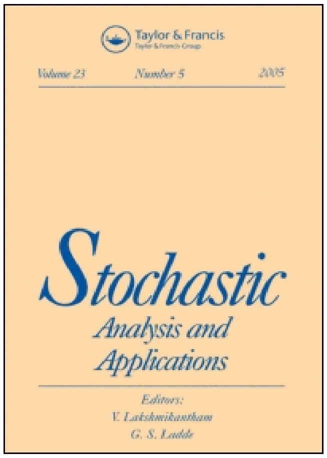

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Solvable systems are usually measurable

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SOLVABLE SYSTEMS ARE USUALLY MEASURABLE

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ABSTRACT

$(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are measurable spaces and $\mathcal{J}^k$, $k = 1, 2, \ldots, N$, are subfields of the product field $\mathcal{B} \otimes (\bigotimes_{i=1}^N \mathcal{U}^i)$. Consider an N-tuple of functions $\gamma := (\gamma^1, \gamma^2, \ldots, \gamma^N)$ for which γ^k, $k = 1, 2, \ldots, N$, is $\mathcal{J}^k/\mathcal{U}^k$-measurable. If for each $\omega \in \Omega$ there exists a unique $u := (u^1, u^2, \ldots, u^N) \in \prod_{i=1}^N U^i$ satisfying the equations $u^k = \gamma^k(\omega, u)$, $k = 1, 2, \ldots, N$, γ induces a unique map Σ^γ from Ω to $\prod_{i=1}^N U^i$.

Is this map necessarily $\mathcal{B}/\bigotimes_{i=1}^N \mathcal{U}^i$-measurable? A generic non-sequential stochastic control problem in which a related question arises is discussed, and the conditions on $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, for which the original question's answer is affirmative are investigated. Specifically, it is shown that Σ^γ is necessarily $\mathcal{B}/\bigotimes_{i=1}^N \mathcal{U}^i$-measurable when either $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are discrete, or $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are Souslin.

1. INTRODUCTION

Let $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, be measurable spaces and let $\mathcal{J}^k$, $k = 1, 2, \ldots, N$, be subfields of the product field $\mathcal{B} \otimes \mathcal{U}$ on $\Omega \times U$, where $\mathcal{U} := \otimes_{i=1}^{N} \mathcal{U}^i$, and $U := \prod_{i=1}^{N} U^i$. Consider an N-tuple of functions $\gamma := (\gamma^1, \gamma^2, \ldots, \gamma^N)$, for which the kth function γ^k, $k = 1, 2, \ldots, N$, is $\mathcal{J}^k/\mathcal{U}^k$-measurable, i.e.,

$$\gamma^k : (\Omega \times U,\ \mathcal{J}^k) \to (U^k, \mathcal{U}^k), \quad \text{for } k = 1, 2, \ldots, N \ . \tag{1}$$

If for each $\omega \in \Omega$ there exists a unique $u := (u^1, u^2, \ldots, u^N) \in U$ satisfying the system of equations

$$u^k = \gamma^k(\omega, u), \quad k = 1, 2, \ldots, N \ , \tag{2}$$

γ induces a unique *solution map*

$$\Sigma^\gamma : \Omega \to U \ , \tag{3}$$

via its solutions $\{\, u_\omega^\gamma \in U : \ u_\omega^\gamma = \gamma(\omega, u_\omega^\gamma) \,\}$, i.e., $\Sigma^\gamma(\omega) = u_\omega^\gamma$ for all $\omega \in \Omega$.

An N-tuple satisfying (1) is said to possess *property S** (solvability*) when it induces a unique solution map Σ^γ, and *property SM** (solvability/measurability*) when this induced map is $\mathcal{B}/\mathcal{U}$-measurable. In this paper we consider the following question:

Q: Under what conditions on $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, does property S* imply property SM*?

This question is closely related to a question, concerning the existence of expected payoffs, that arises when formulating non-causal, non-sequential stochastic control problems. Loosely speaking, most stochastic control problems involving N control actions can be modeled as problems in which: $(\Omega, \mathcal{B}, \mathrm{P})$ is the underlying probability space, $(U^k,\mathcal{U}^k)$ is the kth action space, γ^k is the kth control law, and $\mathcal{J}^k$ is the information field induced on $\Omega \times U$ by the kth observation function. Within this framework (Witsenhausen's intrinsic model for discrete stochastic control [14,16]), γ denotes a control policy; γ's possession of property S* ensures that, for every random input, γ induces a unique N-tuple of control actions; and γ's possession of property SM* ensures that the expected payoff of these control actions can be defined.

