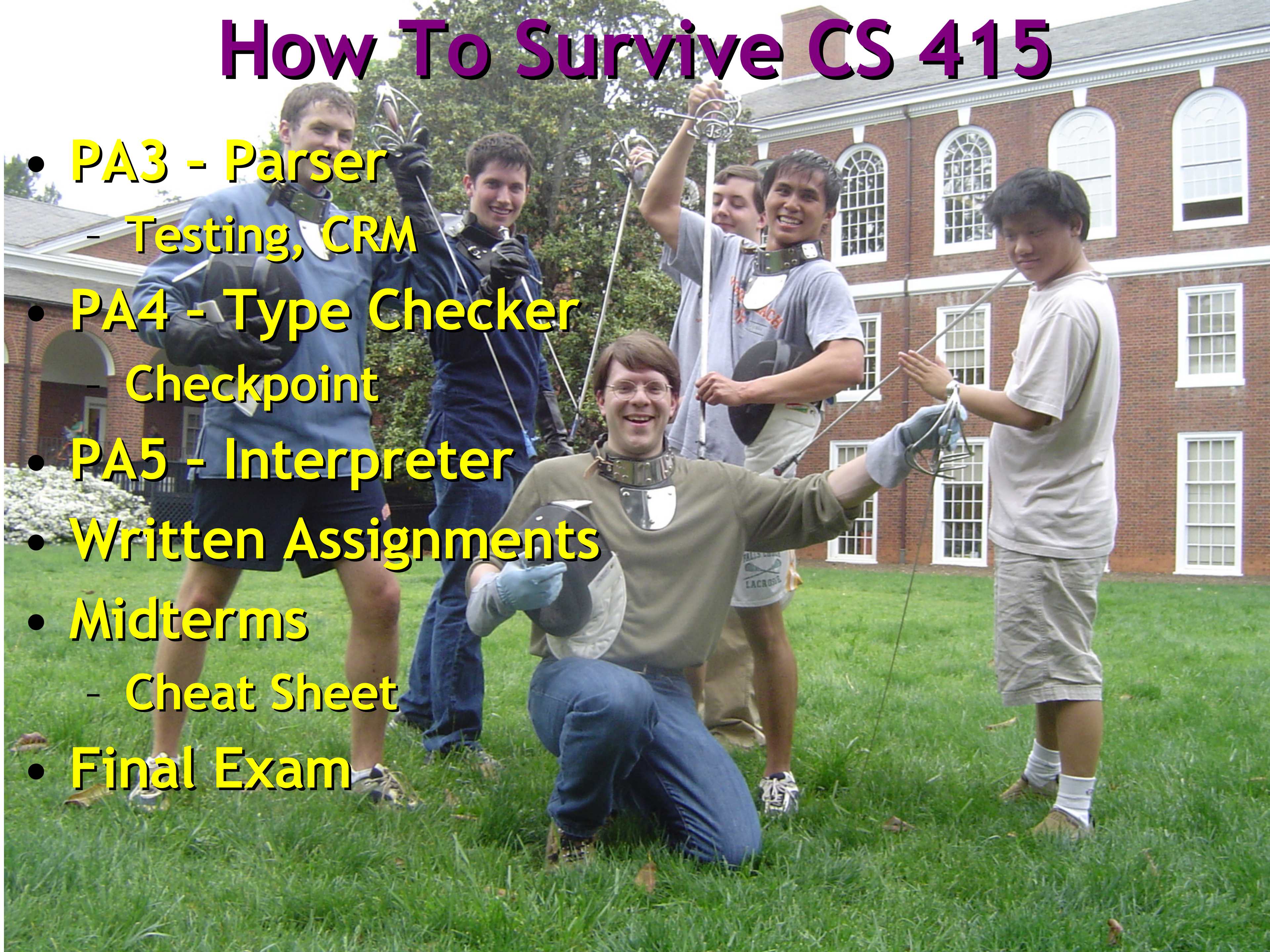
How To Survive CS 415



- PA3 – Parser
  - Testing, CRM
- PA4 – Type Checker
  - Checkpoint
- PA5 – Interpreter
- Written Assignments
- Midterms
  - Cheat Sheet
- Final Exam

The more things change...

LR Parsing

Bottom-Up Parsing

Outline

- No Stopping The Parsing!
  - LL(1) Construction
- Bottom-Up Parsing

- LR Parsing
  - Shift and Reduce
  - LR(1) Parsing Algorithm

- LR(1) Parsing Tables

In One Slide

- If the LL(1) table for a grammar G has multiple entries, then G is **not LL(1)**!
- An **LR(1) parser** reads tokens from left to right and constructs a bottom-up rightmost derivation. LR(1) parsers shift terminals and reduce the input by application productions in reverse. LR(1) parsing is fast and easy, and uses a finite automaton with a stack. LR(1) works fine if the grammar is left-recursive, or not left-factored.

Constructing LL(1) Parsing Tables

- Here is how to construct a parsing table T for context-free grammar G

- For each production $A \to \alpha$ in G do:
  - For each terminal $b \in$ First(α) do
    - $T[A, b] = \alpha$
  - If $\alpha \to^* \varepsilon$, for each $b \in$ Follow(A) do
    - $T[A, b] = \alpha$

LL(1) Table Construction Example

- Recall the grammar

$E \rightarrow T\ X$ $\qquad X \rightarrow +\ E \mid \varepsilon$

$T \rightarrow (\ E\ ) \mid \text{int}\ Y$ $\qquad Y \rightarrow *\ T \mid \varepsilon$

- Where in the row of Y do we put $Y \rightarrow *\ T$?
  - In the columns of First(*T) = { * }

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

LL(1) Table Construction Example

- Recall the grammar

$E \to T\ X \qquad\qquad X \to +\ E \mid \varepsilon$

$T \to (\ E\ ) \mid \text{int}\ Y \qquad\qquad Y \to *\ T \mid \varepsilon$

- Where in the row of Y we put $Y \to \varepsilon$?
  - In the columns of Follow(Y) = { $, +,) }

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

Avoid Multiple Definitions!

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then **G is <u>not</u> LL(1)**
  - If G is ambiguous
  - If G is left recursive
  - If G is not left-factored
  - *And in other cases as well*
- Most programming language grammars are not LL(1) (e.g., Java, Ruby, C++, OCaml, Cool, Perl, ...)
- There are tools that build LL(1) tables

Top-Down Parsing Strategies

- Recursive Descent Parsing
  - But backtracking is too annoying, etc.
- Predictive Parsing, aka. **LL(k)**
  - Predict production from k tokens of lookahead
  - Build LL(1) table
  - Parsing using the table is fast and easy
  - But many grammars are not LL(1) (or even LL(k))
- Next: a more powerful parsing strategy for grammars that are not LL(1)

Bottom-Up Parsing

- **Bottom-up parsing** is more general than top-down parsing
  - And just as efficient
  - Builds on ideas in top-down parsing
  - Preferred method in practice

- Also called **LR parsing**
  - L means that tokens are read left to right
  - R means that it constructs a rightmost derivation

An Introductory Example

- LR parsers don't need left-factored grammars and can also handle left-recursive grammars

- Consider the following grammar:

$$E \rightarrow E + (E) \mid \mathbf{int}$$

  - Why is this not LL(1)? (Guess before I show you!)

