

Correction to “An Efficient Game Form for Multi-Rate Multicast Service Provisioning”

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THIS note corrects an error in our paper, “An Efficient Game Form for Multi-Rate Multicast Service Provisioning” (reference [1]), which presents a rate allocation mechanism for multi-rate multicast service provisioning in networks with arbitrary topology and strategic users. The mechanism presented in [1] includes a tax function which is not differentiable with respect to the rate allocations. To obtain a Nash implementation, however, we need a tax function that is differentiable with respect to these allocations. We correct this error as follows.

We consider the problem formulated in [1]. We use the same notation as in [1]. To help the reader, we repeat here the notation used in this note.

G_i := Multicast group i .

\mathcal{N} := $\{G_1, G_2, \dots, G_N\}$.

(j, G_i) := User j in multicast group i .

$\mathcal{R}_{(j, G_i)}$:= Route of user (j, G_i) .

$\Xi_{(j, G_i)}^l$:= The term that appears in the tax function $t_{(j, G_i)}^l$ and is expressed in terms of indicator functions.

$G_i^{\max}(l)$:= Set of users in G_i that request the maximum amount of bandwidth at link l among all users in G_i .

Q_l := Set of multicast groups that use link l .

$x_{G_i}(l)$:= Bandwidth requested by G_i in link l .

Specification of the game form/mechanism:

Message space: The message space is the same as that of the mechanism presented in [1]. A message of user (j, G_i) , $j \in G_i$, $G_i \in \mathcal{N}$ is of the form

$$m_{(j, G_i)} = (x_{(j, G_i)}, \pi_{(j, G_i)}^{l_{j1}}, \pi_{(j, G_i)}^{l_{j2}}, \dots, \pi_{(j, G_i)}^{l_{|\mathcal{R}_{(j, G_i)}|}}),$$

where $x_{(j, G_i)}$ denotes the (non-negative) bandwidth user (j, G_i) requests at all the links of his route $\mathcal{R}_{(j, G_i)}$, and $\pi_{(j, G_i)}^{l_k} \geq 0$ denotes the price user (j, G_i) is willing to pay at link l_k of his route $\mathcal{R}_{(j, G_i)}$.

Outcome function: For any $m \in \mathcal{M}$, the outcome function is

defined as follows:

$$f(m) = ((x_{(i, G_1)}, t_{(i, G_1)}), \dots, (x_{(k, G_N)}, t_{(k, G_N)}))$$

$$t_{(j, G_i)} = \sum_{l \in \mathcal{R}_{(j, G_i)}} t_{(j, G_i)}^l,$$

where $t_{(j, G_i)}^l$ is tax paid by user (j, G_i) for using link l . The form of $t_{(j, G_i)}^l$ is the same as the tax function defined in [1] excluding the terms denoted by $\Xi_{(j, G_i)}^l$. For example, in **Part DI** where $|G_i^{\max}(l)| \geq 2$, the tax function in **Eq. (14) of [1]** is modified as follows:

Let the label of (j, G_i) in $G_i^{\max}(l)$ be $(k, G_i^{\max}(l))$. Then:

(i). If $(j, G_i) \in G_i^{\max}(l)$,

$$t_{(j, G_i)}^l = \pi_{(k+1, G_i^{\max}(l))} x_{(j, G_i)} + \frac{(P_{G_i^{\max}(l)} - P_{-G_i^{\max}(l)} - \eta_+^l)^2}{|G_i^{\max}(l)|} - 2 \frac{P_{-G_i^{\max}(l)}}{|G_i^{\max}(l)|} (P_{G_i^{\max}(l)} - P_{-G_i^{\max}(l)}) \left[\frac{\mathcal{E}_{-G_i^{\max}(l)} + x_{(j, G_i)}}{\gamma} \right] + \frac{\Gamma_{G_i}^l}{|G_i^{\max}(l)|}, \quad (1)$$

where

$$\eta_+^l = \max\{0, \frac{\sum_{G_i \in Q_l} x_{G_i}(l) - c_l}{\hat{\gamma}}\},$$

c_l is the capacity of link l , $\mathcal{E}_{-G_i^{\max}(l)}$, $P_{G_i^{\max}(l)}$, $P_{-G_i^{\max}(l)}$ and $\Gamma_{G_i}^l$ are defined by equations (18)-(21) in [1], and $\gamma, \hat{\gamma}$ are positive constants.

(ii). If $(j, G_i) \notin G_i^{\max}(l)$ then

$$t_{(j, G_i)}^l = \pi_{(k+1, G_i^{\max}(l))} (\mathcal{E}_{-G_i^{\max}(l)} + x_{G_i}(l)). \quad (2)$$

This completes the specification of the mechanism.

Based on the above specification, the proof of Lemma 3 in [1] is updated as follows.

