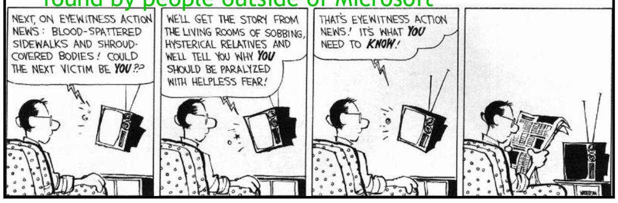
MS Patch Tuesday

- "eEye Digital Security has reported a vulnerability in Windows Media Player … due to a boundary error within the processing of bitmap files (.bmp) and can be exploited to cause a heap-based buffer overflow via a specially crafted bitmap file that declares its size as 0 … exploitation allows execution of arbitrary code"
- Six of seven "critical" or "important" bugs were found by people outside of Microsoft

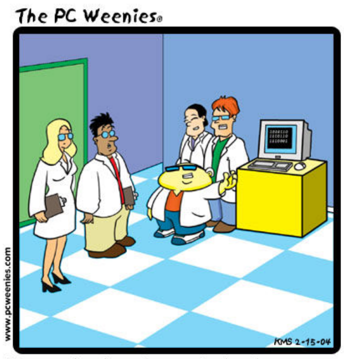


Apologies to Ralph Macchio

- Daniel: You're supposed to teach and I'm supposed to learn. Four homeworks I've been working on IMP, I haven't learned a thing.
- Miyagi: You learn plenty.
- Daniel: I learn plenty, yeah. I learned how to analyze IMP, maybe. I evaluate your commands, derive your judgments, prove your soundness. I learn plenty!
- Miyagi: Not everything is as seems.
- Daniel: You're not even relatively complete! I'm going home, man.
- Miyagi: Daniel-san!
- Daniel: What?
- Miyagi: Come here. Show me "compute the VC".

Homework

- Exciting, practical HW 5 out today
- If you've been skiving, now is a great time to try one out
- Easily applicable to other research



"Our results show that our fault tolerance test program fails 80% of the time, so we've decided to use it for only 20% of our tests."

©2004 Krishna M. Sadasivam

Abstract Interpretation (Non-Standard Semantics)

a.k.a. "Picking The Right Abstraction"

The Problem

- It is extremely useful to predict program behavior *statically* (= without running the program)
  - For optimizing compilers, program analyses, software engineering tools, finding security flaws, etc.
- The semantics we studied so far give us the precise behavior of a program
- However, precise static predictions are impossible
  - The exact semantics is not computable
- We must settle for approximate, but correct, static analyses (e.g. VC vs. WP)

#5

The Plan

- We will introduce abstract interpretation by example
- Starting with a miniscule language we will build up to a fairly realistic application
- Along the way we will see most of the ideas and difficulties that arise in a big class of applications

#6

A Tiny Language

- Consider the following language of arithmetic ("shrIMP"?)

  $e ::= n \mid e_1 * e_2$
- The denotational semantics of this language

  $[\![n]\!] = n$

  $[\![e_1 * e_2]\!] = [\![e_1]\!] \times [\![e_2]\!]$
- We'll take deno-sem as the "ground truth"
- For this language the precise semantics is computable (but in general it's not)

#7

An Abstraction

- Assume that we are interested not in the value of the expression, but only in its sign:
  - positive (+), negative (-), or zero (0)
- We can define an abstract semantics that computes **only** the sign of the result

  σ: Exp $\rightarrow$ {-, 0, +}

  $\sigma(n) = \text{sign}(n)$

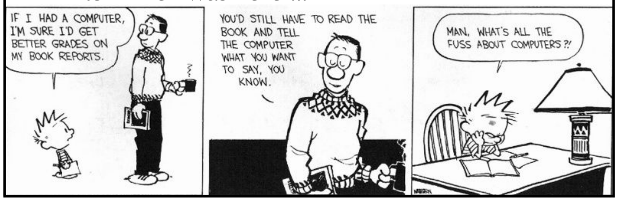
  $\sigma(e_1 * e_2) = \sigma(e_1) \otimes \sigma(e_2)$

$\otimes$	-	0	+
-	+	0	-
0	0	0	0
+	-	0	+

#8

I Saw the Sign

- Why did we want to compute the sign of an expression?
  - One reason: no one will believe you know abstract interp if you haven't seen the sign thing
- What could we be computing instead?
  - Alex Aiken was here …



Correctness of Sign Abstraction

- We can show that the abstraction is correct in the sense that it predicts the sign