- Consider the string: **int + (int) + (int)**

The Idea

- LR parsing **reduces** a string to the start symbol by *inverting* productions:

str $\leftarrow$ input string of terminals

repeat

- Identify β in **str** such that $A \rightarrow \beta$ is a production (i.e., **str** = $\alpha\ \beta\ \gamma$)
- Replace β by A in **str** (i.e., **str** becomes $\alpha\ A\ \gamma$)

until **str** = S

A Bottom-up Parse in Detail (1)

int + (int) + (int)

int + (int) + (int)

A Bottom-up Parse in Detail (2)

int + (int) + (int)

E + (int) + (int)

A Bottom-up Parse in Detail (3)

int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

```
E                E
|                |
int   +   (     int   )   +   (   int   )
```

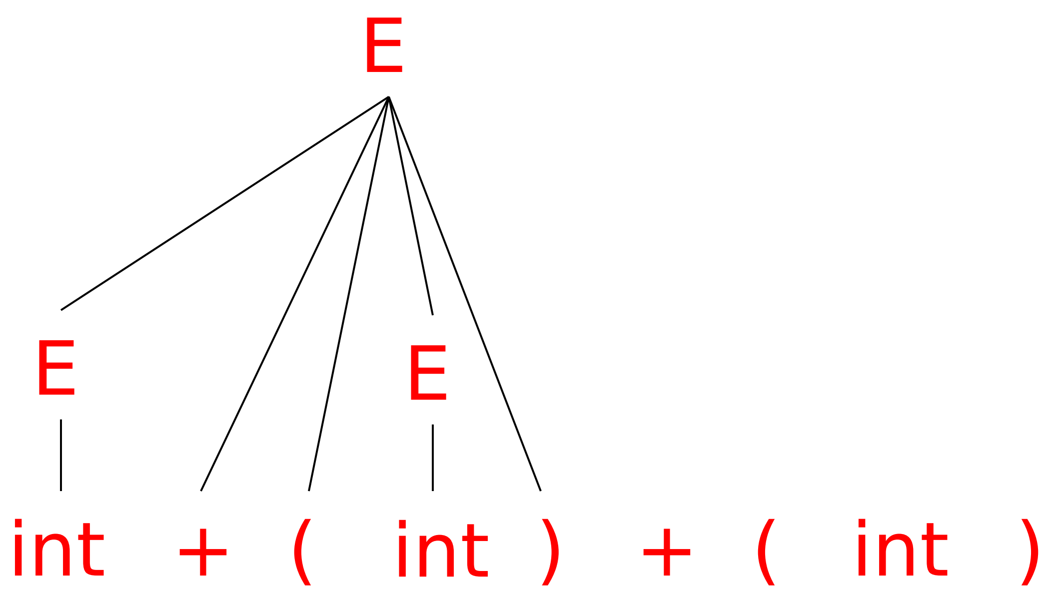
A Bottom-up Parse in Detail (4)

int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

E + (int)



A Bottom-up Parse in Detail (5)

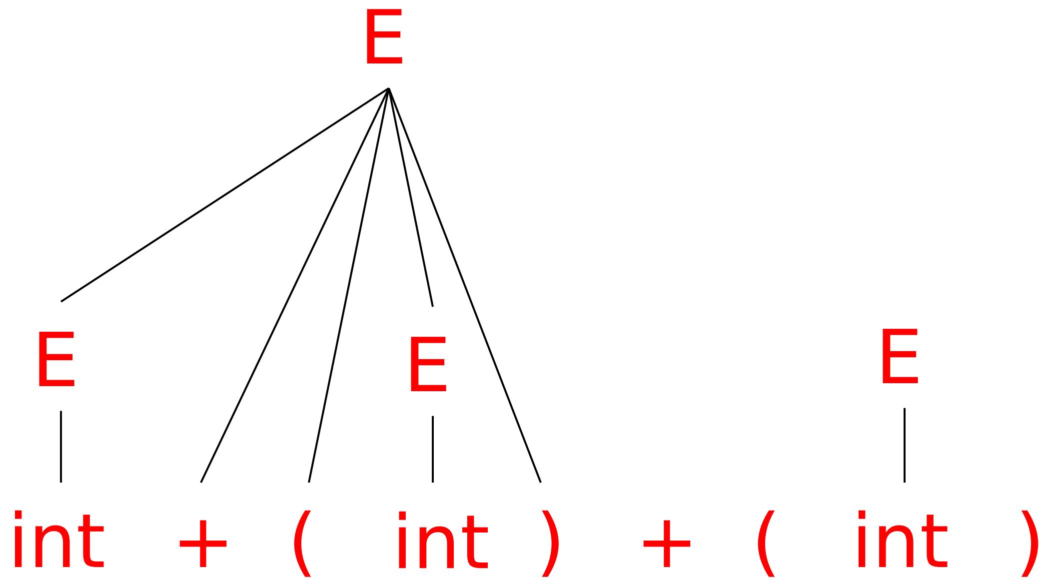
int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

E + (int)

E + (E)



A Bottom-up Parse in Detail (6)

int + (int) + (int)

E + (int) + (int)

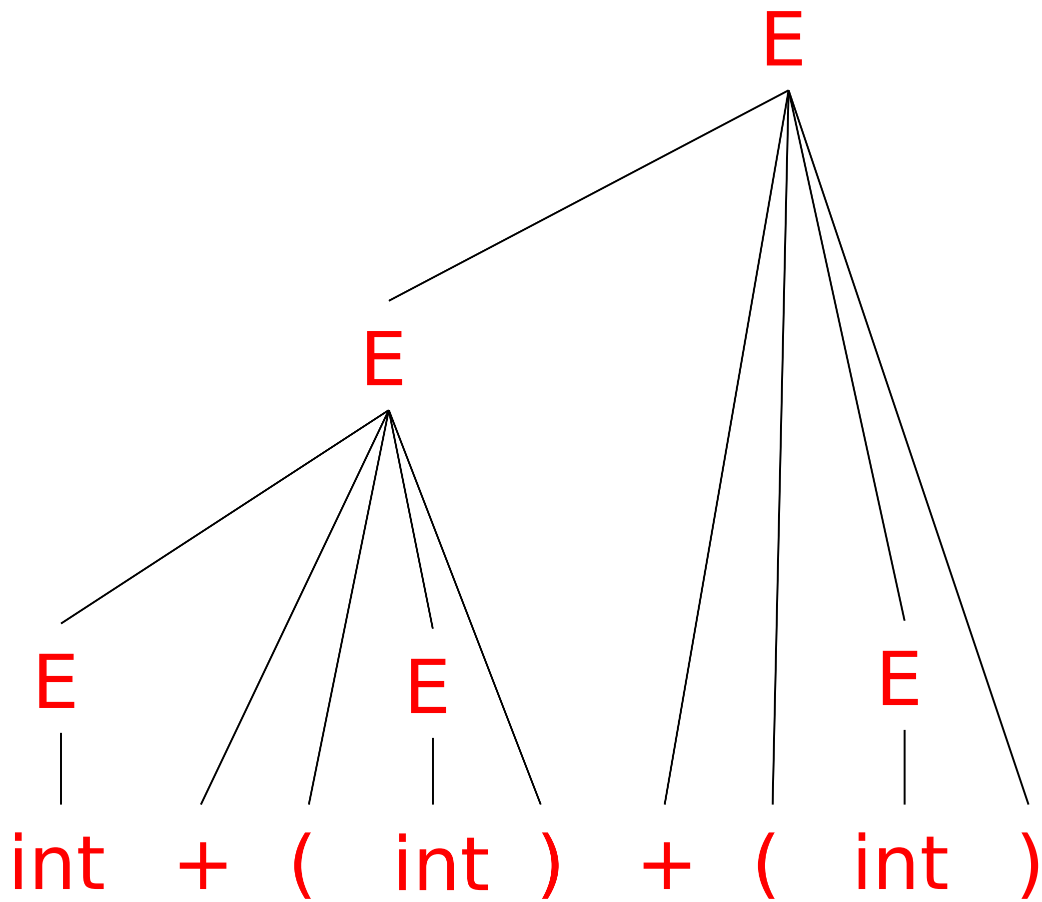
E + (E) + (int)

E + (int)

E + (E)

E

A rightmost derivation *in reverse*



Important Fact

Important Fact #1 about bottom-up parsing:

An LR parser traces a rightmost derivation in reverse.

