Telecommun Syst
DOI 10.1007/s11235-010-9302-6

A game-theoretic approach to decentralized optimal power allocation for cellular networks

Shrutivandana Sharma · Demosthenis Teneketzis

Abstract The rapidly growing demand for wireless communication makes efficient power allocation a critical factor in the network's efficient operation. Power allocation in cellular networks with interference, where users are selfish, has been recently studied by pricing methods. However, pricing methods do not result in efficient/optimal power allocations for such systems for the following reason. Because of interference, the communication between the Base Station (BS) and a given user is affected by that between the BS and all other users. Thus, the power vector consisting of the transmission power in each BS-user link can be viewed as a public good which simultaneously affects the utilities of all the users in the network. It is well known (Mas-Colell et al., Microeconomic Theory, Oxford University Press, London, 2002, Chap. 11.C) that in public good economies, standard efficiency theorems on market equilibrium do not apply and pricing mechanisms do not result in globally optimal allocations. In this paper we study power allocation in the presence of interference for a single cell wireless Code Division Multiple Access (CDMA) network from a game theoretic perspective. We consider a network where each user knows only its own utility and the channel gain from the base station to itself. We formulate the uplink power allocation problem as a public good allocation problem. We present a game form the Nash Equilibria of which yield power allocations that are optimal solutions of the corresponding centralized uplink network.

S. Sharma (✉) · D. Teneketzis
Department of Electrical Engineering and Computer Science,
University of Michigan, Ann Arbor, MI 48109-2122, USA
e-mail: svandana@umich.edu

D. Teneketzis
e-mail: teneket@eecs.umich.edu

Keywords Power allocation · Cellular network · Interference · Decentralized mechanism · Game theory · Mechanism design · Nash implementation

1 Introduction

1.1 Overview and literature survey

With rapidly growing demand for wireless communication the need for efficient use of spectrum has drawn significant attention of researchers. One of the factors that governs the efficiency of spectrum usage is power and interference control. The growth in the size of wireless networks makes it desirable to use decentralized mechanisms for power control because centrally operated mechanisms involve added infrastructure. However, the increasing intelligence of end-user/intermediate network devices which are owned by selfish users, puts decentralized mechanisms at risks of failure against strategic behavior of users. Therefore it is desirable to develop decentralized mechanisms for power allocation which are robust against the strategies of selfish users.

Decentralized mechanisms for power allocation/control in cellular networks that study game-theoretic/strategic behavior issues have received considerable attention in the literature. One of the earliest works which introduced an individual utility maximization formulation for uplink power control in a single cell Code Division Multiple Access (CDMA) data network can be found in [2]. An uplink problem similar to that of [2] in which users' utilities are taken to be functions of their respective Signal to Interference Ratio (SIR) was investigated in [3]; in this paper the existence of an equilibrium was shown and a decentralized algorithm for solving the power control problem

Published online: 03 June 2010

was suggested. The problem formulated in [2] was re investigated in [4] using pricing; it was shown that pricing results in multiple equilibria which are Pareto superior to the equilibria obtained in [2] and [3]. Pricing-based analysis of the uplink power control problem was also done in [5]; in [5] the authors introduced user specific parametric utility functions and proposed two decentralized algorithms, the parallel update and the random update algorithms, that converge to the unique equilibrium of the problem. In [6] pricing-based ideas for uplink power control were extended to multi-cell data networks. The authors of [7] studied uplink power allocation under an Interference Temperature Constraint (ITC); they proposed a power auction run by a manager that achieves a power allocation arbitrarily close to the globally optimal one. The conditions under which the power auction achieves an optimal solution however require in essence, that the manager should know the users' utility functions.

Game theoretic study of downlink CDMA data networks can be found in [8, 9, 12]. In [8] and [9], optimal power allocation strategies were determined for a single class CDMA system under the assumption that the utility functions of the users are common knowledge (see [10, 11] for the definition of common knowledge). The authors of [12] studied a downlink power allocation problem for multi-class CDMA networks; they proposed a decentralized mechanism based on dynamic pricing and partial cooperation between the mobiles and the base station. The mechanism achieves a partial-cooperative optimal power allocation which was shown to be close to a globally-optimal power allocation. In [13] the authors presented a decentralized mechanism for power allocation that works for both uplink and downlink networks, and also takes into account multiple ITCs; the mechanism obtains an optimal power allocation under the assumption that the users are cooperative.

In this paper we consider a single cell wireless CDMA data network. We study power allocation for the uplink communication in a given carrier frequency where the communication between a user and the Base Station (BS) generates interference to that between other users and the BS. We consider a decentralized network where users are selfish, and where each user's utility and the channel gain between the BS and the user are that user's private information. The objective is to develop a game form/decentralized mechanism for determining power allocations such that the allocations obtained at the Nash equilibria of the induced game are optimal solutions of the corresponding centralized problem. Below we explain the motivation for considering the above problem.

1.2 Motivation

A network resource is said to be a public good if the presence of the resource simultaneously affects the utilities of all network users without getting divided among them. For single cell wireless networks, power allocation problems in the presence of interference can be treated as problems of a public good allocation. The public good for the uplink network is the power vector received by the BS from all the users, and the public good for the downlink network is the power vector transmitted by the BS to all the users. Power allocation problems in cellular wireless networks with interference have been previously considered in the literature cited in Sect. 1.1. The solution approach in all the references [2–9, 12] is based on different variations of pricing mechanisms where each user pays some money for the power allocated to it.

In general, in decentralized resource allocation problems involving a public good, pricing mechanisms that fix a common price for the public good for all the users, fail to obtain globally optimal allocations. The reason is that in a public good economy the *same* good is simultaneously consumed by users having different valuations of the good; thus, individual valuations of the public good are different from the system's valuation and this results in inefficiency. This explains why the pricing mechanisms employed in [2–4, 7] do not achieve globally optimal allocations and why the mechanism proposed in [5] does not achieve optimal allocations unless the users vary their utilities according to their target SIRs.

The pricing mechanism proposed in [12] is different from the above references in that it obtains close to globally optimal allocations. The reason for this is the following. The authors of [12] introduce a constraint on the total power transmitted by the BS. Due to this constraint, the original problem, where each user's utility depends on the entire power vector transmitted by the BS, reduces to one where each user's utility depends only on the power transmitted to it. Thus, the problem changes from a public good allocation problem (when explicit interference is present) to a private good allocation problem. This is why the pricing mechanism proposed in [12] results in efficient allocations. In systems where there is no constraint on the maximum sum power, the above-stated reduction is not possible and therefore, pricing mechanisms do not yield optimal allocations. The failure of pricing mechanisms to produce globally optimal power allocations for wireless networks affected by interference, provides the key motivation for the formulation and solution methodology presented in this paper.

In [13] a decentralized power allocation mechanism was proposed that appropriately takes into account the externalities (public good effect) due to the interference from other users. The mechanism overcomes the inefficiency of the pricing mechanisms and obtains optimal power allocations. However, the network studied in [13] assumes cooperative users. The results of [13] motivated us to explore optimal power allocation mechanisms for networks in a non-cooperative setup; this setup is adopted in this paper.

1.3 Contributions of the paper

The key contributions of this paper are: (i) The formulation of the uplink power allocation problem with interference as a public good allocation problem; (ii) The specification of a game form/decentralized power allocation mechanism (based on the public good formulation), the equilibrium analysis of the mechanism, and the proof of its key properties, namely: (1) All Nash equilibria (NE) of the game induced by the mechanism result in allocations that are optimal solutions of the corresponding centralized uplink problem (Nash implementation, cf. Sect. 2.2). (2) All users voluntarily participate in the allocation process (individual rationality, cf. Sect. 2.2). (3) Budget balance at all NE and off equilibrium.

Our proposed mechanism is distinctly different from the pricing mechanisms studied in the aforementioned literature. Our formulation properly captures the valuation of interference by each individual user as well as the system and hence, the proposed mechanism leads to globally optimal power allocations. Because the valuation of interference has to be properly captured, the complexity of the strategy space (also called message space) of our mechanism is significantly larger than that of pricing mechanisms. We discuss issues related to the complexity of our mechanism and its impact on scalability in Sect. 3.3 and in Sect. 4.

The rest of the paper is organized as follows: In Sect. 2 we present the uplink model and formulate the corresponding centralized power allocation problem. In Sect. 2.2 we model the power allocation problem in the framework of implementation theory. In Sect. 3.1 we present a game form that obtains the solution of the centralized power allocation problem at its Nash equilibria, is individually rational and budget-balanced at all Nash equilibria as well as off equilibrium. The proofs of the theorems that assert the above properties of the game form are presented in Appendices A and B. A discussion on users' utility functions when the base station employs multi-user detector decoding is presented in Appendix C. We conclude in Sect. 4.

Before we present the model in Sect. 2, we describe here the notation that we will use throughout the paper.

1.3.1 Notation

We represent vectors by underlined bold letters and scalars by normal letters. The elements of a vector are represented by subscripting the vector symbol. An underlined bold subscripted-symbol means that the vector-element is also a vector e.g. in $\underline{\boldsymbol{x}} = (\underline{\boldsymbol{x}}_1, \underline{\boldsymbol{x}}_2, \ldots, \underline{\boldsymbol{x}}_N)$, each $\underline{\boldsymbol{x}}_i$, $i = 1, 2, \ldots, N$, is a vector; in $\underline{\boldsymbol{x}} = (x_1, x_2, \ldots, x_N)$, each x_i, $i = 1, 2, \ldots, N$, is a scalar. Unless otherwise stated, all vectors are treated as column vectors. Bold $\underline{\mathbf{0}}$ is treated as a zero vector of appropriate size determined by the context. The notation $(x_i, \underline{\boldsymbol{x}}^*/i)$ (or $(\underline{\boldsymbol{x}}_i, \underline{\boldsymbol{x}}^*/i)$) is used to represent the following: $(x_i, \underline{\boldsymbol{x}}^*/i)$ (or $(\underline{\boldsymbol{x}}_i, \underline{\boldsymbol{x}}^*/i)$) is a vector of dimension same as that of $\underline{\boldsymbol{x}}^*$; the ith element of $(x_i, \underline{\boldsymbol{x}}^*/i)$ (or $(\underline{\boldsymbol{x}}_i, \underline{\boldsymbol{x}}^*/i)$) is x_i (or $\underline{\boldsymbol{x}}_i$), all other elements of it are the same as the corresponding elements of $\underline{\boldsymbol{x}}^*$. We represent a diagonal matrix of size $N \times N$ whose diagonal entries are elements of the vector $\underline{\boldsymbol{x}} \in \mathbb{R}^N$ by $\mathrm{diag}(\underline{\boldsymbol{x}})$.

2 The model (M1)

We consider a single cell CDMA wireless data network consisting of a Base Station (BS) and multiple mobile users. In this paper we focus on the uplink transmission from the mobiles to the BS as shown in Fig. 1. Later we briefly discuss how the results for the downlink network can be obtained in a similar way.[1] We assume that there are N mobile users,[2] $N \geq 3$, in the network; we denote the set of users by $\mathcal{N} := \{1, 2, \ldots, N\}$. We consider the transmissions of the users in a given carrier frequency; we assume that the signature codes used by the users are not completely orthogonal,[3] hence the reception of signals from each user experiences interference at the BS due to other users' transmissions to the BS. Each user $i \in \mathcal{N}$ receives a Quality of Service (QoS) from the data decoded by the BS for user i. Due to interference, the QoS of user $i, i \in \mathcal{N}$, depends not only on the transmission power p_i^t of user i but also, on the power p_j^t, $j \in \mathcal{N}\backslash\{i\}$ of other users' transmissions to the BS. User $i, i \in \mathcal{N}$, is capable of transmitting in the power range $\mathcal{P}_i^t := [0, P_i^{t\,max}]$. We assume that,

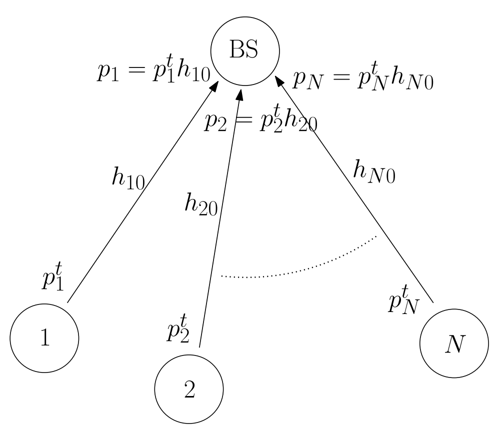

Fig. 1 An uplink network with N mobile users and one base station

[1] In [14] we treat the problem of downlink transmission from the BS to the mobiles in detail. For this problem we derive results similar to the ones for the uplink problem presented in this paper.