In [1] it was shown that all *causal policies* (those whose control actions can be ordered, for each ω, such that each control action only depends on ω and the control actions that precede it), and in particular, all *sequential policies* (those causal policies whose control actions can be ordered a priori) possess property SM*. Consequently, all *causal problems* (those in which all admissible policies are causal), and in particular, all *sequential problems* (those in which all admissible policies are sequential and share the same sequential order[1]) are well-posed in the sense that an expected payoff can be defined for every admissible policy.

Our investigation of question Q is motivated by a desire to formulate (abstractly) non-sequential stochastic control problems in which policies that occasionally deadlock (non-causal policies) are admissible by necessity—i.e., problems in which it is impossible, or too costly, to ensure that all admissible policies are deadlock-free. Such problems can arise, for instance, when scheduling transactions in distributed data bases [6], and when routing, resequencing and acknowledging packets in computer and communication networks [12]. In both cases the detection and restarting (by roll-back or retransmission, for instance) of deadlocked processes (transactions or transmissions) is often preferable to the performance degradation and increased complexity that may result when all policies in the admissible set are constrained to be deadlock-free.

In Section 2 we introduce Witsenhausen's intrinsic model and briefly elaborate on the relationship between question Q and non-sequential stochastic control. In Section 3 we answer question Q. Specifically, we show that, for all practical purposes, property S* implies property SM*. In the process we also confirm Witsenhausen's conjecture ([14], §8) that, "in most special cases of interest," *property S* (which holds when all N-tuples satisfying (1) possess property S*) implies *property SM* (which holds when all N-tuples satisfying (1) possess property SM*). Section 4 contains our conclusion.

2. NON-SEQUENTIALITY AND QUESTION Q

Question Q is motivated by an existence question peculiar to non-sequential stochastic control. Since the "conventional" discrete time, finite horizon models of stochastic control theory presuppose a fixed ordering of a system's control actions, to describe this existence question it is necessary to introduce a more general modeling framework. This framework, Witsenhausen's intrinsic model for discrete stochastic control [14,16], has three components.

1. An *information structure* $\mathcal{I} := \{(\Omega, \mathcal{B}), (U^k, \mathcal{U}^k), \mathcal{J}^k \ : \ 1 \leq k \leq N\}$ specifies the system's admissible controls and distinguishable events.

   (a) $N \in I\!N$ denotes the number of control actions to be taken.

   (b) $(\Omega, \mathcal{B})$ denotes the measurable space from which a random input ω is drawn.

   (c) $(U^k, \mathcal{U}^k)$ denotes the measurable space from which u^k, the kth control action, is selected. It is assumed that the cardinality of U^k is greater than one, and that $\mathcal{U}^k$ contains the singletons of U^k. The product space containing the N-tuple of control actions, $u := (u^1, u^2, \ldots, u^N)$, is denoted by $(U, \mathcal{U}) := (\Pi_{i=1}^N U^i, \otimes_{i=1}^N \mathcal{U}^i)$.

(d) $\mathcal{J}^k$ denotes the information subfield of the product σ-field $\mathcal{B} \otimes \mathcal{U}$ characterizing the maximal information that can be used to select the kth control action.

2. A design constraint set Γ_C constrains the set of admissible N-tuples of control laws, $\gamma := (\gamma^1, \gamma^2, \ldots, \gamma^N)$, called *designs*, to a non-empty subset of $\Gamma := \prod_{i=1}^N \Gamma^i$, where Γ^k, $k = 1, 2, \ldots, N$, denotes the set of all $\mathcal{J}^k/\mathcal{U}^k$-measurable functions.

3. A probability measure P on $(\Omega, \mathcal{B})$ determines the statistics of the system's random input ω.

With respect to the conventional models, this representation entails no loss of generality. A system's random inputs—its initial state, state and observation noises, and so on—can always be viewed as a single random input $\omega \in \Omega$. Moreover, for all k, the system's kth control law—normally assumed to be a measurable function of its $\mathcal{B}/\mathcal{U}$-measurable kth observation—can always be viewed as a $\mathcal{J}^k$-measurable function of the *intrinsic variables* ω and u, where $\mathcal{J}^k \subset \mathcal{B} \otimes \mathcal{U}$ denotes the information field induced on the space of intrinsic variables by the kth observation.