Where Do Reductions Happen

Important Fact #1 has an Interesting Consequence:

- Let $\alpha\beta\gamma$ be a step of a bottom-up parse
- Assume the next reduction is by $A \rightarrow \beta$
- Then γ is a string of **terminals**!

Why? Because $\alpha A\gamma \rightarrow \alpha\beta\gamma$ is a step in a right-most derivation

Notation

- Idea: Split the string into two substrings
  - Right substring (a string of terminals) is as yet unexamined by parser
  - Left substring has terminals and non-terminals

- The dividing point is marked by a ▶
  - The ▶ is not part of the string

- Initially, all input is new: ▶$x_1 x_2 \ldots x_n$

Shift-Reduce Parsing

- Bottom-up parsing uses only two kinds of actions:

Shift

Reduce

Shift

Shift: Move ▶ one place to the right

- Shifts a terminal to the left string

E + (▶ int)

$\Rightarrow$

E + (int ▶)

Reduce

Reduce: Apply an inverse production at the right end of the left string

- If $\mathbf{T} \rightarrow \mathbf{E + (E)}$ is a production, then

$$\mathbf{E + (\underline{E + (E)} \blacktriangleright)}$$

$$\Rightarrow$$

$$\mathbf{E + (\underline{T} \blacktriangleright)}$$

Reductions can only happen here!

Q: Movies (268 / 842)

- In the 1986 movie **Stand By Me**, what do the kids journey to see?

Q: Books (769 / 842)

- Who wrote the 1978 book entitled **What is the Name of this Book?**, obliquely referenced here by critic Andrew Maizels:
  - In fact, between SG&E and the Illuminatus! trilogy, I was sufficiently intrigued that I went out and bought a copy of Ayn Rand's **What is the Name of this Book?**. No, sorry, **Atlas Shrugged**, that's the one. Of course, I haven't actually read it; every so often I pick it up, and say to myself: "it's a lovely day today, I think I shall go for a walk". This book has improved my life immensely.

Q: Movies (287 / 842)

- This 1995 Pixar animated film featured "Forrest Gump" and "Tim The Tool-Man Taylor". Initially jealous and resentful, the two eventually work together.

Shift-Reduce Example

- int + (int) + (int)$ shift

int + (int) + (int)

Shift-Reduce Example

▶ int + (int) + (int)\$ shift

int ▶ + (int) + (int)\$ red. $E \rightarrow int$

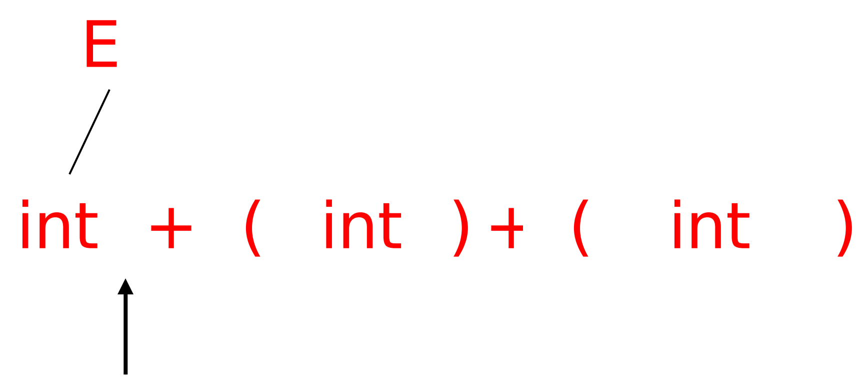
int + (int) + (int)

Shift-Reduce Example

▶ int + (int) + (int)\$ shift

int ▶ + (int) + (int)\$ red. E $\rightarrow$ int

E ▶ + (int) + (int)\$ shift 3 times



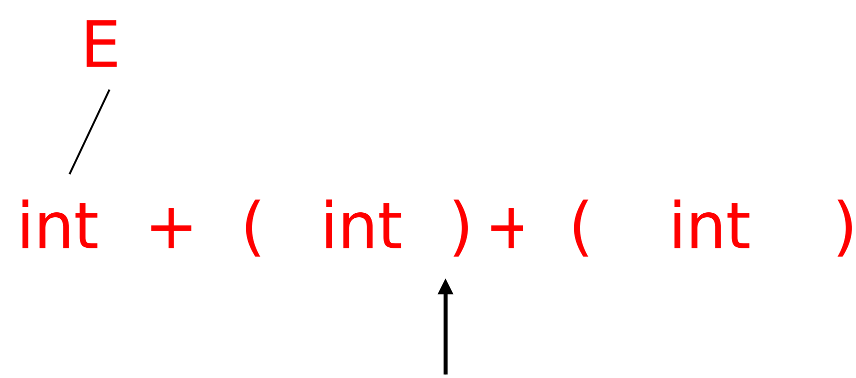
Shift-Reduce Example

▶ int + (int) + (int)\$ shift

int ▶ + (int) + (int)\$ red. E $\rightarrow$ int

E ▶ + (int) + (int)\$ shift 3 times

E + (int ▶) + (int)\$ red. E $\rightarrow$ int



Shift-Reduce Example

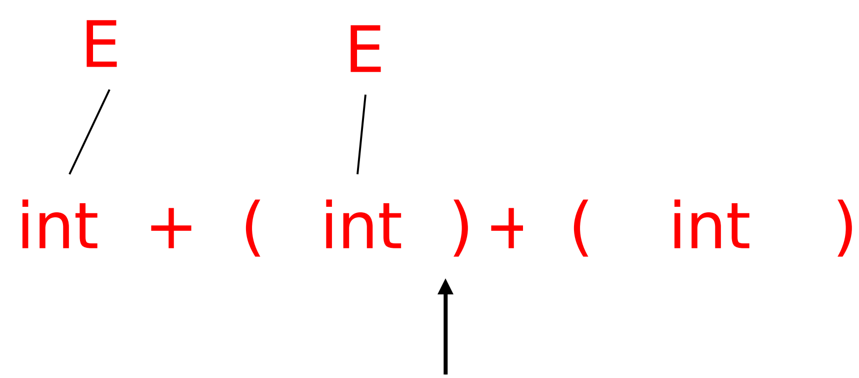
▶ int + (int) + (int)\$ shift

int ▶ + (int) + (int)\$ red. $E \rightarrow$ int

E ▶ + (int) + (int)\$ shift 3 times

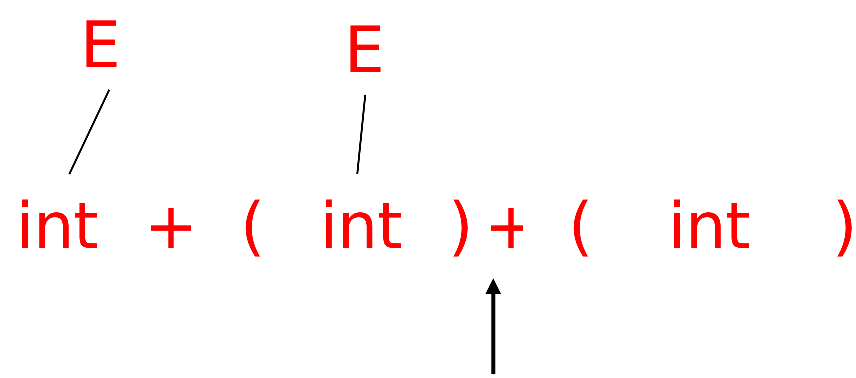
E + (int ▶) + (int)\$ red. $E \rightarrow$ int