[2] Here onwards we will use the terms "mobiles" and "users" interchangeably to mean mobile users.

[3] This helps increase the capacity of the network.

Assumption 1 *The transmission power range $\mathcal{P}_i^t$ is user i's private information.*[4]

Due to the path loss from the mobiles to the BS, the QoS of user i actually depends on the power $p_j := p_j^t h_{j0}$, $j \in \mathcal{N}$, received at the BS from all the users, where h_{j0} is the channel gain from user j to the BS.

The QoS of a user that results from the power transmitted by all the users is quantified by a *utility function*. We denote the utility that user $i \in \mathcal{N}$ obtains when the *power profile* received by the BS is $\underline{\boldsymbol{p}} := (p_1, p_2, \dots, p_N)$ by $u_i(\underline{\boldsymbol{p}})$. The functional form of $u_i : \mathbb{R}^N \to \mathbb{R}$ depends on the technology used by the BS to decode user i's data as well as on the personal preference of (human) user i for the decoded data. We assume that the BS uses a Multi User Detector (MUD) decoder for each user. The BS informs each user a-priori as to which code to use for its data transmission so that the BS can employ an MUD upon receiving the signals from all the users. We note that it is in interest of each user to stick to the code assigned by the BS because otherwise, the BS will not be able to decode their respective data correctly. In Appendix C (see also [13]) we present explicitly the utility function of a user when the BS uses an MUD for each user. We show that such a utility function is almost concave in $\underline{\boldsymbol{p}}$. Hence, we make the following approximation. Let

$$\begin{aligned} S_i &:= \{\underline{\boldsymbol{p}} \mid p_i \in \mathcal{P}_i;\ p_j \in \mathbb{R}_+, j \in \mathcal{N}\backslash\{i\}\}, \\ &\quad \text{where } \mathcal{P}_i := [0, P_i^{t\,max} h_{i0}] =: [0, P_i^{max}]. \end{aligned} \tag{1}$$

Assumption 2 *For each $i \in \mathcal{N}$, $u_i : \mathbb{R}^N \to \mathbb{R}$ is concave in $\underline{\boldsymbol{p}}$ for $\underline{\boldsymbol{p}} \in S_i$ and $u_i(\underline{\boldsymbol{p}}) = 0$ for $\underline{\boldsymbol{p}} \notin S_i$. Also, the function u_i is private information of user i.*

The assumption that $u_i(\underline{\boldsymbol{p}}) = 0$ for $\underline{\boldsymbol{p}} \notin S_i$ is made for the following reason. A power profile $\underline{\boldsymbol{p}} \notin S_i$ implies that either $p_i \notin \mathcal{P}_i$, or $p_j \notin \mathbb{R}_+$ for some $j \in \mathcal{N}\backslash\{i\}$. According to user i's knowledge,[5] it is not possible for the BS to receive such a power profile because it corresponds to transmission powers that are outside the feasible range (as known to user i) of users' transmission powers. Therefore, a power profile $\underline{\boldsymbol{p}} \notin S_i$ cannot provide any QoS to user i and results in zero utility.

We assume that,

Assumption 3 *The network users are non-cooperative and selfish. The BS on the other hand does not have any utility associated with the power allocations/transmissions. It acts like an accountant that redistributes taxes (discussed below) according to the specifications of the allocation mechanism.*

Assumption 3 implies that the users have an incentive to misrepresent their private information, e.g. a user $i \in \mathcal{N}$ may not want to report to other users or to the BS its true preference for the users' transmissions, if by doing so user i obtains a power allocation in its favor.

We note that each user $i \in \mathcal{N}$ needs to know the channel gain h_{i0} in order to know how the power transmitted by it affects its QoS at the BS. The BS can measure the channel gains h_{i0}, $i \in \mathcal{N}$, and announce them to the respective users if the users send some "pre-specified" pilot signals to the BS. However, because the users are selfish, the BS cannot rely upon the pilot signal transmission from the users. Therefore, we assume that the BS periodically transmits pilot signals to the users so that each user $i \in \mathcal{N}$ can measure the channel gain h_{0i} from the BS to itself. Furthermore, we assume that,

Assumption 4 *The channel between the BS and the users is symmetric, i.e. $h_{0i} = h_{i0}\ \forall i \in \mathcal{N}$.*

Because of Assumption 4, each user $i \in \mathcal{N}$ can compute the channel gain h_{i0} from its measurement of h_{0i}. We note that it is in the interest of each user to measure its respective channel gain h_{0i} correctly because this will tell the user correctly the influence of its transmission power on its QoS. We assume that,

Assumption 5 *For each $i \in \mathcal{N}$, the channel gain h_{i0} is user i's private information.*

We would like to mention here that Assumption 4 is made only for convenience and that it is not necessary for the power allocation mechanism we present in this paper to work. We explain the consequence of relaxing this assumption in Sect. 3.2 after we present the power allocation mechanism.

Each user $i \in \mathcal{N}$ pays a *tax* $t_i \in \mathbb{R}$ to the BS. This tax is imposed for the following reasons: (i) For the use of the network by the users. (ii) To provide incentives to the users to transmit powers that result in a network-wide performance objective. The tax for a user can be either positive or negative and is determined by the rules of the power allocation mechanism. With the flexibility of either charging a user (positive tax) or paying compensation/subsidy (negative tax) to a user, it is possible to induce users to behave in such a way that a network wide performance objective is achieved. For example, given the power transmission and interference

[4] Private information of a user is defined as the information that is known only to that user and nobody else in the network.

[5] Assumption 4 that we state later implies that, each user $i \in \mathcal{N}$ knows the channel gain h_{i0} from itself to the BS. As a result it knows the range $\mathcal{P}_i$ as well as the set S_i exactly. On the other hand, the set S_j, $j \in \mathcal{N}\backslash\{i\}$ is private information of user j and user i does not know this set. Therefore, user i perceives S_i to be the set of powers that are feasible for the BS to receive.

constraints in the network, we can satisfy all the users by setting "positive tax" for the users that receive power allocations close to those requested by them and paying "compensation" to the users that receive allocations that are not close to their desirable ones. According to Assumption 3 the BS does not derive any profit from the above tax and the purpose of the above tax collection is to just redistribute the money among network users. This implies that the tax profile $\underline{\boldsymbol{t}} := (t_1, t_2, \ldots, t_N)$ is determined in a way such that,

$$\sum_{i=1}^{N} t_i = 0. \tag{2}$$

To describe the "overall satisfaction" of a user from the QoS it receives from the power profile received by the BS and the tax it pays for this QoS, we define an *aggregate utility function* $u_i^A : \mathbb{R}^{1+N} \to \mathbb{R} \cup \{-\infty\}$ for each user $i \in \mathcal{N}$ as follows:

$$u_i^A(t_i, \underline{\boldsymbol{p}}) := \begin{cases} -t_i + u_i(\underline{\boldsymbol{p}}) & \text{if } \underline{\boldsymbol{p}} \in S_i, \\ -\infty, & \text{otherwise.} \end{cases} \tag{3}$$

Equation (3) signifies that an allocation $(t_i, \underline{\boldsymbol{p}})$ is of no use to user i if $\underline{\boldsymbol{p}} \notin S_i$. This is because, based on its knowledge, user i knows that it is not possible for the BS to receive a power profile $\underline{\boldsymbol{p}} \notin S_i$. Because of Assumption 5 and Assumption 1, the set S_i is user i's private information. This along with Assumption 2 implies that for each $i \in \mathcal{N}$, the aggregate utility u_i^A is user i's private information. As stated in Assumption 3 users are non-cooperative and selfish. Therefore, *the users are self aggregate utility maximizers*.

In this paper we restrict attention to static problems. Specifically we make the following assumption:

Assumption 6 *The set of users $\mathcal{N}$, their utilities and the channel gains between the BS and the users are fixed in advance and they do not change with time.*

We also assume that before any power allocation period, the BS announces the set of users in the network, therefore,

Assumption 7 *The set of users $\mathcal{N}$ is* common knowledge.

In the following section we formulate the power allocation problem for the network model (M1).

2.1 The uplink power allocation problem

For the network model (M1) we want to develop a power and tax determination mechanism that works under the constraints imposed by the model and obtains a solution to the following centralized problem corresponding to it.

Problem ($\boldsymbol{P_{CU}}$)

$$\max_{(\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}})} \sum_{i \in \mathcal{N}} u_i^A(t_i, \underline{\boldsymbol{p}}),$$
$$\text{s.t.} \quad \sum_{i \in \mathcal{N}} t_i = 0. \tag{4}$$

$$\equiv \max_{(\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}}) \in S} \sum_{i \in \mathcal{N}} u_i(\underline{\boldsymbol{p}}), \quad \text{where}$$

$$S := \left\{ (\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}}) \mid \sum_{i \in \mathcal{N}} t_i = 0, \underline{\boldsymbol{t}} \in \mathbb{R}^N;\ p_i \in \mathcal{P}_i, i \in \mathcal{N} \right\}. \tag{5}$$

The optimization problem (4) is equivalent to (5) because for $(\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}}) \notin S$, the objective function in (4) is negative infinity by (3). Thus S is the set of feasible solutions of Problem (P_{CU}). Because of Assumption 2, the objective function in (5) is concave in $\underline{\boldsymbol{p}}$. Moreover, the sets $\mathcal{P}_i, i \in \mathcal{N}$, are convex and compact. Therefore, there exists an optimal power profile $\underline{\boldsymbol{p}}^*$ of Problem (P_{CU}). Furthermore, since the objective function in (5) does not explicitly depend on $\underline{\boldsymbol{t}}$, an optimal solution of Problem (P_{CU}) must be of the form $(\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}}^*)$, where $\underline{\boldsymbol{p}}^*$ is an optimal power profile and $\underline{\boldsymbol{t}}$ is any feasible tax profile for Problem (P_{CU}), i.e. a tax profile that satisfies (2).

Assumptions 1, 2 and 5 imply that there is no entity in the network that knows perfectly all the parameters that describe Problem (P_{CU}). Therefore, we need to develop a mechanism that allows the users and the BS to communicate with one another and that leads to optimal allocations for Problem (P_{CU}). Since a key assumption in Model (M1) is that the users are non-cooperative and selfish, the mechanism we develop must take into account the possible strategic behavior of the users in their communication with the BS.

A systematic approach to the development of resource allocation mechanisms for informationally decentralized networks (as the one described by Model (M1)) where users behave strategically, is provided by *implementation theory*, a branch of Mathematical Economics. In the context of our problem, implementation theory deals with the design of mechanisms that provide rules/guidelines on; (i) how the BS and the mobiles should "communicate" with one another; and (ii) how power allocations and tax allocations should be determined, based on the outcome of communication, so as to induce the desired user/mobile strategic behavior.