  $[\![e]\!] > 0 \Leftrightarrow \sigma(e) = +$

  $[\![e]\!] = 0 \Leftrightarrow \sigma(e) = 0$

  $[\![e]\!] < 0 \Leftrightarrow \sigma(e) = -$
- Our semantics is abstract but precise
- Proof is by structural induction on the expression e
  - Each case repeats similar reasoning

#10

Another View of Soundness

- Link each concrete value to an abstract one:

  $\beta : \mathbb{Z} \rightarrow \{ -, 0, + \}$
- This is called the abstraction function (β)
  - This three-element set is the abstract domain
- Also define the concretization function (γ):

  $\gamma : \{-, 0, +\} \rightarrow \mathcal{P}(\mathbb{Z})$

  $\gamma(+) = \{ n \in \mathbb{Z} \mid n > 0 \}$

  $\gamma(0) = \{ 0 \}$

  $\gamma(-) = \{ n \in \mathbb{Z} \mid n < 0 \}$

#11

Another View of Soundness 2

- Soundness can be stated succinctly

  $\forall e \in \text{Exp}.\ [\![e]\!] \in \gamma(\sigma(e))$

  (the real value of the expression is among the concrete values represented by the abstract value of the expression)
- Let C be the concrete domain (e.g. $\mathbb{Z}$) and A be the abstract domain (e.g. {-, 0, +})
- Commutative diagram:

$$\begin{tikzcd} \text{Exp} \arrow[r, "\sigma"] \arrow[d, "{[\![\cdot]\!]}"'] & A \arrow[d, "\gamma"] \\ C \arrow[r, "\in"] & \mathcal{P}(C) \end{tikzcd}$$

#12

Another View of Soundness 3

- Consider the generic abstraction of an operator

$$\sigma(e_1 \text{ op } e_2) = \sigma(e_1) \underline{\text{op}}\ \sigma(e_2)$$

- This is sound iff

$\forall a_1 \forall a_2.\ \gamma(a_1 \underline{\text{op}}\ a_2) \supseteq \{n_1 \text{ op } n_2 \mid n_1 \in \gamma(a_1), n_2 \in \gamma(a_2)\}$

- e.g. $\gamma(a_1 \otimes a_2) \supseteq \{ n_1 * n_2 \mid n_1 \in \gamma(a_1), n_2 \in \gamma(a_2) \}$

#13

Abstract Interpretation

- This is our first example of an abstract interpretation
- We carry out computation in an abstract domain
- The abstract semantics is a sound approximation of the standard semantics
- The concretization and abstraction functions establish the connection between the two domains

#14

Adding Unary Minus and Addition

- We extend the language to
  $e ::= n \mid e_1 * e_2 \mid - e$
- We define $\sigma(- e) = \ominus\ \sigma(e)$

	-	0	+
$\ominus$	+	0	-

$\oplus$	-	0	+
-	-	-	?
0	-	0	+
+	?	+	+

- Now we add addition:
  $e ::= n \mid e_1 * e_2 \mid - e \mid e_1 + e_2$
- We define $\sigma(e_1 + e_2) = \sigma(e_1) \oplus \sigma(e_2)$

#15

Adding Addition

- The sign values are not closed under addition
- What should be the value of "$+ \oplus -$"?
- Start from the soundness condition:

$$\gamma(+ \oplus -) \supseteq \{ n_1 + n_2 \mid n_1 > 0, n_2 < 0\} = \mathbb{Z}$$

- We don't have an abstract value whose concretization includes $\mathbb{Z}$, so we add one:

  $\top$ ("top" = "don't know")

$\oplus$	-	0	+	$\top$
-	-	-	$\top$	$\top$
0	-	0	+	$\top$
+	$\top$	+	+	$\top$
$\top$	$\top$	$\top$	$\top$	$\top$

#16

Loss of Precision

- Abstract computation might loose information

$$[\![(1 + 2) + -3]\!] = 0$$

but $\sigma((1+2) + -3) =$

$(\sigma(1) \oplus \sigma(2)) \oplus \sigma(-3) =$

$(+ \oplus +) \oplus - = \top$

- We loose some precision
- But this will simplify the computation of the abstract answer in cases when the precise answer is not computable

#17

Adding Division

- Straightforward except for division by 0
  - We say that there is no answer in that case
  - $\gamma(+ \oslash 0) = \{ n \mid n = n_1 / 0 , n_1 > 0 \} = \emptyset$
- Introduce $\perp$ to be the abstraction of the $\emptyset$
  - We also use the same abstraction for non-termination!

