Operational Semantics

One-Slide Summary

- **Operational semantics** are a precise way of specifying how to evaluate a program.
- A **formal semantics** tells you what each expression means.
- Meaning depends on **context**: a **variable environment** will map variables to memory locations and a **store** will map memory locations to values.

Lecture Outline

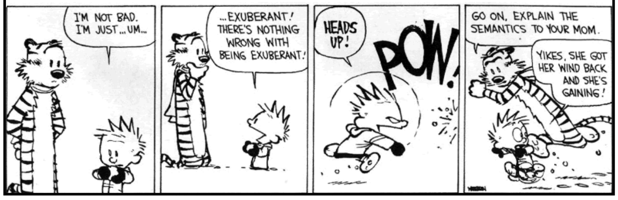
- COOL operational semantics
- Motivation
- Notation
- The rules

Motivation

- We must specify for every Cool expression *what happens when it is evaluated*
  - This is the **meaning** of an expression
- The definition of a programming language:
  - The tokens ⇒ lexical analysis
  - The grammar ⇒ syntactic analysis
  - The typing rules ⇒ semantic analysis
  - The evaluation rules ⇒ interpretation

Evaluation Rules So Far

- So far, we specified the evaluation rules intuitively
  - We described how dynamic dispatch behaved in words (e.g., "just like Java")
  - We talked about scoping, variables, arithmetic expressions (e.g., "they work as expected")
- Why isn't this description good enough?



Assembly Language Description of Semantics

- We might just tell you how to compile it
- But assembly-language descriptions of language implementation have too many irrelevant details
  - Which way the stack grows
  - How integers are represented on a particular machine
  - The particular instruction set of the architecture
- We need a complete but not overly restrictive specification

Programming Language Semantics

- There are many ways to specify programming language semantics
- They are all equivalent but some are more suitable to various tasks than others
- **Operational semantics**
  - Describes the evaluation of programs on an abstract machine
  - Most useful for specifying implementations
  - This is what we will use for Cool

Other Kinds of Semantics

- **Denotational semantics**
  - The meaning of a program is expressed as a mathematical object
  - Elegant but quite complicated
- **Axiomatic semantics**
  - Useful for checking that programs satisfy certain correctness properties
    - e.g., that the quick sort function sorts an array
  - The foundation of many program verification systems

Introduction to Operational Semantics

- Once, again we introduce a formal notation
  - Using logical rules of inference, just like typing
- Recall the typing judgment

$$\textbf{Context} \vdash \textbf{e} : \textbf{C}$$

(in the given context, expression e has type C)

- We try something similar for evaluation

$$\textbf{Context} \vdash \textbf{e} : \textbf{v}$$

(in the given context, expression e evaluates to value v)

Example Operational Semantics Inference Rule

$$\frac{\textbf{Context} \vdash e_1 : 5 \qquad \textbf{Context} \vdash e_2 : 7}{\textbf{Context} \vdash e_1 + e_2 : 12}$$

- In general the result of evaluating an expression depends on the result of evaluating its subexpressions
- The logical rules specify everything that is needed to evaluate an expression

What Contexts Are Needed?

- Contexts are needed to handle variables
- Consider the evaluation of $y \leftarrow x + 1$
  - We need to keep track of values of variables
  - We need to allow variables to change their values during the evaluation
- We track variables and their values with:
  - An **environment** : tells us at what address in memory is the value of a variable stored
  - A **store** : tells us what is the contents of a memory location

Variable Environments

- A variable **environment** is a map from variable names to **locations**
- Tells in what memory location the value of a variable is stored
  - Locations = Memory Addresses
- Environment tracks in-scope variables only
- Example environment:

$$E = [a : l_1,\ b : l_2]$$

- To lookup a variable a in environment E we write $E(a)$

Stores

- A **store** maps memory locations to values
- Example store:

$$S = [l_1 \rightarrow 5,\ l_2 \rightarrow 7]$$

- To lookup the contents of a location l_1 in store S we write $S(l_1)$
- To perform an assignment of 12 to location l_1 we write $S[12/l_1]$
  - This denotes a new store S' such that

$$S'(l_1) = 12 \quad \text{and} \quad S'(l) = S(l) \text{ if } l \neq l_1$$

Cool Values

- All **values** in Cool are objects
  - All objects are instances of some class (the dynamic type of the object)
- To denote a Cool object we use the notation $X(a_1 = l_1, \ldots, a_n = l_n)$ where
  - X is the dynamic type of the object
  - a_i are the attributes (including those inherited)
  - l_i are the locations where the values of attributes are stored

Cool Values (Cont.)