In this paper we use an implementation theory-based approach for the solution of the power allocation problem presented in this section. Therefore, in the next section we provide a brief introduction to implementation theory and set the preliminaries for our solution to the power allocation problem.

2.2 Embedding the power allocation problem for Model (M1) in the framework of implementation theory

Implementation theory is a branch of the theory of *mechanism design* developed by mathematical economists. Mechanism design provides a systematic methodology for the design of decentralized resource allocation mechanisms for informationally decentralized systems; it focuses on the design of decentralized mechanisms that can achieve some prespecified objective, e.g. maximizing some network-wide/social welfare function. In the mechanism design framework, a centralized resource allocation problem is described by the triple $(\mathcal{E}, \mathcal{A}, \gamma)$: the environment space $\mathcal{E}$, the action/allocation space $\mathcal{A}$ and the goal correspondence γ.

The *environment* $\underline{\boldsymbol{e}}$ of a resource allocation problem, centralized or decentralized, is defined to be the set of resources and technologies available to all the users, their utilities, and any other information available to them, taken together. These are circumstances that cannot be changed either by the users in the network or by the designer of the resource allocation mechanism. For the network described by Model (M1), the environment $\underline{\boldsymbol{e}_i}$ of user i, $i \in \mathcal{N}$, consists of the channel gains h_{0i} and h_{i0}, its utility function u_i^A, and the common knowledge about the set of users $\mathcal{N}$ as well as the fact that the set of users, their utilities and the channel gains remain fixed throughout a power allocation period. The environments of all the users collectively define the system environment $\underline{\boldsymbol{e}} := (\underline{\boldsymbol{e}_1}, \underline{\boldsymbol{e}_2}, \dots, \underline{\boldsymbol{e}_N})$. The set of all possible environments $\underline{\boldsymbol{e}_i}$ of a user defines its environment space $\mathcal{E}_i$. The environment spaces of all the users collectively define the environment space $\mathcal{E} := (\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_N)$ of the system/problem.

The *action/allocation space* $\mathcal{A}$ of a resource allocation problem, centralized or decentralized, is defined to be the set of all possible resource allocation/exchange actions that can be taken by the users. For the network described by Model (M1), $\mathcal{A}$ is the set S of all tax and received power profiles $(\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}})$ that the BS can possibly allocate to the users.

The *goal correspondence* γ of a centralized resource allocation problem is a map from $\mathcal{E}$ to $\mathcal{A}$ which assigns for every environment $\underline{\boldsymbol{e}} \in \mathcal{E}$, the set of allocations in $\mathcal{A}$ that are solutions to the centralized resource allocation problem according to some pre-specified system goal. For the centralized power allocation problem (P_{CU}), the system goal is the maximization of the sum $\sum_{i \in \mathcal{N}} u_i^A(t_i, \underline{\boldsymbol{p}})$ of users' utilities, and γ is a mapping that maps every environment $\underline{\boldsymbol{e}} \in \mathcal{E}$, defined in the previous paragraph, to the set of solutions of (P_{CU}). Since in a centralized scenario one of the users (or a controller such as the BS) has complete system information, i.e. it knows $\underline{\boldsymbol{e}}$, it can determine optimal allocations $\gamma(\underline{\boldsymbol{e}})$ in $\mathcal{A}$ corresponding to any given $\underline{\boldsymbol{e}}$ using centralized optimization methods (such as mathematical programming or dynamic programming).

In an informationally decentralized system as the one described by Model (M1), the controller (BS in Model (M1)) does not completely know $\underline{\boldsymbol{e}}$, therefore it can not determine optimal centralized allocations $\gamma(\underline{\boldsymbol{e}})$ by methods similar to those for the centralized problems. Therefore, for resource allocation in a decentralized system, it is desirable to devise a communication/message exchange process between the users and the controller that eventually enables the controller to determine optimal centralized allocations. However, when the users in a system are selfish, they have an incentive to misrepresent their private information while communicating with the controller so as to shift the allocation determined by the controller in their own favor. The users may also choose not to participate in the communication process if they know that the resulting allocation will not be in their favor (or if by not participating they are better off). This may defeat the objective of maximizing the system objective function ($\sum_{i \in \mathcal{N}} u_i^A(t_i, \underline{\boldsymbol{p}})$ for the power allocation problem). Therefore, for the success of a communication process in leading to desirable outcomes it is required that the allocation rule employed by the controller induces the users to behave in a desirable manner (i.e. it ensures voluntary participation of the users in the communication process and furthermore, it induces the users to communicate information that results in system objective maximizing allocations). In the context of mechanism design, a formal treatment of the design of such communication and allocation rules is provided by implementation theory.

In implementation theory, a decentralized resource allocation mechanism is specified by a *game form*. An N-user game form is defined by the pair $(\mathcal{M}, f)$. $\mathcal{M} := \prod_{i=1}^N \mathcal{M}_i$ is the *message space* which specifies for each $i \in \mathcal{N}$ the set of messages $\mathcal{M}_i$ that user i can communicate to other users and the controller. f is the *outcome function* which maps $\mathcal{M} \to \mathcal{A}$; it specifies for each *message profile* $\underline{\boldsymbol{m}} \in \mathcal{M}$, $(\underline{\boldsymbol{m}} := (\underline{\boldsymbol{m}_1}, \underline{\boldsymbol{m}_2}, \dots, \underline{\boldsymbol{m}_N}), \underline{\boldsymbol{m}_i} \in \mathcal{M}_i, i \in \mathcal{N})$, the resulting allocation $f(\underline{\boldsymbol{m}}) \in \mathcal{A}$.

Since the participation of the users in a resource allocation mechanism requires that they be aware of its protocols, it is assumed that the game form is known to all the users in the system. In order for a decentralized mechanism, specified by a game form, to obtain (optimal) centralized solutions when the users in the system are selfish, it is required, as discussed before, that the allocation rule f induces the users to behave in a desirable manner. To specify this requirement on a game form, we must first specify the users' behavior. In the context of implementation theory, the users' behavior is specified by specifying *games* and associated *equilibrium concepts*.

A game is specified by a game form $(\mathcal{M}, f)$ together with the utilities $u_i^A, i \in \mathcal{N}$, specified by an environment

$\underline{e} \in \mathcal{E}$. Such a game is represented by $(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$. In this game the players are the users in $\mathcal{N}$, the set of strategies of a user is its respective message space $\mathcal{M}_i$, $i \in \mathcal{N}$, and the payoff of a user corresponding to a given strategy/message profile $\underline{\boldsymbol{m}}$ is the utility $u_i^A(f(\underline{\boldsymbol{m}}))$, $i \in \mathcal{N}$, it obtains from the resulting allocation $f(\underline{\boldsymbol{m}})$. Given a game, the users can be assumed to behave according to different "behavioral concepts" which lead to different types of "equilibrium concepts". One such equilibrium concept is *Nash Equilibrium* (NE). A Nash Equilibrium of a game is defined as a message profile $\underline{\boldsymbol{m}^*}$ such that none of the users finds it profitable to unilaterally deviate to any other message. Mathematically, $\underline{\boldsymbol{m}^*}$ is a NE of the game $(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$ if,

$$u_i^A(f(\underline{\boldsymbol{m}^*})) \geq u_i^A(f((\underline{\boldsymbol{m}_i}, \underline{\boldsymbol{m}^*}/i))),$$
$$\forall \underline{\boldsymbol{m}_i} \in \mathcal{M}_i, \ \forall i \in \mathcal{N}. \tag{6}$$

Let $NE(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$ represent the set of all Nash equilibria of the game $(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$, and let

$$\mathcal{A}_{NE}(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$$
$$:= \{\underline{\boldsymbol{a}} \in \mathcal{A} \mid \underline{\boldsymbol{a}} = f(\underline{\boldsymbol{m}}) \text{ for some } \underline{\boldsymbol{m}} \in NE(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)\}, \tag{7}$$

that is, $\mathcal{A}_{NE}$ is the set of allocations corresponding to all Nash equilibria of the game.

Now consider a decentralized resource allocation problem. Let $\mathcal{E} = \prod_{i=0}^N \mathcal{E}_i$ be the environment space and $\mathcal{A}$ the allocation space associated with the problem, let $\gamma : \mathcal{E} \to \mathcal{A}$ be a goal correspondence, and let $u_1^A, u_2^A, \ldots, u_N^A$, be the users' utilities corresponding to a given environment $\underline{e} \in \mathcal{E}$. Then, we have the following:

Definition 1 (Implementation in Nash equilibria) A game form $(\mathcal{M}, f)$ is said to "implement in Nash equilibria" the goal correspondence γ if,

$$\mathcal{A}_{NE}(\mathcal{M}, f, \{u_i^A\}_{i=1}^N) \subset \gamma(\underline{e}) \quad \forall \underline{e} \in \mathcal{E},$$

i.e., for any given environment, the set of allocations resulting (through the outcome function f) from the Nash equilibria of the game $(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$ is a subset of the set of allocations $\gamma(\underline{e})$ that are optimal solutions of the corresponding centralized problem $(\underline{e}, \mathcal{A}, \gamma)$.

Definition 1 implies that a game form that implements in NE a goal correspondence, takes into account the users' strategic behavior and obtains centralized solutions, given that the users participate in the message exchange process specified by the game form. However, in order that the users voluntarily participate in a mechanism specified by a game form, the game form must satisfy an additional property defined as follows. Let the *initial endowment* of a user be defined as the amount of resources the user has before participating in a game form; e.g. for the network model (M1), the initial endowment $\underline{\boldsymbol{f}_i^{\boldsymbol{0}}}$ of user i, $i \in \mathcal{N}$, is the tax and transmission power profile before the power allocation mechanism is run, i.e. $\underline{\boldsymbol{f}_i^{\boldsymbol{0}}} = (t_i^0, \underline{\boldsymbol{p}}^0) = (0, \underline{\boldsymbol{0}})$, $\forall i \in \mathcal{N}$. We then have the following,

Definition 2 (Individual rationality) A game form $(\mathcal{M}, f)$ is said to be individually rational if $\forall i \in \mathcal{N}$, $u_i^A(f(\underline{\boldsymbol{m}})) \geq u_i^A(\underline{\boldsymbol{f}_i^{\boldsymbol{0}}})$ for all $\underline{\boldsymbol{m}} \in NE(\mathcal{M}, f, \{u_i^A\}_{i=1}^N)$, i.e. at any NE allocation the utility of each user is at least as much as its utility before participating in the game/allocation process.

Definitions 1 and 2 imply that a game form that is individually rational and implements in NE a goal correspondence, obtains optimal allocations of the corresponding centralized system by having the users voluntarily participate in the allocation process. These are exactly the properties that we want in a tax and power allocation mechanism for the network model (M1). Thus the theory of implementation introduced above provides us with a framework to develop the desired decentralized power allocation mechanism for the network model (M1).

In light of the discussion provided in this section, we now state our objective for the power allocation problem presented in Sect. 2.1.

The objective: Let $\mathcal{E}$ and $\mathcal{A}$ be respectively the environment space and the allocation space corresponding to the uplink network model (M1) as defined in Sect. 2.2. Let $\gamma : \mathcal{E} \to \mathcal{A}$ be the goal correspondence for Problem (P_{CU}) as defined in Sect. 2.2. Our objective is to design an individually rational game form $(\mathcal{M}, f)$ that implements in NE the goal correspondence γ.

In the next section, we present a game form that achieves the above objective. However, before we proceed, we present a brief clarification on the interpretation of NE in the mechanism that we present in the following section. Nash equilibria describe strategic behavior in games of complete information. Since the users in Model (M1) do not know each other's utilities, for any profile of the users' utilities the resulting game is not one of complete information. We can create a game of complete information by increasing the message/strategy space following Maskin's approach [15]. However, such an approach would result in an infinite dimensional message/strategy space for the corresponding game. We do not follow Maskin's approach; instead, we adopt the philosophy of [16]. Specifically, by quoting [16], "we interpret our analysis as applying to an unspecified (message exchange) process in which users grope their way to a stationary message and in which the Nash property is a necessary condition for stationarity."

