$X^{(m)}$, a **sample mean**, is a new covariance stationary time series obtained by averaging the original series X over non-overlapping blocks of size m

X : 1, 3, 6, 2, 5, 1, 8, 3, 2, ...

$X^{(2)}$:

$X^{(3)}$:

$X^{(4)}$:

Self-Similarity

A traffic trace is self-similar if (equivalently):

2. auto-correlation functions (acf) at various time scales are of the form: $r(k) \sim k^{-\beta}, k \rightarrow \infty, 0 < \beta < 1$

\Rightarrow the acf is non-summable, i.e., traffic exhibits **long range dependence (LRD)**: $\sum_k r(k) \rightarrow \infty$

Short-range dependence: $r(k) \sim \rho^k, 0 < \rho < 1 \Rightarrow \sum_k r(k) < \infty$

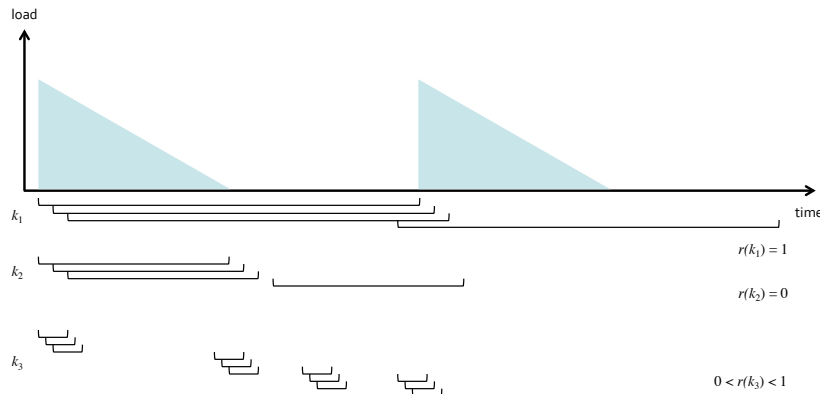
X is (exactly) second-order self-similar if: $r^{(m)}(k) = r(k), k \geq 0$

and (asymptotically) second-order self-similar if:

$$r^{(m)}(k) \rightarrow r(k), m \rightarrow \infty$$

White noise (not self-similar): $r^{(m)}(k) \rightarrow 0, m \rightarrow \infty$

Auto-correlation



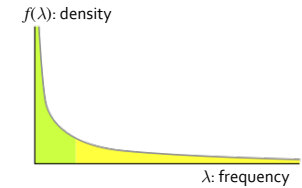
Self-Similarity

A traffic trace is self-similar if (equivalently):

3. taking the time series into the frequency domain (Fourier transform), the low frequency components obeys a power-law near the origin (a low frequency is proportionally denser than its next higher frequency):

$$f(\lambda) \sim a_3 \lambda^{-\gamma}, \lambda \rightarrow 0, 0 < \gamma < 1, a_3 > 0$$

$$\gamma = 1 - \beta$$



Cf. Zipf distribution

Self-Similarity

A traffic trace is self-similar if (equivalently):

4. the expected *rescaled adjusted range statistic*:

$$\mathbb{E}[R(n)/S(n)] \sim a_5 n^H, n \rightarrow \infty, a_5 > 0$$

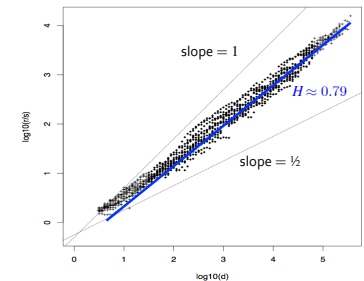
has **Hurst parameter** $\frac{1}{2} < H < 1$

The Hurst parameter expresses the speed of decay of the acf

- $H \leq \frac{1}{2}$: short-range dependent processes, e.g., Poisson, batch-Poisson, Markov-modulated Poisson
- $H > \frac{1}{2}$: non-stationary process