- Special cases (classes without attributes)

Int(5)	the integer 5
Bool(true)	the boolean true
String(4, “Cool”)	the string “Cool” of length 4

- There is a special value void that is a member of all types
  - No operations can be performed on it
  - Except for the test isvoid
  - Concrete implementations might use NULL here

Operational Rules of Cool

- The evaluation judgment is

$$so, E, S \vdash e : v, S'$$

read:
  - Given so the current value of the self object
  - And E the current variable environment
  - And S the current store
  - If the evaluation of e terminates then
  - The returned value is v
  - And the new store is S'

Notes

- The "result" of evaluating an expression is both a value and a new store
- Changes to the store model side-effects
  - side-effects = assignments to variables
- The variable environment does not change
- Nor does the value of "self"
- The operational semantics allows for non-terminating evaluations
- We define one rule for each kind of expression

Operational Semantics for Base Values

$$\frac{}{so, E, S \vdash \text{true} : \text{Bool(true)}, S}$$

$$\frac{}{so, E, S \vdash \text{false} : \text{Bool(false)}, S}$$

$$\frac{i \text{ is an integer literal}}{so, E, S \vdash i : \text{Int}(i), S}$$

$$\frac{\begin{array}{c} s \text{ is a string literal} \\ n \text{ is the length of } s \end{array}}{so, E, S \vdash s : \text{String}(n,s), S}$$

- No side effects in these cases

  (the store does not change)

Operational Semantics of Variable References

$$\frac{\begin{array}{c} E(id) = l_{id} \\ S(l_{id}) = v \end{array}}{so, E, S \vdash id : v, S}$$

- Note the double lookup of variables
  - First from name to location
  - Then from location to value
- The store does not change
- A special case:

$$\frac{}{so, E, S \vdash self : so, S}$$

Operational Semantics of Assignment

$$\frac{\begin{array}{c} so, E, S \vdash e : v, S_1 \\ E(id) = l_{id} \\ S_2 = S_1[v/l_{id}] \end{array}}{so, E, S \vdash id \leftarrow e : v, S_2}$$

- A three step process
  - Evaluate the right hand side
    - ⇒ a value v and a new store S_1
  - Fetch the location of the assigned variable
  - The result is the value v and an updated store
- The environment does not change

Operational Semantics of Conditionals

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : Bool(true), S_1 \\ so, E, S_1 \vdash e_2 : v, S_2 \end{array}}{so, E, S \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : v, S_2}$$

- The “threading” of the store enforces an evaluation sequence
  - e_1 must be evaluated first to produce S_1
  - Then e_2 can be evaluated
- The result of evaluating e_1 is a boolean object
  - The typing rules ensure this
  - There is another, similar, rule for Bool(false)

Operational Semantics of Sequences

$$\frac{\begin{array}{c}\text{so}, E, S \vdash e_1 : v_1, S_1 \\ \text{so}, E, S_1 \vdash e_2 : v_2, S_2 \\ \ldots \\ \text{so}, E, S_{n-1} \vdash e_n : v_n , S_n\end{array}}{\text{so}, E, S \vdash \{\ e_1;\ \ldots;\ e_n;\ \}\ : v_n, S_n}$$

- Again the threading of the store expresses the intended evaluation sequence
- Only the last value is used
- But all the side-effects are collected (how?)