3 Solution of the uplink power allocation problem

In this section we present a game form that provides a decentralized mechanism for solving the uplink power allocation problem presented in Sect. 2.1. We first present the structure of the message space $\mathcal{M}$ and the outcome function f that constitute the game form. We then present theorems that assert that the proposed game form is individually rational and that it fully implements in NE the goal correspondence γ corresponding to Problem (P_{CU}). At the end we present a discussion on the intuition behind the structure of the proposed game form.

3.1 The game form

To obtain an appropriate game form for the power allocation problem it is useful to observe that in the uplink network, the power profile $\underline{\boldsymbol{p}} = (p_1, p_2, \dots, p_N)$ received by the BS can be treated as a *public good* [1]. This is because, analogous to a public good in an economy, the same vector $\underline{\boldsymbol{p}}$ affects the utility of all the users in the network. Furthermore, like a public good, the exact amount of the utility a user obtains from $\underline{\boldsymbol{p}}$ differs from user to user and depends on its individual function $u_i, i \in \mathcal{N}$ that determines its QoS. Game forms that implement in NE efficient allocation of public goods have been proposed by Groves and Ledyard [17], Hurwicz [18] and Walker [19]. In this section we present a game form for the uplink power allocation problem that is inspired from Hurwicz' mechanism [18]. Below we specify each of the elements of the proposed game form, the message space and the outcome function.

3.1.1 The message space

Since for the network model (M1) we are interested in determining the power profile that should be received at the BS and tax that the users should pay, the communication between the users and the BS should contain information that is helpful in determining the optimal amounts of each of commodities. We let each user $i \in \mathcal{N}$ send to the BS a message $\underline{\boldsymbol{m}_i} \in \mathcal{M}_i := \mathbb{R}_+^N \times \mathbb{R}^N$ that has the following form:

$$\underline{\boldsymbol{m}_i} := (\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i}); \qquad \underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N, \; \underline{\boldsymbol{p}_i} \in \mathbb{R}^N. \tag{8}$$

The message $\underline{\boldsymbol{m}_i}$ consists of two elements: $\underline{\boldsymbol{p}_i} = (p_{i1}, p_{i2}, \dots, p_{iN})$ which can be interpreted as the received power profile that user i $(i \in \mathcal{N})$ suggests to be allocated to all the users $j \in \mathcal{N}$; and $\underline{\boldsymbol{\pi}_i} = (\pi_{i1}, \pi_{i2}, \dots, \pi_{iN})$ which can be interpreted as the price that user i $(i \in \mathcal{N})$ suggests to be charged to the users $j \in \mathcal{N}$ for using the network.

3.1.2 The outcome function

Based on the message profile $\underline{\boldsymbol{m}} = (\underline{\boldsymbol{m}_1}, \underline{\boldsymbol{m}_2}, \dots, \underline{\boldsymbol{m}_N})$, the BS sets the taxes $\hat{t}_i(\underline{\boldsymbol{m}}), i \in \mathcal{N}$, and determines powers $\underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}}) = (\hat{p}_1(\underline{\boldsymbol{m}}), \hat{p}_2(\underline{\boldsymbol{m}}), \dots, \hat{p}_N(\underline{\boldsymbol{m}}))$ to be received from the users as follows:

$$\underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}}) = \frac{1}{N} \sum_{i=1}^{N} \underline{\boldsymbol{p}_i}, \tag{9}$$

$$\begin{aligned} \hat{t}_i(\underline{\boldsymbol{m}}) &= \underline{\boldsymbol{l}_i^T}(\underline{\boldsymbol{m}}) \underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}}) + (\underline{\boldsymbol{p}_i} - \underline{\boldsymbol{p}_{i+1}})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_i})(\underline{\boldsymbol{p}_i} - \underline{\boldsymbol{p}_{i+1}}) \\ &\quad - (\underline{\boldsymbol{p}_{i+1}} - \underline{\boldsymbol{p}_{i+2}})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_{i+1}})(\underline{\boldsymbol{p}_{i+1}} - \underline{\boldsymbol{p}_{i+2}}), \quad i \in \mathcal{N}, \end{aligned} \tag{10}$$

where

$$\underline{\boldsymbol{l}_i}(\underline{\boldsymbol{m}}) = \underline{\boldsymbol{\pi}_{i+1}} - \underline{\boldsymbol{\pi}_{i+2}}. \tag{11}$$

In (10) and (11), $i + 2 \equiv 1$ for $i = N - 1$, and for $i = N$, $i + 1 \equiv 1$ and $i + 2 \equiv 2$.

The game form defined by (8)–(11) together with the users' utility functions in (3) specify a game. The strategy of user $i, i \in \mathcal{N}$, in this game is its message $\underline{\boldsymbol{m}_i}$. We note that the message $\underline{\boldsymbol{m}_i}$ of user $i, i \in \mathcal{N}$, is allowed to take any value (which can be unboundedly large) in the space $\mathbb{R}_+^N \times \mathbb{R}^N$; in particular $\underline{\boldsymbol{p}_i}$ is not restricted to lie in S_i. Thus, a Nash equilibrium[6] of the above game is a message profile $\underline{\boldsymbol{m}}^*$ from which no user wants to unilaterally deviate (see (6)) even when arbitrary deviations are possible by unbounded magnitude of messages.

As discussed in Sect. 2.2, our objective is to develop a game form for which the set of tax and received power allocations obtained at all its NE is the same as the set of optimal tax and received power allocations for the centralized problem (P_{CU}). Below we present theorems that assert that the proposed game form achieves this goal.

3.2 Properties of the game form

The main results of this paper are summarized by Theorems 1 and 2 below.

Theorem 1 *Let $\underline{\boldsymbol{m}}^*$ be a NE of the game induced by the game form presented in Sect. 3.1 and the users' utility functions (3). Let $(\underline{\hat{\boldsymbol{t}}}(\underline{\boldsymbol{m}}^*), \underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}}^*)) =: (\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*)$ be the tax and received power allocation at $\underline{\boldsymbol{m}}^*$ determined by the game form. Then,*

(a) *$(\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*)$ is individually rational, i.e. all users weakly prefer $\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*$ to the initial allocation $(\underline{\mathbf{0}}, \underline{\mathbf{0}})$. Mathematically,*

$$u_i^A(\hat{t}_i^*, \underline{\hat{\boldsymbol{p}}}^*) \geq u_i^A(0, \underline{\mathbf{0}}), \qquad \forall i \in \mathcal{N}.$$

(b) *$(\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*)$ is an optimal solution of the centralized problem (P_{CU}).*

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[6] See footnote 8 for a discussion on the interpretation of Nash equilibria.

Theorem 2 *Let $\underline{\hat{p}}^*$ be an optimum received power profile corresponding to Problem (P_{CU}). Then,*

(a) *There exist a set of personalized prices $\underline{l}_i^*$, $i \in \mathcal{N}$, such that*

$$\underline{\hat{p}}^* \in \underset{\underline{p} \in S_i}{\operatorname{argmax}}\{-\underline{l}_i^{*T}\underline{p} + u_i(\underline{p})\}, \quad \forall i \in \mathcal{N}.$$

(b) *There exists at least one NE $\underline{m}^*$ of the game induced by the game form presented in Sect. 3.1 and the users' utility functions (3) such that, $\underline{\hat{p}}(\underline{m}^*) = \underline{\hat{p}}^*$. Furthermore, if $\hat{t}_i^* := \underline{l}_i^{*T}\underline{\hat{p}}^*$, $i \in \mathcal{N}$, the set of all NE $\underline{m}^* = (\underline{m}_1^*, \underline{m}_2^*, \ldots, \underline{m}_N^*)$ (where $\underline{m}_i^* = (\underline{\pi}_i^*, \underline{p}_i^*), i \in \mathcal{N}$) that result in $(\underline{\hat{t}}^*, \underline{\hat{p}}^*)$ is characterized by the solution of the following set of conditions*:

$$\frac{1}{N}\sum_{i\in\mathcal{N}} \underline{p}_i^* = \underline{\hat{p}}^*,$$

$$\underline{\pi}_{i+1}^* - \underline{\pi}_{i+2}^* = \underline{l}_i^*, \quad i \in \mathcal{N},$$

$$(\underline{p}_i^* - \underline{p}_{i+1}^*)^T \operatorname{diag}(\underline{\pi}_i^*)(\underline{p}_i^* - \underline{p}_{i+1}^*) = 0, \quad i \in \mathcal{N},$$

$$\underline{\pi}_i^* \geq \mathbf{0}, \quad i \in \mathcal{N}.$$

Because Theorem 1 is stated for an arbitrary NE $\underline{m}^*$ of the game induced by the game form presented in Sect. 3.1 and the users' utility functions (3), the assertion of the theorem holds for all NE of this game. Thus, part (a) of Theorem 1 establishes that the game form presented in Sect. 3.1 is *individually rational*.

Part (b) of Theorem 1 asserts that all NE of the game induced by the game form presented in Sect. 3.1 and the users' utility functions (3) result in optimal centralized allocations (solutions of Problem (P_{CU})). Thus, the set of NE allocations is a subset of the set of centralized allocations. This establishes that the game form presented in Sect. 3.1 *implements in NE* the goal correspondence γ defined by Problem (P_{CU}) (see Sect. 2.2). Because of this property, the game form guarantees to provide a centralized allocation irrespective of which NE is achieved in the game induced by the game form.

The assertion of Theorem 1 that establishes the above two properties of the game form is based on the assumption that there exists a NE of the game induced by the game form of Sect. 3.1 and the users' utility functions (3). However, Theorem 1 does not say anything about the existence of a NE. Theorem 2 establishes that NE exist in the above game and also characterizes the set of all NE that result in optimal centralized allocations $(\underline{\hat{t}}^*, \underline{\hat{p}}^*) = ((\underline{l}_i^{*T}\underline{\hat{p}}^*)_{i=1}^N, \underline{\hat{p}}^*)$ where $\underline{l}_i^*$, $i = 1, 2, \ldots, N$, are defined in Theorem 2(a).

The proofs of Theorems 1 and 2 are given in Appendices A and B. In the next section we provide a brief discussion on the intuition behind the structure of the proposed game form.

Before we proceed to the next section, we note that the game form presented in Sect. 3.1 determines for the uplink network an optimum power profile that should be "received" at the BS. Once the game form determines an optimum received power profile, each user can determine its respective transmission power that would result in the optimum received power profile since each user knows its respective channel gain h_{i0}, $i \in \mathcal{N}$. Since the optimum received power profile is obtained at the NE of users' messages, no user can gain by unilaterally changing the power received from it at the BS; in other words the user cannot gain by transmitting a power that does not result in the received power determined by the game form. Thus, the game form of Sect. 3.1 not only determines the optimum received powers, but also induces the users to "transmit" with optimum powers.

As we mentioned earlier, Assumption 4 is not necessary for the game form proposed in Sect. 3.1 to result in optimal power allocations. Consider the case when the symmetric channel assumption is relaxed. We note that the game form of Sect. 3.1 requires the users to communicate messages in terms of the power vector received at the BS, not the power vector transmitted by the users. Therefore, once the mechanism determines the power vector that should be received at the BS, the BS can announce it to the users. In the absence of the knowledge of uplink channel gains, the users will have to transmit power based on some estimate of the uplink channel gain; if the power received by the BS is not the same as that determined by the mechanism, the BS can send feedback to the users to adjust their transmission powers. As explained in the previous paragraph, it will be in the interest of the users to make the transmission power adjustment so as to match the received power to the optimal one. Thus, the mechanism would result in the same outcome as in the case with the symmetric channel assumption.