E + (E ▶) + (int)\$ shift



Shift-Reduce Example

▶ int + (int) + (int)\$	shift
int ▶ + (int) + (int)\$	red. $E \rightarrow int$
E ▶ + (int) + (int)\$	shift 3 times
E + (int ▶) + (int)\$	red. $E \rightarrow int$
E + (E ▶) + (int)\$	shift
E + (E) ▶ + (int)\$	red. $E \rightarrow E + (E)$



Shift-Reduce Example

▶ int + (int) + (int)\$ shift

int ▶ + (int) + (int)\$ red. $E \rightarrow int$

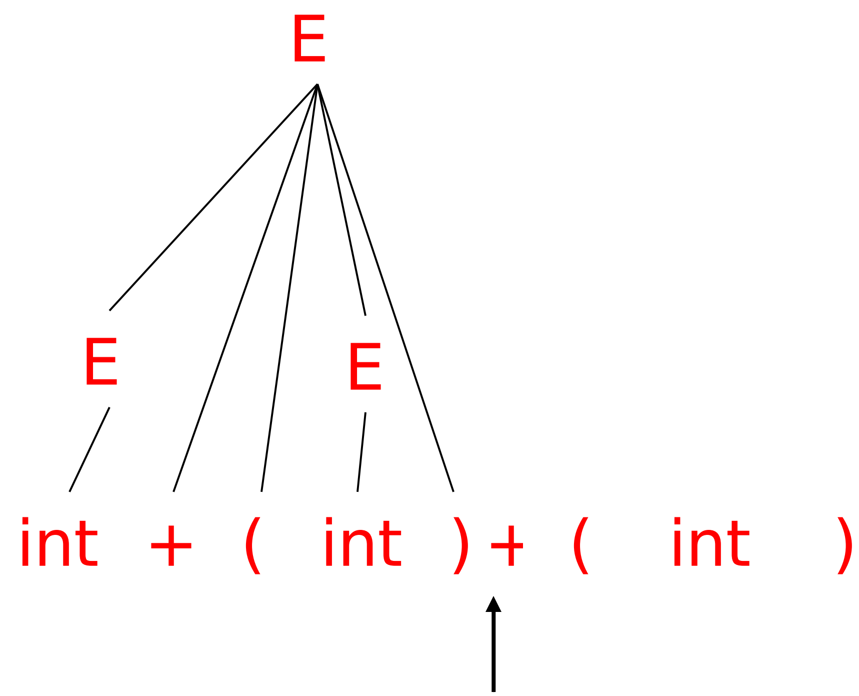
E ▶ + (int) + (int)\$ shift 3 times

E + (int ▶) + (int)\$ red. $E \rightarrow int$

E + (E ▶) + (int)\$ shift

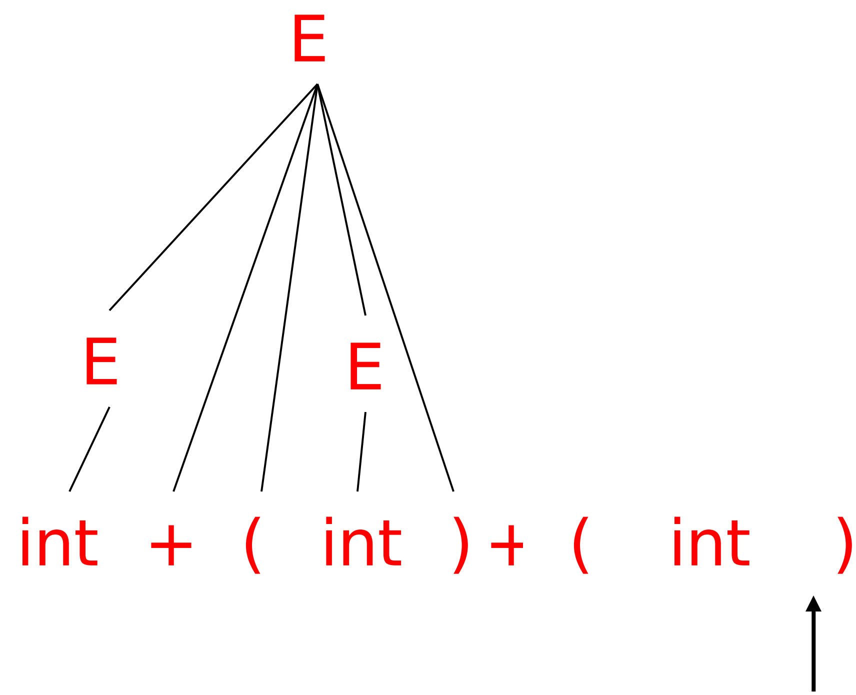
E + (E) ▶ + (int)\$ red. $E \rightarrow E + (E)$

E ▶ + (int)\$ shift 3 times



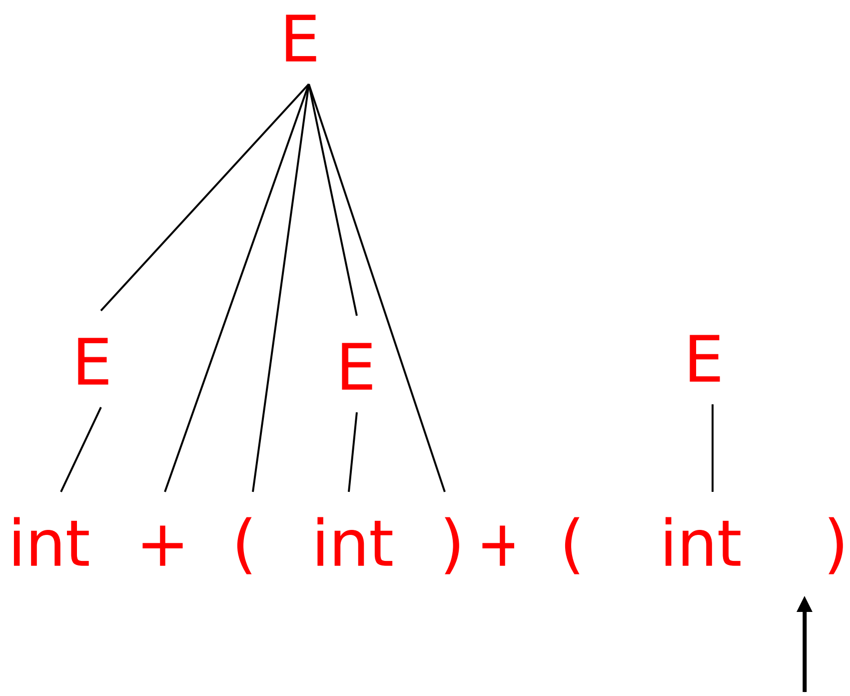
Shift-Reduce Example

▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	red. E $\rightarrow$ int
E ▶ + (int) + (int)$	shift 3 times
E + (int ▶) + (int)$	red. E $\rightarrow$ int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	red. E $\rightarrow$ E + (E)
E ▶ + (int)$	shift 3 times
E + (int ▶)$	red. E $\rightarrow$ int