Proof of Lemma 3 in [1]: Let m^* be a NE of the game induced by the mechanism. Consider $G_i \in Q_l$, and $(k, G_i^{\max}(l)) \in G_i^{\max}(l)$. Since user $(k, G_i^{\max}(l))$ does not control $\Gamma_{G_i}^l$ (as in [1], page 2101), $\frac{\partial \Gamma_{G_i}^l}{\partial \pi_{(k, G_i^{\max}(l))}} = 0$. By following the same steps as in equations (32-40) of [1], we

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obtain:

$$\begin{aligned} & \left. \frac{\partial t_{(k, G_i^{\max}(l))}^l}{\partial \pi_{(k, G_i^{\max}(l))}} \right|_{m=m^*} \\ &= \frac{-2}{|G_i^{\max}(l)|} P_{-G_i^{\max}(l)}^* \left(\frac{\mathcal{E}_{-G_i^{\max}(l)}^* + x_{G_i}^*(l)}{\gamma} \right) \\ &+ \frac{2}{|G_i^{\max}(l)|} \left(P_{G_i^{\max}(l)}^* - P_{-G_i^{\max}(l)}^* - \eta_+^{*l} \right) \\ &= 0. \end{aligned} \quad (3)$$

Furthermore (as in Eq. (36) in [1]) we note that $\sum_{G_i \in Q_l} P_{G_i^{\max}(l)} = \sum_{G_i \in Q_l} P_{-G_i^{\max}(l)}$. Thus, summing (3) over all $G_i \in Q_l$, and $(k, G_i^{\max}(l)) \in G_i^{\max}(l)$, we get

$$\begin{aligned} & \sum_{G_i \in Q_l} \sum_{(k, G_i^{\max}(l)) \in G_i^{\max}(l)} \frac{\partial t_{(k, G_i^{\max}(l))}^l}{\partial \pi_{(k, G_i^{\max}(l))}} \\ &= -|Q_l| \eta_+^{*l} - \sum_{G_i \in Q_l} P_{-G_i^{\max}(l)}^* \left(\frac{\mathcal{E}_{-G_i^{\max}(l)}^* + x_{G_i}^*(l)}{\gamma} \right) \\ &= 0. \end{aligned} \quad (4)$$

Suppose $\sum_{G_i \in Q_l} x_{G_i}^*(l) - c_l > 0$. Then we must have $\eta_+^{*l} > 0$ and $\sum_{G_i \in Q_l} P_{-G_i^{\max}(l)}^* \left(\frac{\mathcal{E}_{-G_i^{\max}(l)}^* + x_{G_i}^*(l)}{\gamma} \right) \geq 0$. But this contradicts (4). Therefore, we must have

$$\sum_{G_i \in Q_l} x_{G_i}^*(l) \leq c_l. \quad (5)$$

Inequality (5) implies,

$$\eta_+^{*l} = 0. \quad (6)$$

Combining (6) along with (4) we obtain

$$\sum_{G_i \in Q_l} P_{-G_i^{\max}(l)}^* \left(\frac{\mathcal{E}_{-G_i^{\max}(l)}^* + x_{G_i}^*(l)}{\gamma} \right) = 0. \quad (7)$$

Moreover, combining (6) and (7) we obtain

$$P_{-G_i^{\max}(l)}^* \left(\frac{\mathcal{E}_{-G_i^{\max}(l)}^* + x_{G_i}^*(l)}{\gamma} \right) = 0. \quad (8)$$

for any $G_i \in Q_l$. Using (8) and (6) in (3) we obtain

$$P_{-G_i^{\max}(l)}^* = P_{G_i^{\max}(l)}^*. \quad (9)$$

Since (9) is true for all $G_i \in Q_l$, it implies

$$P_{-G_i^{\max}(l)}^* = P_{G_i^{\max}(l)}^* = P_{G_j^{\max}(l)}^* =: P_{G^{\max}(l)}^*, \quad \forall G_i \in Q_l \quad (10)$$

and along with (8) it implies

$$P_{G^{\max}(l)}^* \left(\mathcal{E}_{-G_i^{\max}(l)}^* + x_{G_i}^*(l) \right) = 0. \quad (11)$$

Furthermore, because of

$$\frac{\partial \Gamma_{(k, G_i^{\max}(l))}}{\partial x_{G_i}(l)} = 0 \quad (12)$$

(as in [1], page 2101), and (6), (8), (10), Eqs. (1) and (2) give

$$\left. \frac{\partial t_{(k, G_i^{\max}(l))}^l}{\partial x_{G_i}(l)} \right|_{m=m^*} = \pi_{(k+1, G_i^{\max}(l))}^*. \quad (13)$$

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I. PROPERTIES OF THE MECHANISM

Existence of Nash equilibria (NE): The proof of existence of NE of the game induced by the mechanism is the same as in Theorem 1 of [1].

Feasibility of allocations at NE: Because of the specification of the mechanism and Eq. (6), the allocations corresponding to all NE are in the feasible set.

Budget Balance at any NE: Budget balance at any NE follows by Lemma 4 of [1].

Individual Rationality: Individual rationality follows by Theorem 5 of [1].

Nash implementation: Nash implementation follows by Theorem 6 of [1].

REFERENCES

- [1] A. Kakhbod and D. Teneketzis, "An efficient game form for multi-rate multicast service provisioning", *IEEE J. on Selected Areas in Communications: Special Issue on the Economics of Communication Networks and Systems*, vol. 30, no. 11, pp. 2093-2104, December 2012.