Operational Semantics of while (1)

$$\frac{\text{so}, E, S \vdash e_1 : \text{Bool(false)}, S_1}{\text{so}, E, S \vdash \textbf{while}\ e_1\ \textbf{loop}\ e_2\ \textbf{pool} : \text{void}, S_1}$$

- If e_1 evaluates to Bool(false) then the loop terminates immediately
  - With the side-effects from the evaluation of e_1
  - And with result value void
- The typing rules ensure that e_1 evaluates to a boolean object

Operational Semantics of while (2)

$$\frac{\begin{array}{c}\text{so}, E, S \vdash e_1 : \text{Bool(true)}, S_1 \\ \text{so}, E, S_1 \vdash e_2 : v, S_2 \\ \text{so}, E, S_2 \vdash \textbf{while}\ e_1\ \textbf{loop}\ e_2\ \textbf{pool} : \text{void}, S_3\end{array}}{\text{so}, E, S \vdash \textbf{while}\ e_1\ \textbf{loop}\ e_2\ \textbf{pool} : \text{void}, S_3}$$

- Note the sequencing ($S \rightarrow S_1 \rightarrow S_2 \rightarrow S_3$)
- Note how looping is expressed
  - Evaluation of "while …" is expressed in terms of the evaluation of itself in another state
- The result of evaluating e_2 is discarded
  - Only the side-effect is preserved

Operational Semantics of let Expressions (1)

$$\frac{\begin{array}{c} \mathbf{so, E, S \vdash e_1 : v_1, S_1} \\ \mathbf{so, ?, ? \vdash e_2 : v, S_2} \end{array}}{\mathbf{so, E, S \vdash let\ id : T \leftarrow e_1\ in\ e_2 : v_2, S_2}}$$

- What is the context in which e_2 must be evaluated?
  - Environment like E but with a new binding of id to a fresh location l_{new}
  - Store like S_1 but with l_{new} mapped to v_1

Operational Semantics of let Expressions (II)

- We write $l_{new} = newloc(S)$ to say that l_{new} is a location that is not already used in S
  - Think of newloc as the dynamic memory allocation function
- The operational rule for let:

$$\frac{\begin{array}{c} \mathbf{so, E, S \vdash e_1 : v_1, S_1} \\ \mathbf{l_{new} = newloc(S_1)} \\ \mathbf{so, E[l_{new}/id]\ ,\ S_1[v_1/l_{new}] \vdash e_2 : v_2, S_2} \end{array}}{\mathbf{so, E, S \vdash let\ id : T \leftarrow e_1\ in\ e_2 : v_2, S_2}}$$

Operational Semantics of new

- Consider the expression new T
- Informal semantics
  - Allocate new locations to hold the values for all attributes of an object of class T
    - Essentially, allocate a new object
  - Initialize those locations with the default values of attributes
  - Evaluate the initializers and set the resulting attribute values
  - Return the newly allocated object

Default Values

- For each class A there is a default value denoted by D_A
  - D_{int} = Int(0)
  - D_{bool} = Bool(false)
  - D_{string} = String(0, “”)
  - D_A = void (for all others classes A)

More Notation

- For a class A we write

  $$\text{class}(A) = (a_1 : T_1 \leftarrow e_1, \ldots, a_n : T_n \leftarrow e_n)$$

where
  - a_i are the attributes (including inherited ones)
  - T_i are their declared types
  - e_i are the initializers

- This is the class map from PA4!

Operational Semantics of new

- Observation: new SELF_TYPE allocates an object with the same dynamic type as self

$$\frac{\begin{array}{l}
T_0 = \text{if } T == \text{SELF\_TYPE and } so = X(\ldots) \text{ then } X \text{ else } T \\
\text{class}(T_0) = (a_1 : T_1 \leftarrow e_1, \ldots, a_n : T_n \leftarrow e_n) \\
l_i = \text{newloc}(S) \text{ for } i = 1,\ldots,n \\
v = T_0(a_1 = l_1, \ldots, a_n = l_n) \\
E' = [a_1 : l_1, \ldots, a_n : l_n] \\
S_1 = S[D_{T1}/l_1, \ldots, D_{Tn}/l_n] \\
v, E', S_1 \vdash \{\ a_1 \leftarrow e_1; \ldots; a_n \leftarrow e_n; \ \} : v_n, S_2
\end{array}}{so, E, S \vdash \text{new } T : v, S_2}$$