3.3 Key features of and intuition behind the game form

The key feature of our problem is that the action/transmission power of a user directly affects the utility of every other user. Thus, every user's action creates an externality for every other user. Consequently, we have to view the power allocation problem with strategic users as the decentralized resource allocation of a public good, where the public good is the power profile $\underline{p} := (p_1, p_2, \ldots, p_N)$ received at the BS. Since the users are strategic, the dimensionality of the message space of any "efficient"[7] mechanism must be at least as large as the dimensionality of any "efficient" mechanism for non-strategic users [20]. Under the condition that users are non-strategic, the minimum dimensionality of any "efficient" public good mechanism is of the order

[7] We define a mechanism to be "efficient" if it implements in Nash equilibria the solution of the corresponding centralized power allocation problem.

$O(N^2)$ (see [21]). Therefore, any "efficient" mechanism for our problem must have a message space whose dimensionality is at least of the order $O(N^2)$.

In our mechanism each of the N users announces a $2N$ dimensional message consisting of an N dimensional power profile proposal and an N dimensional price profile proposal. Thus, the dimensionality of the message space of our mechanism/game form is $2N^2$. From the above discussion it is clear that the use of high dimensional mechanism is inevitable if one wants to have full implementation in Nash equilibria.

To understand how the proposed structure of the game form achieves the desired goal, let us now look at the properties the game form induces in its NE. A NE of the game corresponding to the proposed game form can be interpreted as follows: Note that the allocated received power profile, given the users' messages $\underline{\boldsymbol{m}}_j$, $j \in \mathcal{N}$, is $1/N \sum_{i=1}^{N} \underline{\boldsymbol{p}}_i$. Therefore user i's proposal $\underline{\boldsymbol{p}}_i$ can be interpreted as the increment user i desires in the power received from each user over the sum of other users' proposals so as to bring the allocated received power profile $\hat{\underline{\boldsymbol{p}}}(\underline{\boldsymbol{m}})$ to i's desired value. Thus, if the average of the received power profiles proposed by users other than user i does not lie in S_i, user i can propose an appropriate received power profile and bring the allocated profile within S_i. It should be noted that the flexibility of proposing any received power profile in $\mathbb{R}^N$ gives each user $i \in \mathcal{N}$ the capability to make the constraint $\underline{\boldsymbol{p}} \in S_i$ be satisfied by unilateral deviation. It follows that any NE received power profile must lie in $\bigcap_{i\in\mathcal{N}} S_i$. Furthermore, it can be seen from (10) that the game form formulation ensures that the allocated tax profile satisfies (2) (even at off-NE messages). The above two features imply that all NE allocations $(\underline{\boldsymbol{t}}, \underline{\boldsymbol{p}})$ lie in S and hence are feasible solutions of Problem (P_{CU}).

To see why NE allocations are optimal, let us look at the form of the tax (10). The tax for user i consists of three types of terms. Type-1 is $\underline{\boldsymbol{l}}_i^T(\underline{\boldsymbol{m}})\hat{\underline{\boldsymbol{p}}}(\underline{\boldsymbol{m}})$ that depends on the power proposals of all the users, and the price proposals of users other than user i. Type-2 term is the one that depends on $\underline{\boldsymbol{p}}_i$ as well as $\underline{\boldsymbol{\pi}}_i$, and type-3 term is the one that depends only on the messages of users other than user i. Since $\underline{\boldsymbol{\pi}}_i$ does not affect the received power allocation and affects only the type-2 term in t_i, the NE strategy of user $i, i \in \mathcal{N}$, that minimizes its tax is to propose for each $j \in \mathcal{N}$, $\pi_{ij} = 0$ unless at the NE, $p_{ij} = p_{i+1\,j}$. Since all the users $i \in \mathcal{N}$ choose the aforementioned strategy at the NE, the type-2 and type-3 terms vanish from every user's tax $t_i, i \in \mathcal{N}$, at the NE. Thus, the tax that users pay at a NE $\underline{\boldsymbol{m}}^*$ is of the form $\underline{\boldsymbol{l}}_i^T(\underline{\boldsymbol{m}}^*)\hat{\underline{\boldsymbol{p}}}(\underline{\boldsymbol{m}}^*)$, $i \in \mathcal{N}$. The NE price term $\underline{\boldsymbol{l}}_i^T(\underline{\boldsymbol{m}}^*) =: \underline{\boldsymbol{l}}_i^{*T}$, $i \in \mathcal{N}$, can therefore be interpreted as the "personalized price" [8] of the NE received power profile $\hat{\underline{\boldsymbol{p}}}(\underline{\boldsymbol{m}}^*) =: \hat{\underline{\boldsymbol{p}}}^*$ (treated as a public good) for user i; at the NE this price for user i is not controlled by i's message. The above reduction of tax terms in terms of the allocated received power profile implies that, at the NE, the utilities of the users $i \in \mathcal{N}$ effectively depend only on the allocated received power profile. Since each user has the capability (by choosing appropriate $\underline{\boldsymbol{p}}_i \in \mathbb{R}^N$) to shift the allocated received power profile to its desired value given that the proposals of all other users are fixed, the NE strategy of each user is to propose a power profile that results in an allocation that maximizes its corresponding utility. Thus, each user maximizes its net utility at the NE, and this results in the maximization of the system objective function at the NE.

It is worth mentioning at this point the significance of type-2 and type-3 terms in the users' tax. As explained above, these terms vanish at NE. However, if these terms are not present in t_i, user $i, i \in \mathcal{N}$, can propose arbitrarily high price for other users in $\underline{\boldsymbol{\pi}}_i$ as $\underline{\boldsymbol{\pi}}_i$ would not affect user i's utility at all.[9] It is also important that the NE price $\underline{\boldsymbol{l}}_i$ is not affected by $\underline{\boldsymbol{\pi}}_i$, otherwise user i may influence its own price in an unfair manner. However, since $\underline{\boldsymbol{\pi}}_i$ would affect other users' price, it is necessary to prevent user i from proposing unfair prices for other users. Type-2 and type-3 terms in t_i do the above job by imposing a penalty on user i at off-equilibrium messages if user i proposes a high value of $\underline{\boldsymbol{\pi}}_i$ or if it deviates too much from other users in its power profile proposal.

4 Conclusion

In this paper we studied power allocation for a single cell wireless CDMA network with interference and selfish users. The problems of power allocation in cellular networks in the presence of interference are analogous to public good allocation problems. Thus, pricing mechanisms that are useful in developing decentralized optimal power allocation algorithms for networks without interference (which are analogous to private good economies), do not result in globally optimal power allocations for cellular networks with interference. The main contributions of this paper are: (i) The formulation of the uplink power allocation problem as a public good allocation problem. (ii) The construction of a game form that has the following properties: (1) It implements in Nash equilibria the solution of the corresponding centralized power allocation problem; (2) It is individually rational; (3) It is budget-balanced at all Nash equilibria and off equilibrium. (iii) The characterization of all Nash equilibria of the games corresponding to the proposed game form.

[8] In Economics literature, these personalized prices for the public goods are called "Lindahl" prices.

[9] Note that $\boldsymbol{l}_i$ depends on $\boldsymbol{\pi}_{i+1}$ and $\boldsymbol{\pi}_{i+2}$ and not $\boldsymbol{\pi}_i$.

For simplicity of presentation and explanation, we presented only the uplink problem in this paper, however, the downlink power allocation in the presence of interference and selfish users can be solved in the same way as the uplink problem. Unlike the uplink power allocation problem in which we treat the power vector received at the BS as a public good, in the downlink problem we treat the power vector transmitted by the BS to the users as a public good because in the downlink case, the power vector transmitted by the BS is the common commodity that affects the utilities of all the users. We refer the interested reader to [14, 22] for details of the results on the downlink problem.

We conclude by discussing some complexity, implementation and scalability issues associated with our mechanism. We have already discussed the complexity of the message space of our mechanism in Sect. 3.3 and have pointed out that the use of high dimensional mechanisms is inevitable if one wants to have full implementation in NE.

Given that the dimensionality of the message space of any "efficient" mechanism must be high (at least $O(N^2)$), we now discuss its impact on scalability. If each user's transmission affects every other user, then the mechanism we propose is not scalable. On the other hand, in the situation where each user's transmission affects only a subset of the network users, it is possible to have, for large-scale networks, mechanisms that are efficient, scalable and have characteristics similar to our mechanism. An efficient and scalable mechanism for this situation is presented in [25].

Even though in our solution to the power allocation problem we have implementation in NE and have obtained a complete characterization of all Nash equilibria, at present we do not have an algorithm for the computation of these equilibria. For our problem, best response dynamics do not guarantee convergence to Nash equilibria because the games induced by the proposed game form are not, in general, supermodular. For development of efficient mechanisms that can compute NE, there can be two different approaches. (i) The development of algorithms that guarantee convergence to Nash equilibria of the games constructed in this paper. (ii) The development of alternative mechanisms/game forms that lead to supermodular games. Both of the above problems are open research problems of paramount importance.

The development of mechanisms/game forms for power allocation in dynamic systems in the presence of selfish users is another very important open research problem.

Acknowledgements This work was supported in part by National Science Foundation (NSF) grant CCR-0325571 and National Aeronautics and Space Administration (NASA) grant NNX06AD47G. The authors are grateful to A. Anastasopoulos for stimulating discussions.

In the appendices that follow, we present the proof of Theorems 1 and 2. We divide the proof into several claims to organize the presentation.

Appendix A: Proof of Theorem 1

Claim 1 *If $\underline{\boldsymbol{m}}^*$ is a NE of the game induced by the game form presented in Sect. 3.1 and the users' utility functions (3), then the allocation $(\underline{\hat{\boldsymbol{t}}}(\underline{\boldsymbol{m}}^*), \underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}}^*)) =: (\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*)$ is a feasible solution of Problem (P_{CU}), i.e. $(\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*) \in S$.*

Proof By construction of the game form, the allocated tax (10) satisfies (2) which implies that the NE tax profile $\underline{\hat{\boldsymbol{t}}}^*$ also satisfies (2). Therefore to prove the claim, we need to show that the NE power profile $\underline{\hat{\boldsymbol{p}}}^* \in \bigcap_{i\in\mathcal{N}} S_i$ (where $S_i, i \in \mathcal{N}$, is defined by (1)). We will prove this by showing that, if $\underline{\hat{\boldsymbol{p}}}^* \notin S_i$ for some $i \in \mathcal{N}$, then there exists a profitable unilateral deviation for user i.

Suppose $\underline{\hat{\boldsymbol{p}}}^* \notin S_i$ for some $i \in \mathcal{N}$. Then, from (3), $u_i^A(\hat{t}_i^*, \underline{\hat{\boldsymbol{p}}}^*) = -\infty$. Consider $\underline{\widetilde{\boldsymbol{m}}_i} = (\underline{\boldsymbol{\pi}_i^*}, \underline{\widetilde{\boldsymbol{p}}_i})$ where $\underline{\boldsymbol{\pi}_i^*}$ is the NE price profile and $\underline{\widetilde{\boldsymbol{p}}_i}$ ($\underline{\widetilde{\boldsymbol{p}}_i} \in \mathbb{R}^N$) is such that,

$$\underline{\hat{\boldsymbol{p}}}(\underline{\widetilde{\boldsymbol{m}}_i}, \underline{\boldsymbol{m}^*}/i) = \frac{1}{N}\left(\sum_{\substack{j\in\mathcal{N}\\ j\neq i}} \underline{\boldsymbol{p}_j^*} + \underline{\widetilde{\boldsymbol{p}}_i}\right) = \underline{\boldsymbol{0}} \in S_i.$$

Then,

$$u_i^A\big(\hat{t}_i(\underline{\widetilde{\boldsymbol{m}}_i}, \underline{\boldsymbol{m}^*}/i), \underline{\hat{\boldsymbol{p}}}(\underline{\widetilde{\boldsymbol{m}}_i}, \underline{\boldsymbol{m}^*}/i)\big) = -\hat{t}_i(\underline{\widetilde{\boldsymbol{m}}_i}, \underline{\boldsymbol{m}^*}/i) + u_i(\underline{\boldsymbol{0}})$$
$$> -\infty = u_i^A(\hat{t}_i^*, \underline{\hat{\boldsymbol{p}}}^*). \qquad (12)$$

Thus user i will find it profitable to deviate to $\underline{\widetilde{\boldsymbol{m}}_i}$.