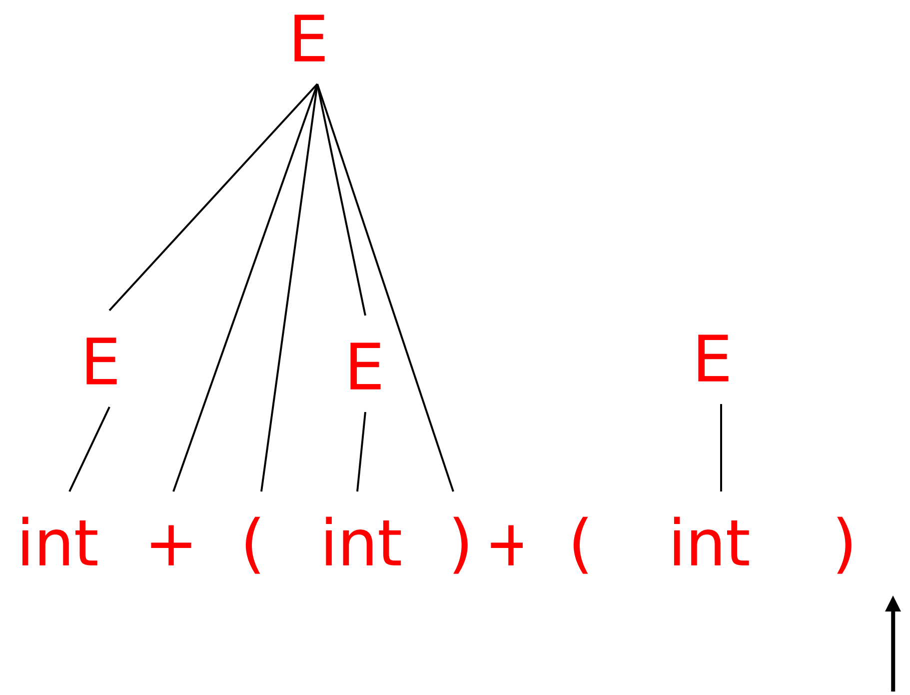
Shift-Reduce Example

▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	red. E → int
E ▶ + (int) + (int)$	shift 3 times
E + (int ▶) + (int)$	red. E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	red. E → E + (E)
E ▶ + (int)$	shift 3 times
E + (int ▶)$	red. E → int
E + (E ▶)$	shift



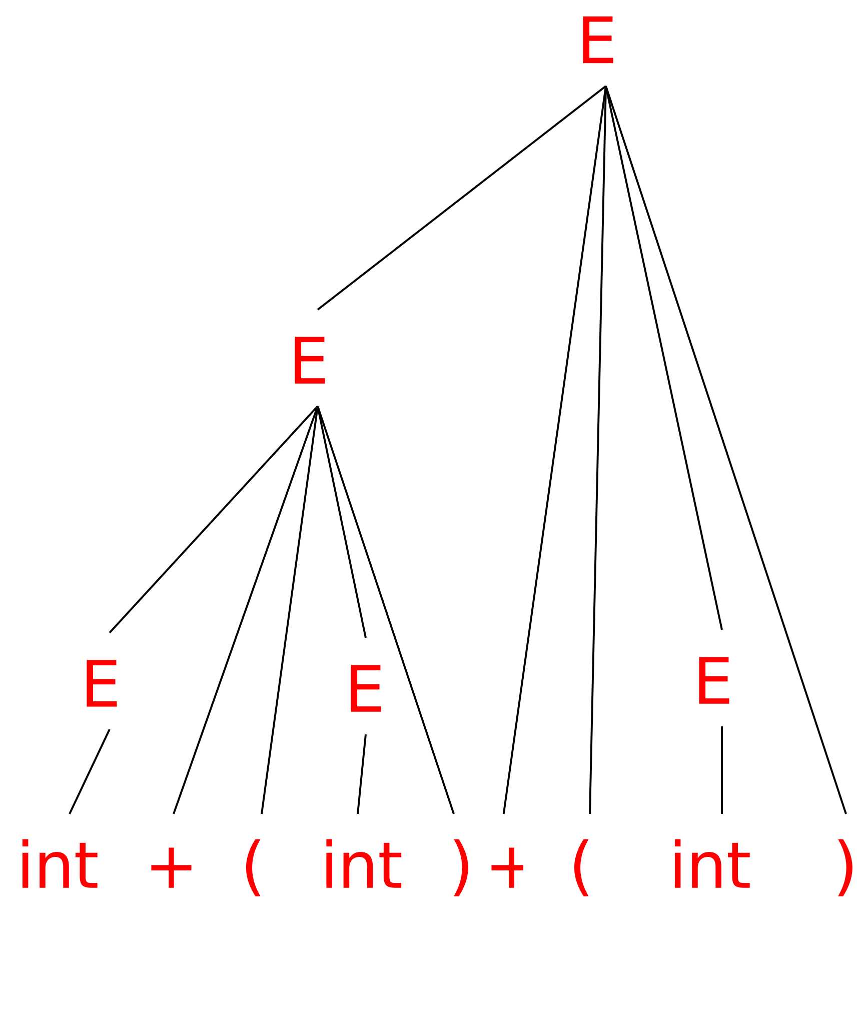
Shift-Reduce Example

▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	red. $E \rightarrow$ int
E ▶ + (int) + (int)$	shift 3 times
E + (int ▶) + (int)$	red. $E \rightarrow$ int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	red. $E \rightarrow E$ + (E)
E ▶ + (int)$	shift 3 times
E + (int ▶)$	red. $E \rightarrow$ int
E + (E ▶)$	shift
E + (E) ▶ $	red. $E \rightarrow E$ + (E)



Shift-Reduce Example

▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	red. $E \rightarrow$ int
E ▶ + (int) + (int)$	shift 3 times
E + (int ▶) + (int)$	red. $E \rightarrow$ int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	red. $E \rightarrow E + (E)$
E ▶ + (int)$	shift 3 times
E + (int ▶)$	red. $E \rightarrow$ int
E + (E ▶)$	shift
E + (E) ▶ $	red. $E \rightarrow E + (E)$
E ▶ $	accept