Operational Semantics of new

- The first three lines allocate the object
- The rest of the lines initialize it
  - By evaluating a sequence of assignments
- State in which the initializers are evaluated
  - Self is the current object
  - Only the attributes are in scope (same as in typing)
  - Starting value of attributes are the default ones
- Side-effects of initialization are preserved

Operational Semantics of Method Dispatch

- Consider the expression **$e_0.f(e_1,…,e_n)$**
- Informal semantics:
  - Evaluate the arguments in order $e_1,…,e_n$
  - Evaluate e_0 to the target object
  - Let X be the dynamic type of the target object
  - Fetch from X the definition of f (with n args)
  - Create n new locations and an environment that maps f’s formal arguments to those locations
  - Initialize the locations with the actual arguments
  - Set self to the target object and evaluate f’s body

More Notation

- For a class A and a method f of A (possibly inherited) we write:

$$imp(A, f) = (x_1, …, x_n, e_{body})$$

where

- x_i are the names of the formal arguments
- e_{body} is the body of the method

- This is the **imp map** from PA4!

Operational Semantics of Dispatch

$$\dfrac{\begin{array}{ll}
\left.\begin{array}{l}
so, E, S \vdash e_1 : v_1, S_1 \\
so, E, S_1 \vdash e_2 : v_2, S_2 \\
\ldots \\
so, E, S_{n-1} \vdash e_n : v_n, S_n
\end{array}\right\} & \textit{Evaluate arguments} \\
\left. so, E, S_n \vdash e_0 : v_0, S_{n+1} \right\} & \textit{Evaluate receiver object} \\
\left. v_0 = X(a_1 = l_1, \ldots, a_m = l_m) \right\} & \textit{Find type and attribtues} \\
\left. imp(X, f) = (x_1, \ldots, x_n, e_{body}) \right\} & \textit{Find formals and body} \\
l_{xi} = newloc(S_{n+1}) \text{ for } i = 1, \ldots, n & \\
\left. E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1, \ldots, a_m : l_m] \right\} & \textit{New environment} \\
\left. S_{n+2} = S_{n+1}[v_1/l_{x1}, \ldots, v_n/l_{xn}] \right\} & \textit{New store} \\
\left. v_0, E', S_{n+2} \vdash e_{body} : v, S_{n+3} \right\} & \textit{Evaluate body}
\end{array}}{so, E, S \vdash e_0.f(e_1, \ldots, e_n) : v, S_{n+3}}$$

Operational Semantics of Dispatch

- The body of the method is invoked with
  - E mapping formal arguments and self's attributes
  - S like the caller's except with actual arguments bound to the locations allocated for formals
- The notion of the activation frame is implicit
  - New locations are allocated for actual arguments
- The semantics of static dispatch is similar except the implementation of f is taken from the specified class

Runtime Errors

Operational rules do not cover all cases

Consider for example the rule for dispatch:

$$\dfrac{\begin{array}{l}
\ldots \\
so, E, S_n \vdash e_0 : v_0, S_{n+1} \\
v_0 = X(a_1 = l_1, \ldots, a_m = l_m) \\
imp(X, f) = (x_1, \ldots, x_n, e_{body}) \\
\ldots
\end{array}}{so, E, S \vdash e_0.f(e_1, \ldots, e_n) : v, S_{n+3}}$$

What happens if imp(X, f) is not defined?

Cannot happen in a well-typed program

(because of the Type Safety Theorem)

Runtime Errors

- There are some runtime errors that the type checker does not try to prevent
  - A dispatch on void
  - Division by zero
  - Substring out of range
  - Heap overflow
- In such case the execution must abort gracefully
  - With an error message not with segfault

Conclusions

- Operational rules are very precise
  - Nothing is left unspecified
- Operational rules contain a lot of details
  - Read them carefully
- Most languages do not have a well specified operational semantics
- When portability is important an operational semantics becomes essential
  - But not always using the exact notation we used for Cool

Homework

- WA5 due this Today at 1pm
- PA4 due Friday March 30th (8 days)
- For Tuesday:
  - Read Dataflow and Basic Block articles