Inequality (12) implies that $\underline{\boldsymbol{m}^*}$ cannot be a NE, which is a contradiction. Therefore we must have that, $\underline{\hat{\boldsymbol{p}}}^* \in \bigcap_{i\in\mathcal{N}} S_i$ and hence, $(\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*) \in S$. □

Claim 2 *If $\underline{\boldsymbol{m}}^*$ is a NE of the game induced by the game form presented in Sect. 3.1 and the users' utility functions (3), then, the tax $\hat{t}_i(\underline{\boldsymbol{m}}^*) =: \hat{t}_i^*$ paid by user i, $i \in \mathcal{N}$, at NE $\underline{\boldsymbol{m}}^*$ is of the form, $\hat{t}_i^* = \underline{\boldsymbol{l}_i^*}^T \underline{\hat{\boldsymbol{p}}}^*$, where $\underline{\boldsymbol{l}_i^*} := \underline{\boldsymbol{l}_i}(\underline{\boldsymbol{m}}^*)$.*

Proof Let $\underline{\boldsymbol{m}}^*$ be a NE described in Claim 2. Then, for each $i \in \mathcal{N}$,

$$u_i^A(\hat{t}_i(\underline{\boldsymbol{m}_i}, \underline{\boldsymbol{m}^*}/i), \underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}_i}, \underline{\boldsymbol{m}^*}/i)) \le u_i^A(\hat{t}_i^*, \underline{\hat{\boldsymbol{p}}}^*),$$
$$\forall \underline{\boldsymbol{m}_i} \in \mathcal{M}_i. \qquad (13)$$

Substituting $\underline{\boldsymbol{m}_i} = (\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i^*})$, $\underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N$, in (13) and using (9) implies that

$$u_i^A(\hat{t}_i((\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i^*}), \underline{\boldsymbol{m}^*}/i), \underline{\hat{\boldsymbol{p}}}^*) \le u_i^A\big(\hat{t}_i^*, \underline{\hat{\boldsymbol{p}}}^*\big), \quad \forall \underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N. \qquad (14)$$

Since u_i^A decreases in t_i (see (3)), (14) implies that

$$\hat{t}_i((\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i^*}), \underline{\boldsymbol{m}^*}/i) \geq \hat{t}_i^*, \quad \forall \underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N. \tag{15}$$

Substituting (10) in (15) implies that

$$\begin{aligned}
&\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*} + (\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_i})(\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*}) \\
&\quad - (\underline{\boldsymbol{p}_{i+1}^*} - \underline{\boldsymbol{p}_{i+2}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_{i+1}^*})(\underline{\boldsymbol{p}_{i+1}^*} - \underline{\boldsymbol{p}_{i+2}^*}) \\
&\geq \underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*} + (\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_i^*})(\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*}) \\
&\quad - (\underline{\boldsymbol{p}_{i+1}^*} - \underline{\boldsymbol{p}_{i+2}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_{i+1}^*})(\underline{\boldsymbol{p}_{i+1}^*} - \underline{\boldsymbol{p}_{i+2}^*}), \\
&\forall \underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N.
\end{aligned} \tag{16}$$

Canceling the common terms in (16) implies

$$\begin{aligned}
&(\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_i} - \underline{\boldsymbol{\pi}_i^*})(\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*}) \geq 0, \\
&\quad \forall \underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N.
\end{aligned} \tag{17}$$

Since (17) must hold for all $\underline{\boldsymbol{\pi}_i} \geq \underline{\boldsymbol{0}}$, it implies that for each $j \in \mathcal{N}$,

$$\text{either } p_{i\,j}^* = p_{i+1\,j}^*, \text{ or } \pi_{i\,j}^* = 0. \tag{18}$$

From (18) it follows that at any NE $\underline{\boldsymbol{m}^*}$,

$$(\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_i^*})(\underline{\boldsymbol{p}_i^*} - \underline{\boldsymbol{p}_{i+1}^*}) = 0, \quad \forall i \in \mathcal{N}. \tag{19}$$

Using (19) in (10) we obtain that any NE tax profile must be of the form

$$\hat{t}_i^* = \underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*}, \quad \forall i \in \mathcal{N}. \tag{20}$$

□

Claim 3 *The game form given in Sect. 3.1 is individually rational, i.e. for every NE $\underline{\boldsymbol{m}^*}$ corresponding to it, the allocation $(\underline{\hat{\boldsymbol{t}}^*}, \underline{\hat{\boldsymbol{p}}^*})$ is weakly preferred by all the users to the initial allocation $(\underline{\boldsymbol{0}}, \underline{\boldsymbol{0}})$, i.e.,*

$$u_i^A(0, \underline{\boldsymbol{0}}) \leq u_i^A(\hat{t}_i^*, \underline{\hat{\boldsymbol{p}}^*}), \quad \forall i \in \mathcal{N}.$$

Proof Suppose $\underline{\boldsymbol{m}^*}$ is a NE of the game specified by the game form presented in Sect. 3.1 and the users' utility functions (3). From Claim 2 we know the form of the tax at $\underline{\boldsymbol{m}^*}$. Substituting that from (20) into (13) we obtain that, for each $i \in \mathcal{N}$,

$$\begin{aligned}
&u_i^A(\hat{t}_i((\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i}), \underline{\boldsymbol{m}^*}/i), \underline{\hat{\boldsymbol{p}}}((\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i}), \underline{\boldsymbol{m}^*}/i)) \\
&\quad \leq u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*}, \underline{\hat{\boldsymbol{p}}^*}), \quad \forall \underline{\boldsymbol{m}_i} = (\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i}) \in \mathcal{M}_i.
\end{aligned} \tag{21}$$

Substituting for $\hat{t}_i$ in (21) from (10) and using equality (19) we obtain

$$\begin{aligned}
&u_i^A\Big(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}}((\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i}), \underline{\boldsymbol{m}^*}/i) \\
&\quad + (\underline{\boldsymbol{p}_i} - \underline{\boldsymbol{p}_{i+1}^*})^T \operatorname{diag}(\underline{\boldsymbol{\pi}_i})(\underline{\boldsymbol{p}_i} - \underline{\boldsymbol{p}_{i+1}^*}), \\
&\quad \underline{\hat{\boldsymbol{p}}}((\underline{\boldsymbol{\pi}_i}, \underline{\boldsymbol{p}_i}), \underline{\boldsymbol{m}^*}/i)\Big) \\
&\leq u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*}, \underline{\hat{\boldsymbol{p}}^*}), \quad \forall \underline{\boldsymbol{\pi}_i} \in \mathbb{R}_+^N, \ \forall \underline{\boldsymbol{p}_i} \in \mathbb{R}^N.
\end{aligned} \tag{22}$$

In particular, $\underline{\boldsymbol{\pi}_i} = \underline{\boldsymbol{0}}$ in (22) implies that

$$\begin{aligned}
&u_i^A\Big(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}}((\underline{\boldsymbol{0}}, \underline{\boldsymbol{p}_i}), \underline{\boldsymbol{m}^*}/i), \ \underline{\hat{\boldsymbol{p}}}((\underline{\boldsymbol{0}}, \underline{\boldsymbol{p}_i}), \underline{\boldsymbol{m}^*}/i)\Big) \\
&\leq u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*}, \underline{\hat{\boldsymbol{p}}^*}), \quad \forall \underline{\boldsymbol{p}_i} \in \mathbb{R}^N.
\end{aligned} \tag{23}$$

Substituting $1/N(\underline{\boldsymbol{p}_i} + \sum_{j \in \mathcal{N} \setminus \{i\}} \underline{\boldsymbol{p}_j^*}) = \underline{\overline{\boldsymbol{p}}}$ in (23) and using the fact that (23) holds for all $\underline{\boldsymbol{p}_i} \in \mathbb{R}^N$ gives

$$u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\overline{\boldsymbol{p}}}, \ \underline{\overline{\boldsymbol{p}}}) \leq u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*}, \underline{\hat{\boldsymbol{p}}^*}), \quad \forall \underline{\overline{\boldsymbol{p}}} \in \mathbb{R}^N. \tag{24}$$

For $\underline{\overline{\boldsymbol{p}}} = \underline{\boldsymbol{0}}$, (24) implies that

$$u_i^A(0, \underline{\boldsymbol{0}}) \leq u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\hat{\boldsymbol{p}}^*}, \underline{\hat{\boldsymbol{p}}^*}), \quad \forall i \in \mathcal{N}. \tag{25}$$

□

Claim 4 *A NE allocation $(\underline{\hat{\boldsymbol{t}}^*}, \underline{\hat{\boldsymbol{p}}^*})$ is an optimal solution of the centralized problem (P_{CU}).*

Proof For each $i \in \mathcal{N}$, (24) can be equivalently written as

$$\begin{aligned}
\underline{\hat{\boldsymbol{p}}^*} &\in \operatorname*{argmax}_{\underline{\overline{\boldsymbol{p}}} \in \mathbb{R}^N} u_i^A(\underline{\boldsymbol{l}_i^{*}}^T \underline{\overline{\boldsymbol{p}}}, \underline{\overline{\boldsymbol{p}}}) \\
&= \operatorname*{argmax}_{\underline{\overline{\boldsymbol{p}}} \in S_i} \big(-\underline{\boldsymbol{l}_i^{*}}^T \underline{\overline{\boldsymbol{p}}} + u_i(\underline{\overline{\boldsymbol{p}}})\big).
\end{aligned} \tag{26}$$

Since for each $i \in \mathcal{N}$, $u_i(\underline{\overline{\boldsymbol{p}}})$ is assumed to be concave in $\underline{\overline{\boldsymbol{p}}}$ over S_i and the set S_i is convex, Karush Kuhn Tucker (KKT) conditions [23, Chap. 11] are necessary and sufficient for $\underline{\hat{\boldsymbol{p}}^*}$ to be a maximizer in (26). Thus, for each $i \in \mathcal{N}$, $\exists \, \underline{\boldsymbol{\lambda}_1^i} \in \mathbb{R}_+^N$ and $\underline{\boldsymbol{\lambda}_2^i} \in \mathbb{R}_+^N$ such that, $\underline{\hat{\boldsymbol{p}}^*}$, $\underline{\boldsymbol{\lambda}_1^i}$ and $\underline{\boldsymbol{\lambda}_2^i}$ satisfy the KKT conditions given below:

$$\underline{\boldsymbol{l}_i^*} - \nabla u_i(\underline{\hat{\boldsymbol{p}}^*}) - \underline{\boldsymbol{\lambda}_1^i} + \underline{\boldsymbol{\lambda}_2^i} = \underline{\boldsymbol{0}}, \tag{27}$$

$$\underline{\boldsymbol{\lambda}_1^{i}}^T \underline{\hat{\boldsymbol{p}}^*} = 0, \tag{28}$$

$$\underline{\boldsymbol{\lambda}_2^{i}}^T (\underline{\hat{\boldsymbol{p}}^*} - P_0^{max} \underline{\boldsymbol{1}}) = 0, \tag{29}$$

where,

$$\underline{\boldsymbol{1}} = \underbrace{(1, 1, \ldots, 1)}_{N \text{ times}} \in \mathbb{R}^{N \times 1}.$$