$\perp$ = "nothing"

$\top$ = "something unknown"

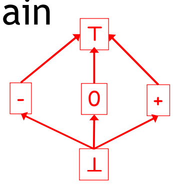
$\oslash$	-	0	+	$\top$	$\perp$
-	+	0	-	$\top$	$\perp$
0	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
+	-	0	+	$\top$	$\perp$
$\top$	$\top$	$\top$	$\top$	$\top$	$\perp$
$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$

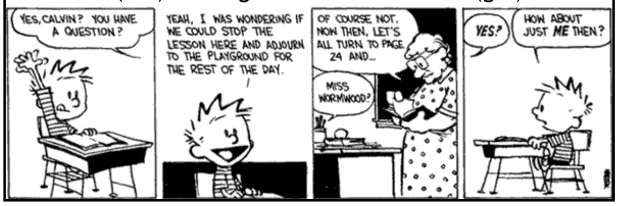
#18

The Abstract Domain

- Our abstract domain forms a lattice
- A partial order is induced by γ

  $a_1 \leq a_2$ iff $\gamma(a_1) \subseteq \gamma(a_2)$
  - We say that a_1 is more precise than a_2!
- Every finite subset has a least-upper bound (lub) and a greatest-lower bound (glb)





Lattice Facts

- A lattice is complete when every subset has a lub and a gub
  - Even infinite subsets!
- Every finite lattice is (trivially) complete
- Every complete lattice is a complete partial order (recall: denotational semantics!)
  - Since a chain is a subset
- Not every CPO is a complete lattice
  - Might not even be a lattice

Lattice History

- Early work in denotational semantics used lattices
  - But it was later seen that only chains need to have lubs
  - And there was no need for $\top$ and glb
- In abstract interpretation we'll use $\top$ to denote *"I don't know"*
  - Corresponds to all values in the concrete domain

From One, Many

- We can start with the abstraction function β

  $\beta : C \to A$

  (maps a concrete value to the best abstract value)
  - A must be a lattice
- We can derive the concretization function γ

  $\gamma : A \to \mathcal{P}(C)$

  $\gamma(a) = \{ x \in C \mid \beta(x) \leq a \}$
- And the abstraction for sets α

  $\alpha : \mathcal{P}(C) \to A$

  $\alpha(S) = \text{lub} \{ \beta(x) \mid x \in S \}$

Example

- Consider our sign lattice

$$\beta(n) = \begin{cases} + & \text{if } n > 0 \\ 0 & \text{if } n = 0 \\ - & \text{if } n < 0 \end{cases}$$

- $\alpha(S) = \text{lub} \{ \beta(x) \mid x \in S\}$
  - Example: $\alpha(\{1, 2\}) = \text{lub} \{ + \} = +$

    $\alpha(\{1, 0\}) = \text{lub} \{ +, 0\} = \top$

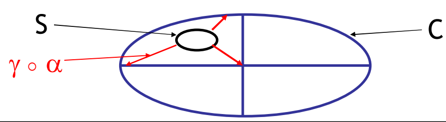
    $\alpha(\{\}) = \text{lub} \{\} = \bot$
- $\gamma(a) = \{ n \mid \beta(n) \leq a \}$
  - Example: $\gamma(+) = \{ n \mid \beta(n) \leq +\} = \{ n \mid \beta(n) = +\} = \{ n \mid n > 0 \}$

    $\gamma(\top) = \{ n \mid \beta(n) \leq \top \} = \mathbb{Z}$

    $\gamma(\bot) = \{ n \mid \beta(n) \leq \bot\} = \emptyset$

Galois Connections

- We can show that
  - γ and α are monotonic (with $\subseteq$ ordering on $\mathcal{P}(C)$)
  - $\alpha(\gamma(a)) = a$ for all $a \in A$
  - $\gamma(\alpha(S)) \supseteq S$ for all $S \in \mathcal{P}(C)$
- Such a pair of functions is called a Galois connection
  - Between the lattices A and $\mathcal{P}(C)$



Correctness Condition

- In general, abstract interpretation satisfies the following (amazingly common) diagram

$$\begin{tikzcd}
\mathrm{Exp} \arrow[r, "\sigma"] \arrow[d, "[\![\cdot]\!]"'] & A \arrow[d, "\gamma"] \\
C \arrow[r, "\in"'] & \mathcal{P}(C) \arrow[u, bend right=60, "\alpha\ (\leq)"']
\end{tikzcd}$$

Correctness Conditions

- Three conditions define a correct abstract interpretation

1. α and γ are monotonic
2. α and γ form a Galois connection
   = "α and γ are almost inverses"
3. Abstraction of operations is correct
   $a_1\ \underline{op}\ a_2 = \alpha(\gamma(a_1)\ op\ \gamma(a_2))$

Homework

- Homework 4 Due Today
- Homework 5 Out Today
- Read Ken Thompson Turing Award
- Project Proposal Due On Tuesday