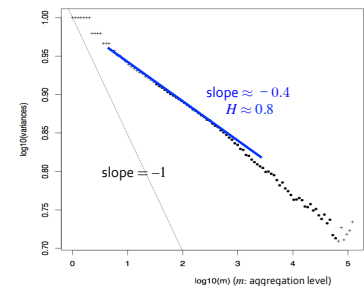
Detecting LRD in Ethernet Trace

H can estimated directly from R/S statistic



Or from variance time plot

- slope $-\beta, 0 < \beta < 1$
- $H = 1 - \beta/2$

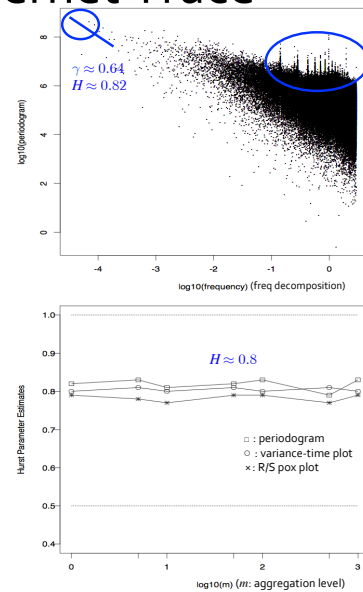


Detecting LRD in Ethernet Trace

Or from periodogram, slope of 10% of the lowest frequencies, near 0

$$H = (1 + \gamma) / 2$$

Hurst parameter stays constant across traffic aggregation levels



Detecting LRD in Ethernet Trace

H increases as traffic load increases!

Remain true over time (89-92)

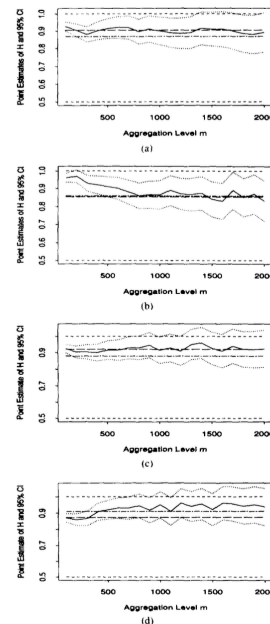
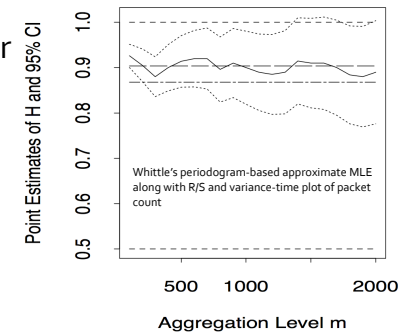


Fig. 6. Periodogram-based MLE/aggregation method for the sequences AUG89.MP, OCT89.MP, JAN90.MP, and FEB92.MP.

Detecting LRD in Ethernet Trace

H can also be estimated with maximum-likelihood estimator (MLE) based on the periodogram (Whittle estimator) with the advantage of computing 95% confidence interval

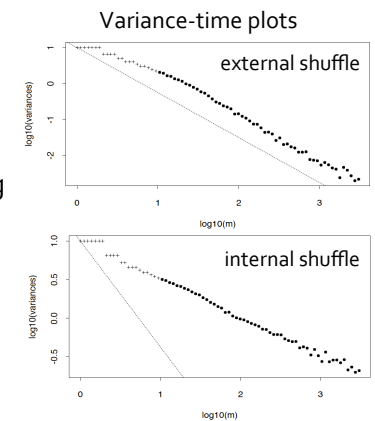


Implication

Long-range dependent traffic effects queueing delay: it makes buffer sizing ineffectual

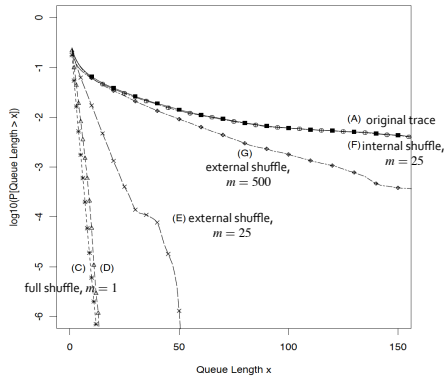
How do we know LRD causes ineffectual buffering [ENW96]?

- external shuffle experiment: divide traffic into m blocks and shuffle the blocks around preserving the sequence inside each block: destroys LRD, preserves SRD
- internal shuffle experiment: same blocks, shuffle traffic inside each block, keeping the block order: destroys SRD, preserves LRD



Implication

Resulting queue occupancy statistics:



What can we do about it?

- frequency domain view: traffic can be decomposed into high (spikes), mid (ripples), and low (swells) frequencies
- network must have enough capacity to handle peak rate of low frequency
- buffer space should be used only to handle high-frequency traffic

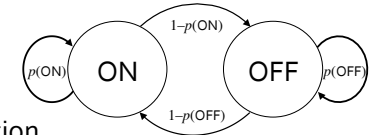
[ENW96]

Causes of LRD

Aggregation of ON/OFF traffic with **heavy-tailed** OFF time distribution [W+97]

- human "think" time
- effect of TCP congestion avoidance (cwnd)
- multimedia sources can also be modeled as ON/OFF

Why does **long-tailed** ON/OFF distributions cause LRD?