Combining the KKT conditions of all the users, i.e. summing (27) for all $i \in \mathcal{N}$, and using the fact that $\sum_{i \in \mathcal{N}} \underline{\boldsymbol{l}_i^*} = \underline{\boldsymbol{0}}$ (see (11)), we obtain

$$\sum_{i \in \mathcal{N}} (-\nabla u_i(\underline{\hat{\boldsymbol{p}}^*}) - \underline{\boldsymbol{\lambda}_1^i} + \underline{\boldsymbol{\lambda}_2^i}) = \underline{\boldsymbol{0}}. \tag{30}$$

Equation (30) along with (28) and (29) for all i, and the non-negativity of $\boldsymbol{\lambda}_1^i, \boldsymbol{\lambda}_2^i$, $i \in \mathcal{N}$, specify the KKT conditions (for variable $\underline{\boldsymbol{p}}$) for (5). Since (5) is a concave optimization problem, the KKT conditions are necessary and sufficient for its optimum. Since $\underline{\hat{\boldsymbol{p}}}^*$ satisfies these KKT conditions, it is a maximizer of the objective function in (5). Therefore, as described in Sect. 2.1, an optimal solution of Problem (P_{CU}) is of the form $(\underline{\boldsymbol{t}}, \underline{\hat{\boldsymbol{p}}}^*)$, where $\underline{\boldsymbol{t}} \in \mathbb{R}^N$ is any tax profile that satisfies (2). Since by construction of the tax the NE allocation $\underline{\hat{\boldsymbol{t}}}^*$ satisfies (2), we conclude that $(\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*)$ is an optimal solution of (P_{CU}). □

Theorem 1 shows that if there exists a NE corresponding to the game of Sect. 3.1, then the allocation at the NE is an optimal solution of the centralized problem (P_{CU}). However, Theorem 1 does not guarantee the existence of a NE; in other words, it does not guarantee that a centralized optimum power profile is attainable through NE. This is guaranteed by Theorem 2 which is proved next.

Appendix B: Proof of Theorem 2

We prove Theorem 2 in two steps. In the first step we show that if $\underline{\hat{\boldsymbol{p}}}^*$ is an optimal power profile for the centralized problem (P_{CU}), there exist a set of personalized prices, one for each user $i \in \mathcal{N}$, such that when every user individually maximizes its own utility taking the above prices as given, then each of them obtains $\underline{\hat{\boldsymbol{p}}}^*$ as its optimal power profile. In the second step we show that $\underline{\hat{\boldsymbol{p}}}^*$ and the corresponding set of personalized prices can be used to construct message profiles that are NE of the game induced by the game form of Sect. 3.1 and the users' utility functions (3).

Claim 5 *If $\underline{\hat{\boldsymbol{p}}}^*$ is an optimum power profile corresponding to Problem (P_{CU}), there exist a set of personalized prices $\underline{\boldsymbol{l}}_i^*$, $i \in \mathcal{N}$, such that*

$$\underline{\hat{\boldsymbol{p}}}^* \in \underset{\underline{\boldsymbol{p}} \in S_i}{\operatorname{argmax}} \; -\underline{\boldsymbol{l}}_i^{*T} \underline{\boldsymbol{p}} + u_i(\underline{\boldsymbol{p}}), \quad \forall i \in \mathcal{N}. \tag{31}$$

Proof Suppose $\underline{\hat{\boldsymbol{p}}}^*$ is an optimal power profile corresponding to Problem (P_{CU}). Problem (P_{CU}) does have a solution since it involves maximization of a concave function in $\underline{\boldsymbol{p}}$ over a convex and compact set in $\underline{\boldsymbol{p}}$ (the solution in $\underline{\boldsymbol{t}}$ trivially exists). Writing the optimization problem (P_{CU}) for $\underline{\boldsymbol{p}}$ we have,

$$\underline{\hat{\boldsymbol{p}}}^* \in \underset{\underline{\boldsymbol{p}}}{\operatorname{argmax}} \sum_{i \in \mathcal{N}} u_i(\underline{\boldsymbol{p}})$$

$$\text{s.t.} \quad \underline{\boldsymbol{p}} \in S_i, \quad \forall i \in \mathcal{N}.$$

An optimal solution of the above problem must satisfy the KKT conditions. Therefore there exist $\underline{\boldsymbol{\lambda}}_1^i \in \mathbb{R}_+^N$ and $\underline{\boldsymbol{\lambda}}_2^i \in \mathbb{R}_+^N$, $i \in \mathcal{N}$, such that $\underline{\hat{\boldsymbol{p}}}^*$, $\underline{\boldsymbol{\lambda}}_1^i$ and $\underline{\boldsymbol{\lambda}}_2^i$, $i \in \mathcal{N}$, satisfy

$$\sum_{i \in \mathcal{N}} \left(-\nabla u_i(\underline{\hat{\boldsymbol{p}}}^*) - \underline{\boldsymbol{\lambda}}_1^i + \underline{\boldsymbol{\lambda}}_2^i\right) = \underline{\boldsymbol{0}}, \tag{32}$$

$$\underline{\boldsymbol{\lambda}}_1^{i\,T} \underline{\hat{\boldsymbol{p}}}^* = 0, \quad \forall i \in \mathcal{N}, \tag{33}$$

and

$$\underline{\boldsymbol{\lambda}}_2^{i\,T} (\underline{\hat{\boldsymbol{p}}}^* - P_0^{max} \underline{\boldsymbol{1}}) = 0, \quad \forall i \in \mathcal{N}. \tag{34}$$

We define for each $i \in \mathcal{N}$,

$$\underline{\boldsymbol{l}}_i^* := \nabla u_i(\underline{\hat{\boldsymbol{p}}}^*) + \underline{\boldsymbol{\lambda}}_1^i - \underline{\boldsymbol{\lambda}}_2^i. \tag{35}$$

Then,

$$\underline{\boldsymbol{l}}_i^* - \nabla u_i(\underline{\hat{\boldsymbol{p}}}^*) - \underline{\boldsymbol{\lambda}}_1^i + \underline{\boldsymbol{\lambda}}_2^i = \underline{\boldsymbol{0}}, \quad \forall i \in \mathcal{N}. \tag{36}$$

Equations (36), (33) and (34) together imply that for each $i \in \mathcal{N}$, $\underline{\hat{\boldsymbol{p}}}^*$, $\underline{\boldsymbol{\lambda}}_1^i \in \mathbb{R}_+^N$ and $\underline{\boldsymbol{\lambda}}_2^i \in \mathbb{R}_+^N$ satisfy the KKT conditions for the following maximization problem:

$$\max_{\underline{\boldsymbol{p}} \in S_i} -\underline{\boldsymbol{l}}_i^{*T} \underline{\boldsymbol{p}} + u_i(\underline{\boldsymbol{p}}). \tag{37}$$

Since (37) is a concave optimization problem, KKT conditions are necessary and sufficient for its optimum. Therefore, from (33), (34) and (36) we conclude that

$$\underline{\hat{\boldsymbol{p}}}^* \in \underset{\underline{\boldsymbol{p}} \in S_i}{\operatorname{argmax}} -\underline{\boldsymbol{l}}_i^{*T} \underline{\boldsymbol{p}} + u_i(\underline{\boldsymbol{p}}). \tag{38}$$

□

Claim 6 *Let $\underline{\hat{\boldsymbol{p}}}^*$ be an optimal power profile corresponding to Problem (P_{CU}), let $\underline{\boldsymbol{l}}_i^*$, $i \in \mathcal{N}$, be the personalized prices defined in Claim 5, and let $\hat{t}_i^* := \underline{\boldsymbol{l}}_i^{*T} \underline{\hat{\boldsymbol{p}}}^*, i \in \mathcal{N}$. Let $\underline{\boldsymbol{m}}_i^* := (\underline{\boldsymbol{\pi}}_i^*, \underline{\boldsymbol{p}}_i^*), i \in \mathcal{N}$, be a solution to the following set of relations*:

$$\frac{1}{N} \sum_{i \in \mathcal{N}} \underline{\boldsymbol{p}}_i^* = \underline{\hat{\boldsymbol{p}}}^*, \tag{39}$$

$$\underline{\boldsymbol{\pi}}_{i+1}^* - \underline{\boldsymbol{\pi}}_{i+2}^* = \underline{\boldsymbol{l}}_i^*, \quad i \in \mathcal{N}, \tag{40}$$

$$(\underline{\boldsymbol{p}}_i^* - \underline{\boldsymbol{p}}_{i+1}^*)^T \operatorname{diag}(\underline{\boldsymbol{\pi}}_i^*)(\underline{\boldsymbol{p}}_i^* - \underline{\boldsymbol{p}}_{i+1}^*) = 0, \quad i \in \mathcal{N}, \tag{41}$$

$$\underline{\boldsymbol{\pi}}_i^* \geq \underline{\boldsymbol{0}}, \quad i \in \mathcal{N}. \tag{42}$$

Then, $\underline{\boldsymbol{m}}^* := (\underline{\boldsymbol{m}}_1^*, \underline{\boldsymbol{m}}_2^*, \ldots, \underline{\boldsymbol{m}}_N^*)$ is a NE of the game induced by the game form of Sect. 3.1 and the users' utility functions (3). Furthermore, $\underline{\hat{\boldsymbol{p}}}(\underline{\boldsymbol{m}}^*) = \underline{\hat{\boldsymbol{p}}}^*$, and for each $i \in \mathcal{N}$, $\underline{\boldsymbol{l}}_i(\underline{\boldsymbol{m}}^*) = \underline{\boldsymbol{l}}_i^*$ and $\hat{t}_i(\underline{\boldsymbol{m}}^*) = \hat{t}_i^*$.

Proof Note that, (39)–(42) are necessary conditions for any NE $\underline{\boldsymbol{m}}^*$ corresponding to the game of Sect. 3.1 to result in the allocation $(\underline{\hat{\boldsymbol{t}}}^*, \underline{\hat{\boldsymbol{p}}}^*)$ (this follows from (9), (11) and (19)).

Therefore, the set of solutions of (39)–(42), if one exists, is a superset of the set of all NE that result in $(\underline{\hat{t}}^*, \underline{\hat{p}}^*)$. Below we show that the solution set of (39)–(42) is in fact exactly the set of NE that result in $(\underline{\hat{t}}^*, \underline{\hat{p}}^*)$.

To prove this we first show that the set of relations (39)–(42) do have a solution. Notice that by setting $\underline{p}_i^* = \underline{\hat{p}}^*$ $\forall i \in \mathcal{N}$, (39) and (41) are satisfied. Notice also that the right hand side of (40) sums to $\underline{\mathbf{0}}$ by taking the sum over $i \in \mathcal{N}$. Therefore, (40) has a solution in $\underline{\pi}_i^*, i \in \mathcal{N}$. Furthermore, for any solution $\underline{\pi}_i^*, i \in \mathcal{N}$, of (40), $\underline{\pi}_i^* + \underline{c}, i \in \mathcal{N}$, where $\underline{c}$ is some constant, is also a solution of (40). Therefore by appropriately choosing $\underline{c}$, we can select a solution of (40) such that (42) is satisfied.