The advantage of this intrinsic representation, as opposed to that of the conventional models, is that it permits interdependence among a problem's control actions (e.g., given a fixed γ, u^j may depend on u^k for some ω, and vice versa for other ω). Consequently, it is possible to model *non-sequential* problems—problems in which a causal ordering for the control actions can not be determined a priori because: 1) it varies from design to design, 2) it may be ω-dependent under some designs, and 3) impossible under others (i.e., a deadlock occurs [1]). Game theorists ([11], Fig. 1) and computer scientists [7] have long been aware that such non-sequential problems exist. More recently, important non-sequential problems have been identified in distributed data and communication networks (for specific examples, see [3,6,12]). Although several control theoretic models for such problems have been proposed (see, for instance, [2,10,13,16]) only in Witsenhausen's framework is the modeling of uncertainty and information compatible with that of the usual state–observation models of control theory.

Within this compatible framework one can pose the following generic stochastic control problem.

P: Given an information structure $\mathcal{I}$, a design constraint set Γ_C, a probability measure P, and a real, upper bounded, $\mathcal{B}/\mathcal{U}$-measurable payoff function V,

$$\text{Identify a design } \gamma \text{ in } \Gamma_C \text{ that achieves}$$
$$\sup_{\gamma \in \Gamma_C} E_\omega[V(w, u^\gamma)] \text{ exactly, or within } \epsilon > 0. \blacksquare$$

Is this generic problem well-defined? It is this question that gives rise to question Q. Since the problem may be non-sequential there are two issues: "mathematical well-posedness" (Does every design $\gamma \in \Gamma_C$ possess an expected payoff?) and "real-world causality" (Is every design $\gamma \in \Gamma_C$ deadlock-free?). To ensure well-posedness, it suffices to require that each $\gamma \in \Gamma_C$ possess property SM*. Then, for each $\gamma \in \Gamma_C$, $V(\cdot, \Sigma^\gamma(\cdot))$ is $\mathcal{B}$-measurable, and consequently, $E_\omega[V(w, \Sigma^\gamma(w))]$ exists. To ensure causality, it suffices to require that for each $\gamma \in \Gamma_C$, and for all $\omega \in \Omega$, there exist an ordering of γ's N control laws, say $\gamma^{s_1(\omega)}, \gamma^{s_2(\omega)}, \ldots, \gamma^{s_N(\omega)}$, such that each control action only depends on ω and the control actions that precede it. Then for each $\gamma \in \Gamma_C$ and all $\omega \in \Omega$, γ is deadlock-free in the sense that, given ω, $u^{s_1(\omega)}$ can be determined; given ω and $u^{s_1(\omega)}$, $u^{s_2(\omega)}$ can be determined; given ω, $u^{s_1(\omega)}$ and $u^{s_2(\omega)}$, $u^{s_3(\omega)}$ can be determined; and so on [1].

Clearly, question Q concerns the generic problem's well-posedness. Any conditions on $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, sufficient to ensure that property S* implies property SM*, are equally sufficient to ensure that the generic problem is well-posed when all $\gamma \in \Gamma_C$ possess property S*. Well-posedness, however, is also ensured by causality (because all deadlock-free designs possess property SM*—[1], Thm. 4.3); consequently, with respect to the generic problem, question Q's answers are only interesting when at least one of the designs in Γ_C is not deadlock-free.

In practice, this is not an unreasonable assumption. To reduce a system's complexity, and or improve its aggregate performance, it may be desirable to enlarge Γ_C to include designs that occasionally deadlock. Indeed, in highly distributed data bases [6], and in computer and communication networks [12], it appears difficult to achieve any level of concurrency without permitting occasional deadlocks. Our answers to question Q, as described in Section 3, provide conditions on $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, sufficient to ensure that such non-causal, non-sequential stochastic control problems are well-posed when all designs $\gamma \in \Gamma_C$ possess property S*.

3. PROPERTY S* USUALLY IMPLIES PROPERTY SM*

In this section it is shown that property S* usually implies property SM*. Henceforth, a measurable space $(X, \mathcal{X})$ will be termed *discrete* (cf. [4], Ex. 2.8) when X is countable and $\mathcal{X}$ contains the singletons of X. A measurable space will be termed *Souslin* ([8], Def. III.16b) when it is σ-isomorphic[2] to a measurable space $(Y, B(Y))$,[3] where Y is a Souslin metrizable space.[4]

Theorem: Property S* implies property SM* when either of the following conditions are satisfied:

(i) $(U^k,\mathcal{U}^k)$, $k=1,2,\ldots,N$, are discrete measurable spaces.

(ii) $(\Omega,\mathcal{B})$ and $(U^k,\mathcal{U}^k)$, $k=1,2,\ldots,N$, are Souslin measurable spaces.