The Stack

- Left string can be implemented **as a stack**
  - Top of the stack is the ▸

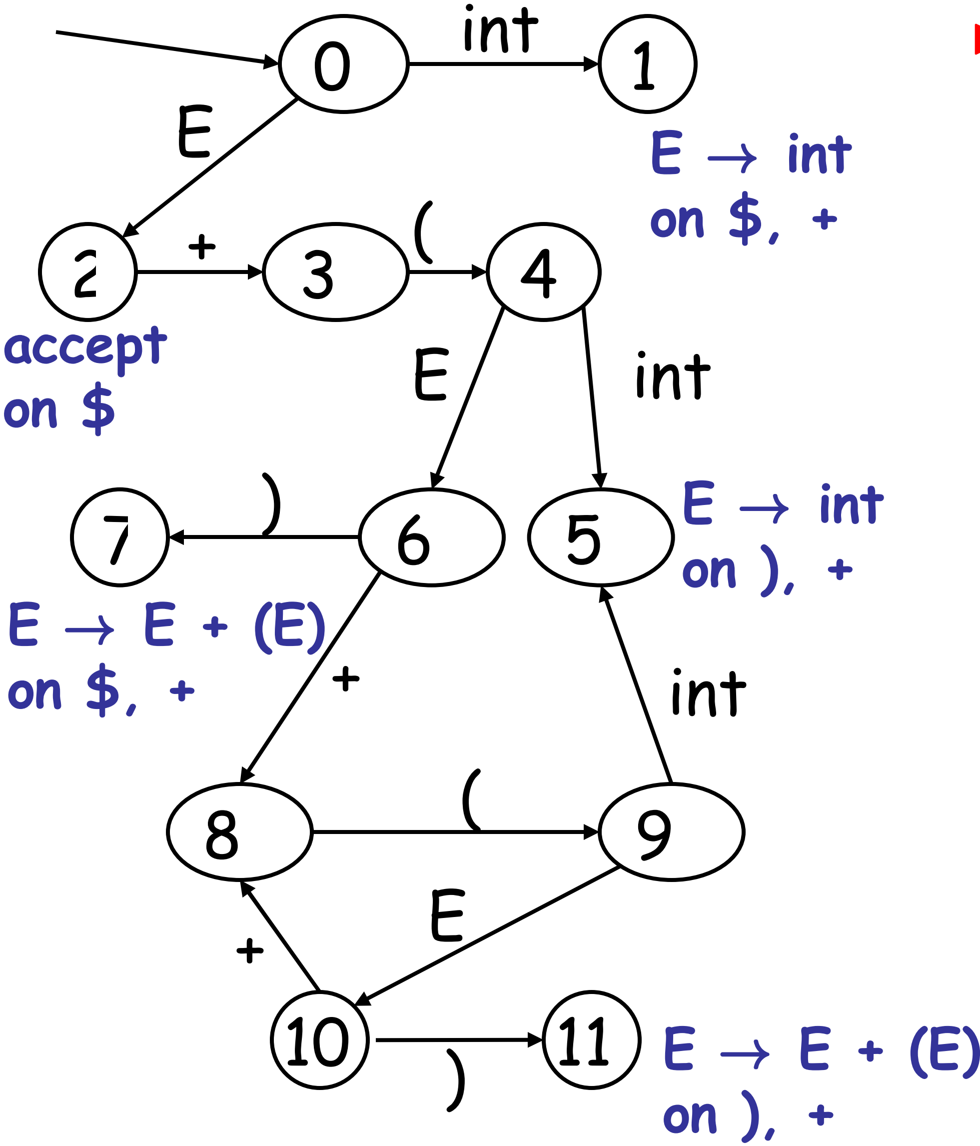
- Shift pushes a terminal on the stack

- Reduce pops 0 or more symbols from the stack (production RHS) and pushes a non-terminal on the stack (production LHS)

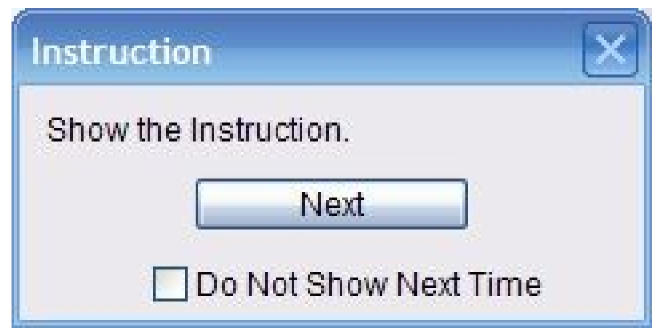
Key Issue: When to Shift or Reduce?

- Decide based on the **left string** (**the stack**)
- Idea: use a finite automaton (DFA) to decide when to shift or reduce
  - The DFA input is the stack
  - DFA language consists of terminals and nonterminals
- We run the DFA on the stack and we examine the resulting state X and the token tok after ▸
  - If X has a transition labeled **tok** then **shift**
  - If X is labeled with "**$A \rightarrow \beta$ on tok**" then **reduce**