It is clear from above that (39)–(42) have multiple solutions. We now show that the set of solutions $\underline{m}^*$ of (39)–(42) is the set of NE that result in the given centralized solution $(\underline{\hat{t}}^*, \underline{\hat{p}}^*)$. From Claim 5, (31) can be equivalently written as

$$\underline{\hat{p}}^* \in \underset{\underline{p} \in \mathbb{R}^N}{\operatorname{argmax}} \; u_i^A(\underline{l}_i^{*T} \underline{p}, \underline{p}), \qquad \forall i \in \mathcal{N}. \tag{43}$$

A change of variable $N\underline{p} - \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \underline{p}_j^* = \underline{p}_i$ in (43) gives

$$\underline{p}_i^* \in \underset{\underline{p}_i \in \mathbb{R}^N}{\operatorname{argmax}} \; u_i^A\Bigg(\underline{l}_i^{*T} \frac{1}{N}\Bigg(\underline{p}_i + \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \underline{p}_j^*\Bigg), \frac{1}{N}\Bigg(\underline{p}_i + \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \underline{p}_j^*\Bigg)\Bigg). \tag{44}$$

Because of (41), (44) also implies the following:

$$\begin{aligned}
&(\underline{\pi}_i^*, \underline{p}_i^*) \\
&\quad \in \underset{(\underline{\pi}_i, \underline{p}_i) \in \mathbb{R}_+^N \times \mathbb{R}^N}{\operatorname{argmax}} \; u_i^A\Big(\underline{l}_i^{*T} \underline{\hat{p}}((\underline{\pi}_i, \underline{p}_i), \underline{m}^*/i) \\
&\qquad - (\underline{p}_{i+1}^* - \underline{p}_{i+2}^*)^T \operatorname{diag}(\underline{\pi}_{i+1}^*)(\underline{p}_{i+1}^* - \underline{p}_{i+2}^*), \\
&\qquad \underline{\hat{p}}((\underline{\pi}_i, \underline{p}_i), \underline{m}^*/i)\Big).
\end{aligned} \tag{45}$$

Furthermore, since u_i^A is strictly decreasing in the tax (see (3)), (45) also implies the following:

$$\begin{aligned}
(\underline{\pi}_i^*, \underline{p}_i^*) \in \underset{(\underline{\pi}_i, \underline{p}_i) \in \mathbb{R}_+^N \times \mathbb{R}^N}{\operatorname{argmax}} \; & u_i^A\Big(\underline{l}_i^{*T} \underline{\hat{p}}((\underline{\pi}_i, \underline{p}_i), \underline{m}^*/i) \\
&+ (\underline{p}_i - \underline{p}_{i+1}^*)^T \operatorname{diag}(\underline{\pi}_i)(\underline{p}_i - \underline{p}_{i+1}^*) \\
&- (\underline{p}_{i+1}^* - \underline{p}_{i+2}^*)^T \operatorname{diag}(\underline{\pi}_{i+1}^*)(\underline{p}_{i+1}^* - \underline{p}_{i+2}^*), \\
&\underline{\hat{p}}((\underline{\pi}_i, \underline{p}_i), \underline{m}^*/i)\Big), \quad i \in \mathcal{N}.
\end{aligned} \tag{46}$$

Equation (46) implies that, if the message exchange and allocation is done according to the game form defined in Sect. 3.1, then user $i, i \in \mathcal{N}$, maximizes its utility at $\underline{m}_i^*$ given that all other users $j \in \mathcal{N}\backslash\{i\}$ use their respective messages $\underline{m}_j^*$, $j \in \mathcal{N}\backslash\{i\}$. This implies that a message profile $\underline{m}^*$ that is a solution to (39)–(42) is a NE corresponding to the aforementioned game. Furthermore, it follows from (39)–(42) that the allocation at $\underline{m}^*$ is

$$\underline{\hat{p}}(\underline{m}^*) = \frac{1}{N} \sum_{i \in \mathcal{N}} \underline{p}_i^* = \underline{\hat{p}}^*, \tag{47}$$

and for each $i \in \mathcal{N}$,

$$\underline{l}_i(\underline{m}^*) = \underline{\pi}_{i+1}^* - \underline{\pi}_{i+2}^* = \underline{l}_i^*, \tag{48}$$

$$\begin{aligned}
\hat{t}_i(\underline{m}^*) &= \underline{l}_i^T(\underline{m}^*)\underline{\hat{p}}(\underline{m}^*) + (\underline{p}_i^* - \underline{p}_{i+1}^*)^T \\
&\quad \times \operatorname{diag}(\underline{\pi}_i^*)(\underline{p}_i^* - \underline{p}_{i+1}^*) \\
&\quad - (\underline{p}_{i+1}^* - \underline{p}_{i+2}^*)^T \operatorname{diag}(\underline{\pi}_{i+1}^*)(\underline{p}_{i+1}^* - \underline{p}_{i+2}^*) \\
&= \underline{l}_i^{*T} \underline{\hat{p}}^* = \hat{t}_i^*.
\end{aligned} \tag{49}$$

It follows from (46)–(49) that the set of solutions $\underline{m}^*$ of (39)–(42) is exactly the set of NE corresponding to the game of Sect. 3.1 that result in the allocation $(\underline{\hat{t}}^*, \underline{\hat{p}}^*)$. This completes the proof of Claim 6 and hence the proof of Theorem 2. □

Appendix C: Utility function for a MMSE-MUD decoder

In this section we present the explicit form of user's utility functions for the case when the BS uses a Mimimum Mean Square Error Multi User Detector (MMSE-MUD) decoder for each user.

Suppose there are N users (transmitter-receiver pairs) in a network, and the BS uses MMSE-MUD receivers to decode the received data for each user. The MMSE at the output of receiver for user $i (i \in \mathcal{N})$ is then given by (see [24, Chap. 6])

$$MMSE_i = \min_{\underline{c}_i^T \in \mathbb{R}^{1\times M}} E[\|b_i - \underline{c}_i^T \underline{y}_i\|^2] = \left[\left(\boldsymbol{I} + \frac{2}{N_0}\boldsymbol{ARA}\right)^{-1}\right]_{ii}. \tag{50}$$

In (50) b_i is the data symbol transmitted by user i, $\underline{y}_i$ is the output of matched filter corresponding to user i, $\boldsymbol{I}$ is the identity matrix of size $M \times M$, $N_0/2$ is the two sided power spectral density of the thermal noise, $\boldsymbol{A} := \operatorname{diag}(A_1, A_2, \ldots, A_N)$ is the diagonal matrix consisting of the received signal amplitudes of users 1 through N, and $\boldsymbol{R}$ is the cross-correlation matrix of the users' signature waveforms. For simplicity of analysis and for analytical tractability we consider the case of two users below.

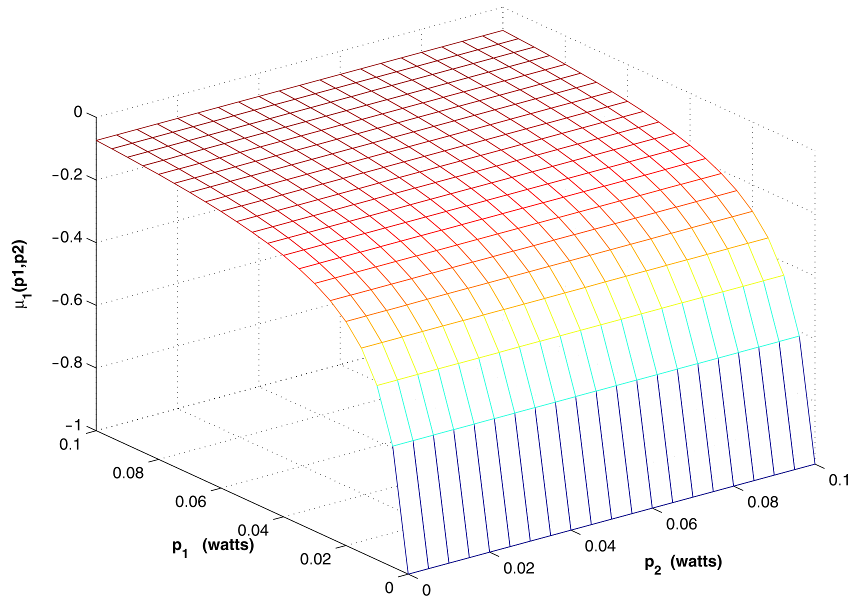

Fig. 2 $u_1(p_1, p_2)$ vs. (p_1, p_2) for $N_0/2 = 10^{-1.2}$ and $\rho = 0.01$

For the two-user ($N = 2$) case, the expression for the MMSE in (50) becomes

$$MMSE_i(\underline{\boldsymbol{p}}) = \frac{\frac{N_0}{2}}{\frac{N_0}{2} + p_i(1 - \frac{\rho^2(p_j)}{(N_0/2+p_j)})}, \quad i, j \in \{1, 2\}, \quad j \neq i. \tag{51}$$

In (51) p_i is the power received at the BS from user $i \in \{1, 2\}$, $p_i = A_i^2$ and $p_j = A_j^2$, $i, j \in \{1, 2\}$, $j \neq i$; and ρ is the cross correlation between the signature waveforms of users 1 and 2.

We define the users' utility functions to be

$$u_i(\underline{\boldsymbol{p}}) = -MMSE_i(\underline{\boldsymbol{p}}), \quad i \in \{1, 2\}. \tag{52}$$

Below we investigate the properties of function u_i defined in (52). From (51) and (52) we see that for a given power spectral density $N_0/2$ of the thermal noise, the cross correlation ρ, and the received power p_2 from user 2, the function u_1 is of the form $\frac{-1}{c_1+c_2 p_1}$ for some constants c_1 and c_2. Thus u_1 is concave in p_1. On the other hand, for a given p_1, if ρ is very small which is usually the case in practical wireless systems, the coefficient ρ^2 in the denominator of (51) makes the variation of u_1 with p_2 very small. Thus p_1 dominantly determines the curvature of function u_1. To illustrate this we plot $u_1(\underline{\boldsymbol{p}})$ vs. (p_1, p_2) in Fig. 2. It can clearly be seen from Fig. 2 that u_1 is a concave function of p_1 and varies very little with p_2. Therefore, u_1 is close to concave in $\underline{\boldsymbol{p}} = (p_1, p_2)$. To check the utility of user 2, we use similar arguments as above by interchanging the indices 1 and 2 and we get that u_2 is also close to concave in $\underline{\boldsymbol{p}}$.

For larger networks with $N > 2$, it is difficult to give a general expression for u_i similar to (51). However, when the cross correlation among the users' waveforms is small, the curvature of function u_i is dominantly determined by p_i. Similar to the case for $N = 2$, the function u_i is concave in p_i, and varies very little with other components of $\underline{\boldsymbol{p}} := (p_1, p_2, \ldots, p_N)$, thus, making it close to concave in $\underline{\boldsymbol{p}}$.

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Shrutivandana Sharma is a Senior Associate Scientist at Yahoo! Labs, Bangalore. She received her B.Tech. in Electrical Engineering from the Indian Institute of Technology, Kanpur, in 2004, and her M.S. and Ph.D., both in Electrical Engineering: Systems, from the University of Michigan, Ann Arbor, in 2007 and 2009 respectively. Her research interests are in decentralized decision and resource allocation problems arising in networks and multi-agent systems and in the development of the theory of mechanism design and games to address these problems.

Demosthenis Teneketzis is a professor of Electrical Engineering and Computer Science at the University of Michigan, Ann Arbor. Prior to joining the University of Michigan he worked for Systems Control Inc., Palo Alto, California, and Alphatech Inc., Burlington Massachusetts. In Winter and Spring 1992, he was a visiting professor at the Swiss Federal Institute of Technology, Zurich, Switzerland. He received his diploma in electrical engineering from the University of Patras, Greece, and his M.S., E.E., and Ph.D. degrees, all in electrical engineering from the Massachusetts Institute of Technology, Cambridge. His research interests are in stochastic control, decentralized systems, queuing and communication networks, stochastic scheduling and resource allocation problems, mathematical economics, and discrete event systems.