Proof: Fix $\gamma\in\Gamma$. By assumption (property S*), γ induces a unique mapping $\Sigma^\gamma:\Omega\to U$. Let π_Ω denote the canonical projection of $\Omega\times U$ onto Ω (i.e., $\pi_\Omega(\omega,u)=\omega$) and let

$$\mathrm{G}^\gamma := \{\,(\omega,u):\ \Sigma^\gamma(\omega)=u\,\} \tag{4}$$

denote the graph of Σ^γ. Since

$$[\Sigma^\gamma]^{-1}(A)=\pi_\Omega((\Omega\times A)\bigcap \mathrm{G}^\gamma) \tag{5}$$

for all $A\subset U$, to prove that Σ^γ is $\mathcal{B}/\mathcal{U}$-measurable (property SM*) it suffices to show that $\pi_\Omega((\Omega\times A)\bigcap\mathrm{G}^\gamma)$, the projection on Ω of the restriction of G^γ to $\Omega\times A$, is $\mathcal{B}$-measurable for all $A\in\mathcal{U}$.

To this end, the following lemma is helpful.

Lemma 1: Assuming that property S* holds, when $(U,\mathcal{U})$ is discrete, G^γ is $\mathcal{B}\otimes\mathcal{U}$-measurable.

Proof: Fix $\gamma\in\Gamma$, let π_U denote the canonical projection of $\Omega\times U$ onto U (i.e., $\pi_U(\omega,u)=u$), and note that by property S*

$$\mathrm{G}^\gamma=\{\,(\omega,u)\in\Omega\times U:\ \gamma(\omega,u)=\pi_U(\omega,u)\,\}\ . \tag{6}$$

By definition, π_U is $\mathcal{B}\otimes\mathcal{U}/\mathcal{U}$-measurable. Likewise, γ^k is $\mathcal{J}^k/\mathcal{U}^k$-measurable for all $k=1,2,\ldots,N$; accordingly, $\gamma:=(\gamma^1,\gamma^2,\ldots,\gamma^N)$ is $\mathcal{B}\otimes\mathcal{U}/\mathcal{U}$-measurable (since $\mathcal{J}^k\subset\mathcal{B}\otimes\mathcal{U}$ for all k). The result follows from Theorem I.12 of [8] which says that $\{\,x\in X:\ f(x)=g(x)\,\}$ is $\mathcal{X}$-measurable when: $(X,\mathcal{X})$ and $(Y,\mathcal{Y})$ are measurable spaces, $(Y,\mathcal{Y})$ is discrete, and f and g are $\mathcal{X}/\mathcal{Y}$-measurable functions (cf. [9], Prop. I.3A.4, and Thm. I.3B.5). ∎

Proof of (i): Fix $\gamma \in \Gamma$. Since $(U^k, \mathcal{U}^k)$ is discrete for all $k = 1, 2, \ldots, N$, for all k, U^k is countable and $\mathcal{U}^k$ contains the singletons of U^k. Since N is finite, $U = \Pi_{i=1}^{N} U^i$ is also countable, and $\mathcal{U} = \Pi_{i=1}^{N} \mathcal{U}^i$ contains the singletons of U. It follows that $\mathcal{U}$ is generated by the singletons of U, a countable set (i.e., $\mathcal{U} = \sigma(\{u\}$:G$^\gamma$ is $\mathcal{B} \otimes \mathcal{U}$-measurable. Since all sections of measurable sets are measurable ([4], Thm. 18.1),

$$\pi_\Omega((\Omega \times \{u\}) \bigcap \mathrm{G}^\gamma) = \{\omega \in \Omega : (\omega, u) \in \mathrm{G}^\gamma\} \tag{7}$$

is $\mathcal{B}$-measurable for all $u \in U$. But the singletons of U generate $\mathcal{U}$; accordingly

$$[\Sigma^\gamma]^{-1}(A) = \pi_\Omega((\Omega \times A) \bigcap \mathrm{G}^\gamma) \tag{8}$$

is $\mathcal{B}$-measurable for all $A \in \mathcal{U}$. This proves (i). ■

The proof of (ii) requires the following lemma.

Lemma 2: If $(X^k, \mathcal{X}^k)$, $k = 1, 2, \ldots, M$, are Souslin measurable spaces, then $(\Pi_{i=1}^{M} X^i, \otimes_{i=1}^{M} \mathcal{X}^i)$ is a Souslin measurable space.