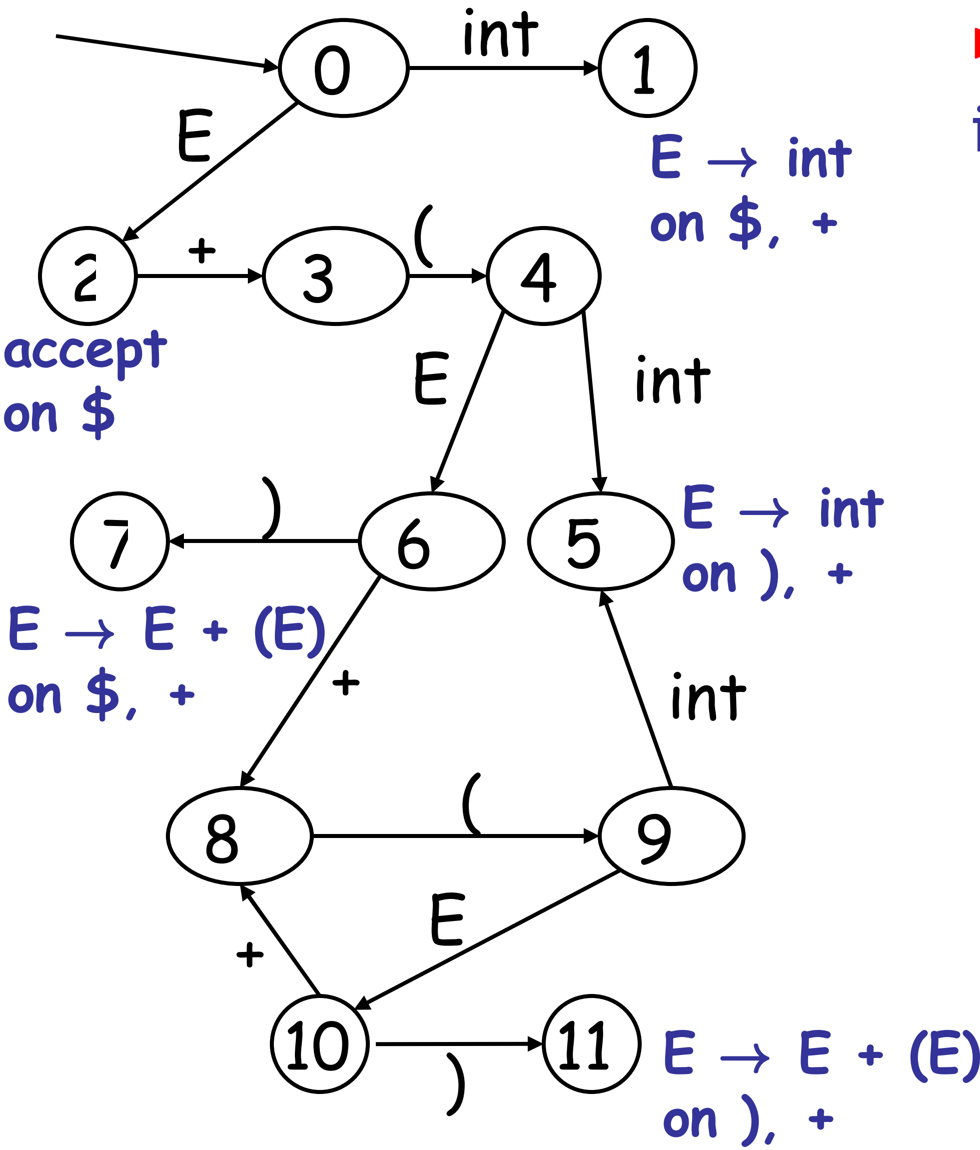
LR(1) Parsing Example



- int + (int) + (int)$



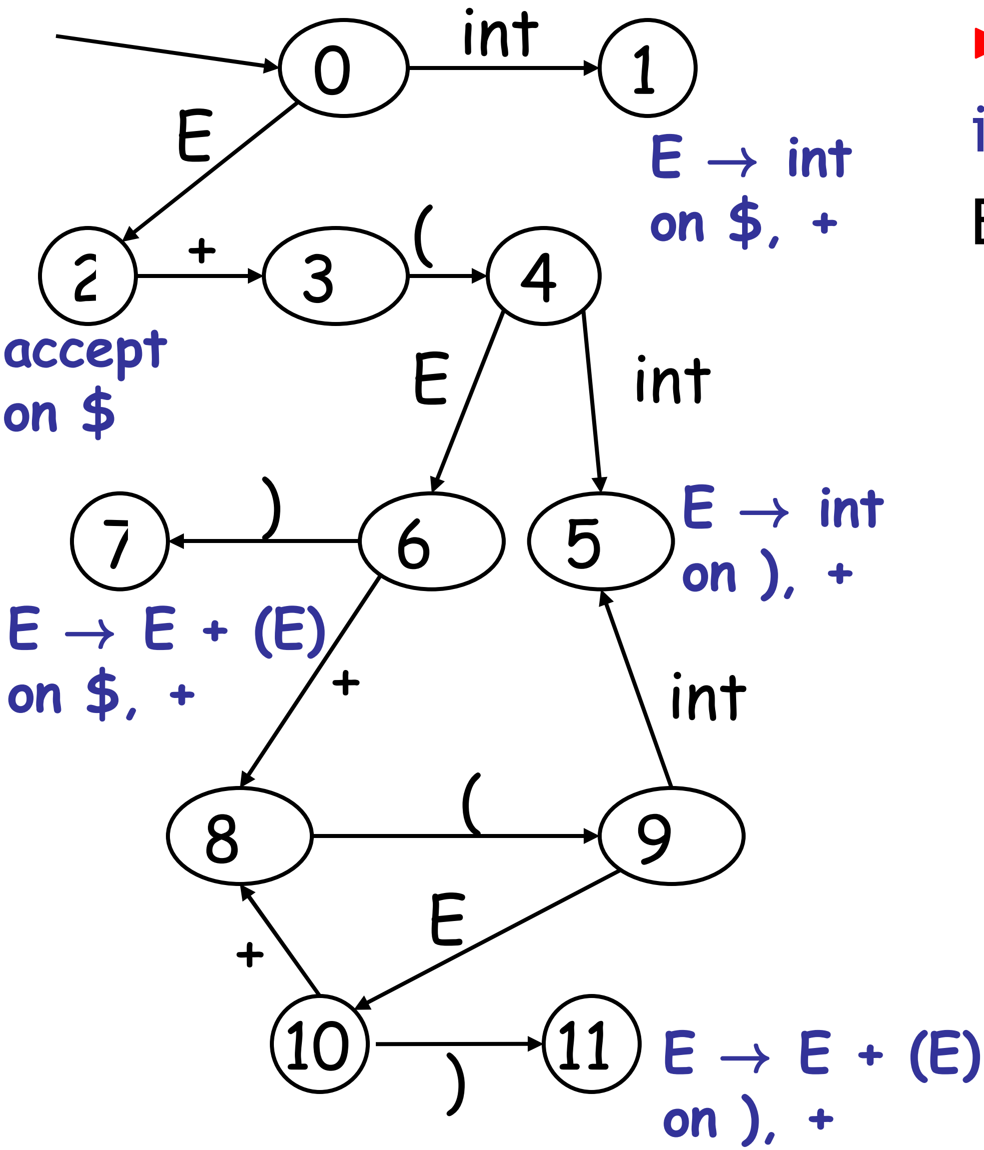
LR(1) Parsing Example



▶ int + (int) + (int)$ shift

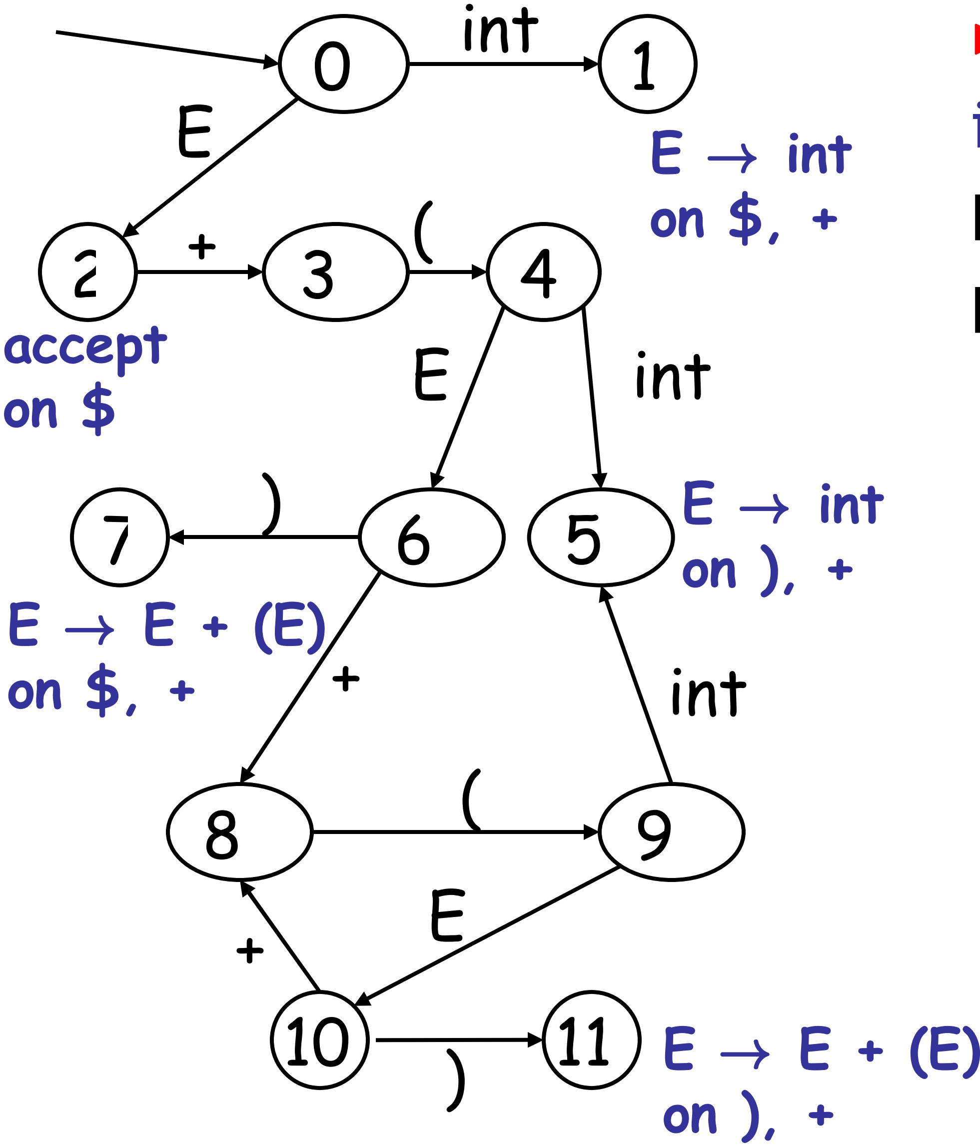
int ▶ + (int) + (int)$

