Written Assignment 2

This assignment asks you to prepare written answers to questions on context-free grammars. Each of the questions has a short answer. You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work.

1. Let L_1 be the language consisting of all non-empty palindromes over the alphabet $\Sigma = \{a, b\}$. That is, L_1 consists of all sequences of a's and b's that read the same forward or backward. For example, $aba \in L_1$ and and $bb \in L_1$ and $aabbbaa \in L_1$, but $abb \notin L_1$.

   Let L_2 be the language over $\Sigma = \{a, b\}$ denotated by the regular expression $a(a|b)*$.

   The language $L_3 = L_1 \cap L_2$ is context-free. A string s is in L_3 if $s \in L_1$ and $s \in L_2$. Write a context-free grammar for the language L_3.

   Optional Thing To Think About: Is the intersection of a context-free language and a regular language always context-free?

2. Consider the following grammar:

$$\begin{aligned} S &\rightarrow aSb \\ S &\rightarrow Sb \\ S &\rightarrow \epsilon \end{aligned}$$

   (a) Give a one-sentence description of the language generated by this grammar.

   (b) Show that this grammar is ambiguous by giving a single string that can be parsed in two different ways. Draw both parse trees.

   (c) Give an unambiguous grammar that accepts the same language as the grammar above.

3. Using the context-free grammar for Cool given in Section 11 of the Cool manual, draw a parse tree for the following expression.

```
while not (x <- z <- 0) loop
  y <- z + 2 * x + 1
pool
```

Note that the context-free grammar by itself is ambiguous, so you will need to use the precedence and associativity rules in Section 11.1 to get the correct tree.

4. Give an example of a grammar that is $LL(3)$ but not $LL(2)$.