Proof: By definition, every Souslin measurable (SM) space $(X^k, \mathcal{X}^k)$ is σ-isomorphic to a measurable space $(Y^k, B(Y^k))$, where Y^k is a Souslin metrizable (Sm) space; consequently, $(\Pi_{i=1}^{M} X^i, \otimes_{i=1}^{M} \mathcal{X}^i)$ is σ-isomorphic to $(\Pi_{i=1}^{M} Y^i, \otimes_{i=1}^{M} B(Y^i))$. The result follows since $\otimes_{i=1}^{M} B(Y^i) = B(\Pi_{i=1}^{M} Y^i)$ ([8], Prop. I.6.4 and Thm. III.1.2(1)), and $\Pi_{i=1}^{M} Y^i$ is an Sm space ([5], Prop. IX.6.7), when Y^k, $k = 1, 2, \ldots,$ M, are Sm spaces. ■

Proof of (ii): Fix $\gamma \in \Gamma$. By Lemma 2, $(U, \mathcal{U})$ is a Souslin measurable (SM) space; accordingly, $(U, \mathcal{U})$ is σ-isomorphic to a Souslin metrizable (Sm) space. Since the topology of every Sm space has a countable base ([5], Prop. IX.6.4), and since every singleton of a metrizable space is closed ([5], Props. IX.4.2, and I.8.4), by σ-isomorphism the σ-field $\mathcal{U}$ must be countably generated and must contain its singletons; hence, $(U, \mathcal{U})$ is discrete. It follows, from Lemma 1, that G^γ is, once again, $\mathcal{B} \otimes \mathcal{U}$-measurable.

By Lemma 2, $(\Omega \times U, \mathcal{B} \otimes \mathcal{U})$ is also a SM space; consequently, every set in $\mathcal{B} \otimes \mathcal{U}$, including G^γ, is a *Souslin* set ([8], Def. III.16c, Thm. III.8, and Thm. III.19(2)), and $(G^\gamma, (\mathcal{B} \otimes \mathcal{U}) \bigcap G^\gamma)$[5] is a SM space. Since the restriction of the canonical projection π_Ω to G^γ, $\pi_\Omega|_{G^\gamma}$, is an injective $(\mathcal{B} \otimes \mathcal{U}) \bigcap G^\gamma / \mathcal{B}$-measurable mapping, the result follows from the Souslin-Lusin Theorem ([8], III.21a) which says that an injective $\mathcal{X}/\mathcal{Y}$-measurable mapping $f : X \to Y$ is a σ-isomorphism of $(X, \mathcal{X})$ onto $(f(X), \mathcal{Y} \bigcap f(X))$ when $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ are SM spaces. That is, by the Souslin-Lusin Theorem $[\pi_\Omega|_{G^\gamma}]^{-1}$ is a $\mathcal{B}/(\mathcal{B} \otimes \mathcal{U}) \bigcap G^\gamma$-measurable function; consequently,

$$[\Sigma^\gamma]^{-1}(A) = \pi_\Omega((\Omega \times A) \bigcap G^\gamma) \tag{9}$$

is $\mathcal{B}$-measurable for all $A \in \mathcal{U}$.[6] This proves (ii). ∎

Corollary: Property S implies property SM (see §1) when either of the following conditions are satisfied:

(i) $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are discrete measurable spaces.

(ii) $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are Souslin measurable spaces.

4. CONCLUSION

Motivated by a question concerning the well-posedness of a class of non-causal, non-sequential stochastic control problems, the conditions under which the solvability of a design (property S*) implies the measurability of its induced solution map (property SM*) have been investigated. Specifically, it has been shown that property S* implies property SM* when the measurable spaces

$(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are discrete, and when the measurable spaces $(\Omega, \mathcal{B})$ and $(U^k, \mathcal{U}^k)$, $k = 1, 2, \ldots, N$, are Souslin. Since most measurable spaces are Souslin—e.g., countable spaces; spaces of the form $(A, B(A))$ where A an analytic subset of $\mathbb{R}^n$; standard Borel spaces; and Blackwell spaces in which all singletons are measurable—solvable systems are usually measurable.

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FOOTNOTES

[1] This definition ([1], §4.4) is a refinement of that in [15].

[2] Two measurable spaces are said to be *σ-isomorphic* when there exists a bijection between them that is measurable and has a measurable inverse.

[3] $B(Y)$ denotes the Borel σ-field of the topological space Y.

[4] A metrizable space Y, is *Souslin* ([5], Chap. IX, Def. 2—see also [8], Thm. AIII.78 and Def. III.16a) when there exists a continuous mapping from a complete separable metric space (a Polish space) onto Y.

[5] $(\mathcal{B} \otimes \mathcal{U}) \cap G^\gamma := \{A \cap G^\gamma : A \in \mathcal{B} \otimes \mathcal{U}\}$ denotes the trace of $\mathcal{B} \otimes \mathcal{U}$ on G^γ.

[6] The same result can be proved under slightly weaker conditions (i.e., when $(\Omega, \mathcal{B})$ is a *semi-compact* measurable space) using Theorem II.4.1 of [9].

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