- long OFF time means autocorrelation of bursts at large k , hence $\sum_k r(k) \rightarrow \infty$
- long ON time increases the probability of seeing other traffic

Heavy-tailed Distributions

$$P[X > x] \sim x^{-\alpha}, x \rightarrow \infty, 0 < \alpha < 2$$

Examples: Pareto, Weibul, Zipf

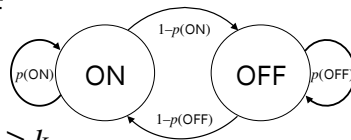
Pareto distribution:

$$p(x) = \alpha k^\alpha x^{-\alpha-1}, \alpha \text{ and } k > 0, x \geq k$$

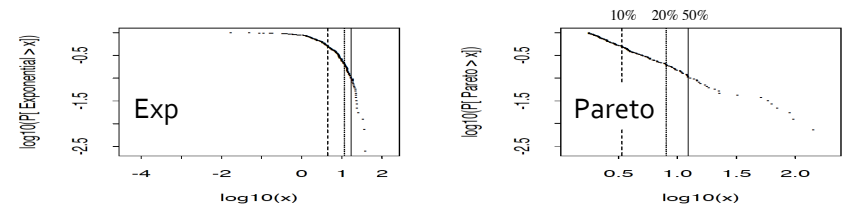
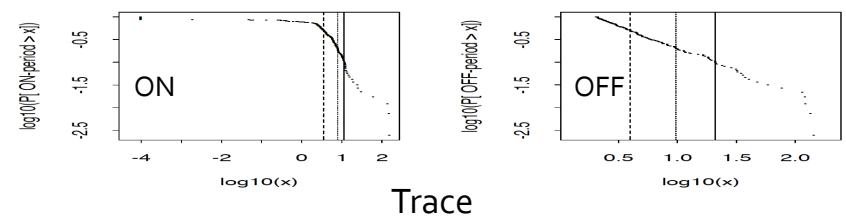
$$F(X) = P[X \leq x] = 1 - (k/x)^\alpha$$

$\alpha < 2$: trace has infinite variance

$\alpha < 1$: trace has infinite mean

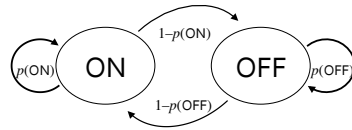


Heavy-tailed Distributions



Generated

Modeling LRD



Aggregation of ON/OFF traffic with heavy-tailed OFF time distribution [W+97]

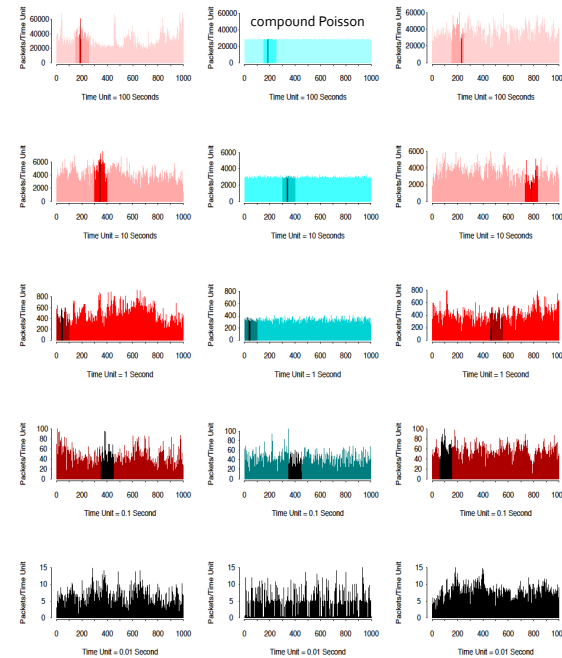
Advantage: parsimonious, only one parameter, α

$$P[X > x] \sim x^{-\alpha}, x \rightarrow \infty, 0 < \alpha < 2$$

Alternative models:

- by fitting multiple short-range dependent processes: parameter explosion, no physically meaningful interpretations
- fractional Gaussian noise: does not model short-range dependencies
- fractional ARIMA:
 - can model both short- and long-range dependencies,
 - but still does not provide physical explanation of self-similarity
 - plus, known parameter estimation techniques too expensive

Ethernet Traffic



Generated Traffic
ON/OFF Sources

[W+97]

Discussions

Gold standard of measurement study and analysis

Prior to this paper, traffic modeling assumes Poisson distribution

After this paper, traffic modeling uses power law distributions (Pareto, Weibul, Zipf)

A flurry of follow-on papers found power-law distribution everywhere in the network . . .