LR(1) Parsing Example



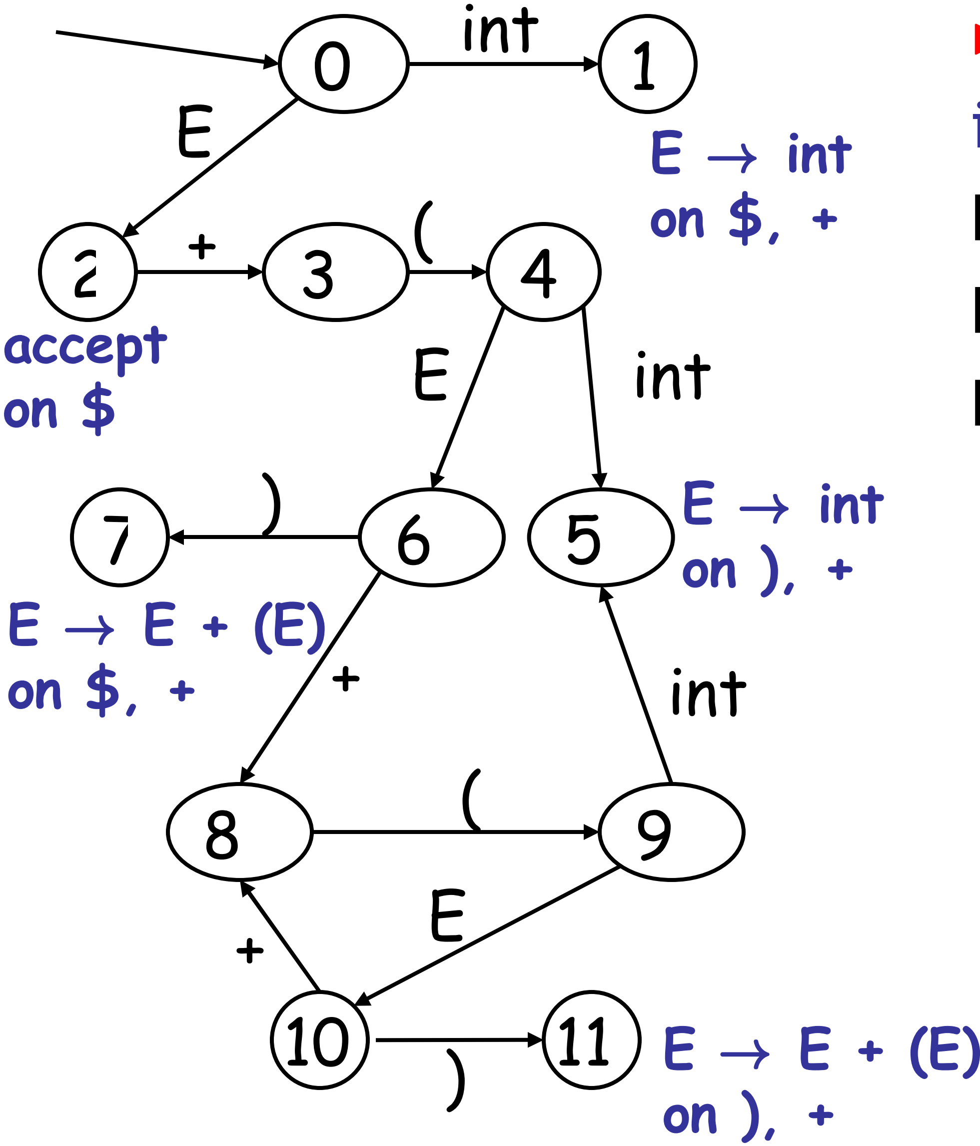
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	

LR(1) Parsing Example



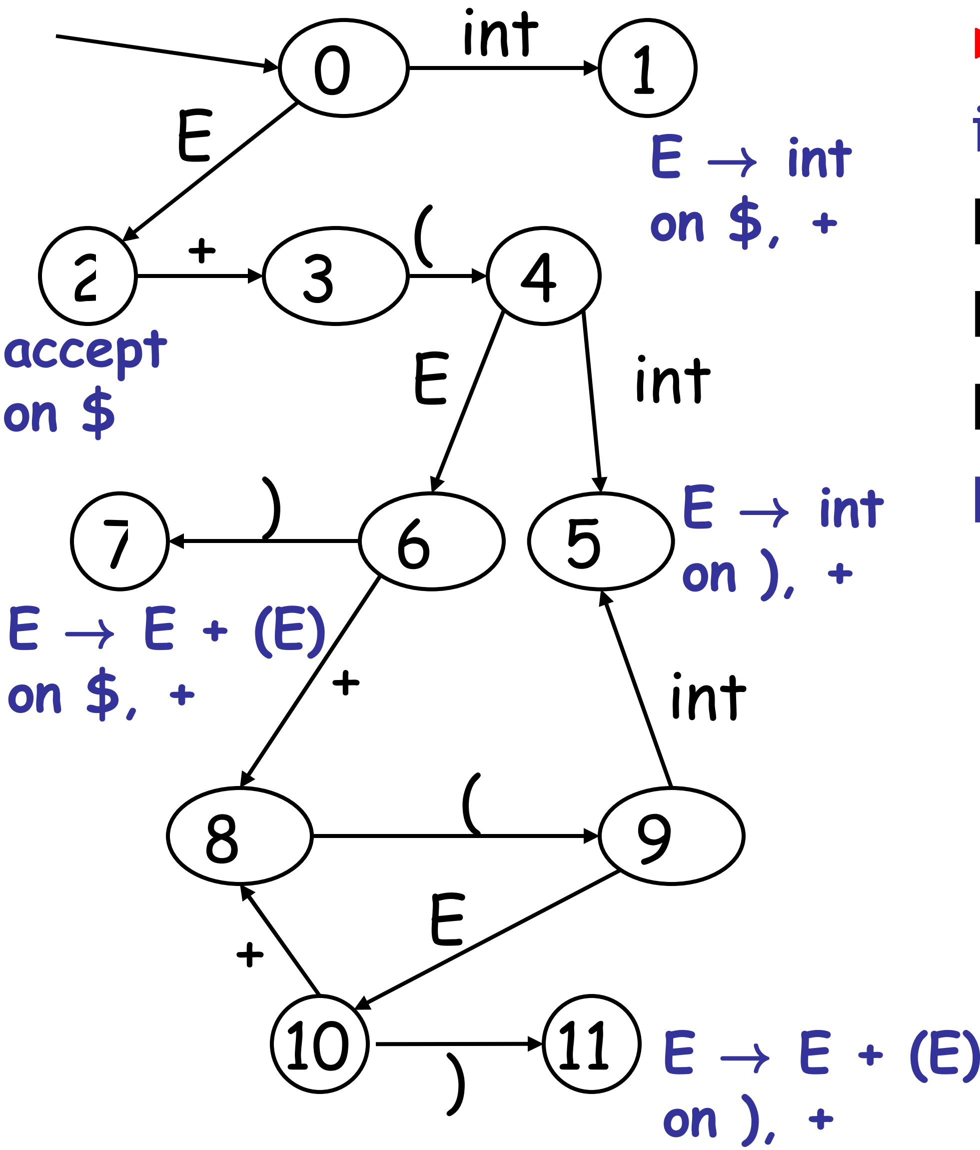
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	

LR(1) Parsing Example



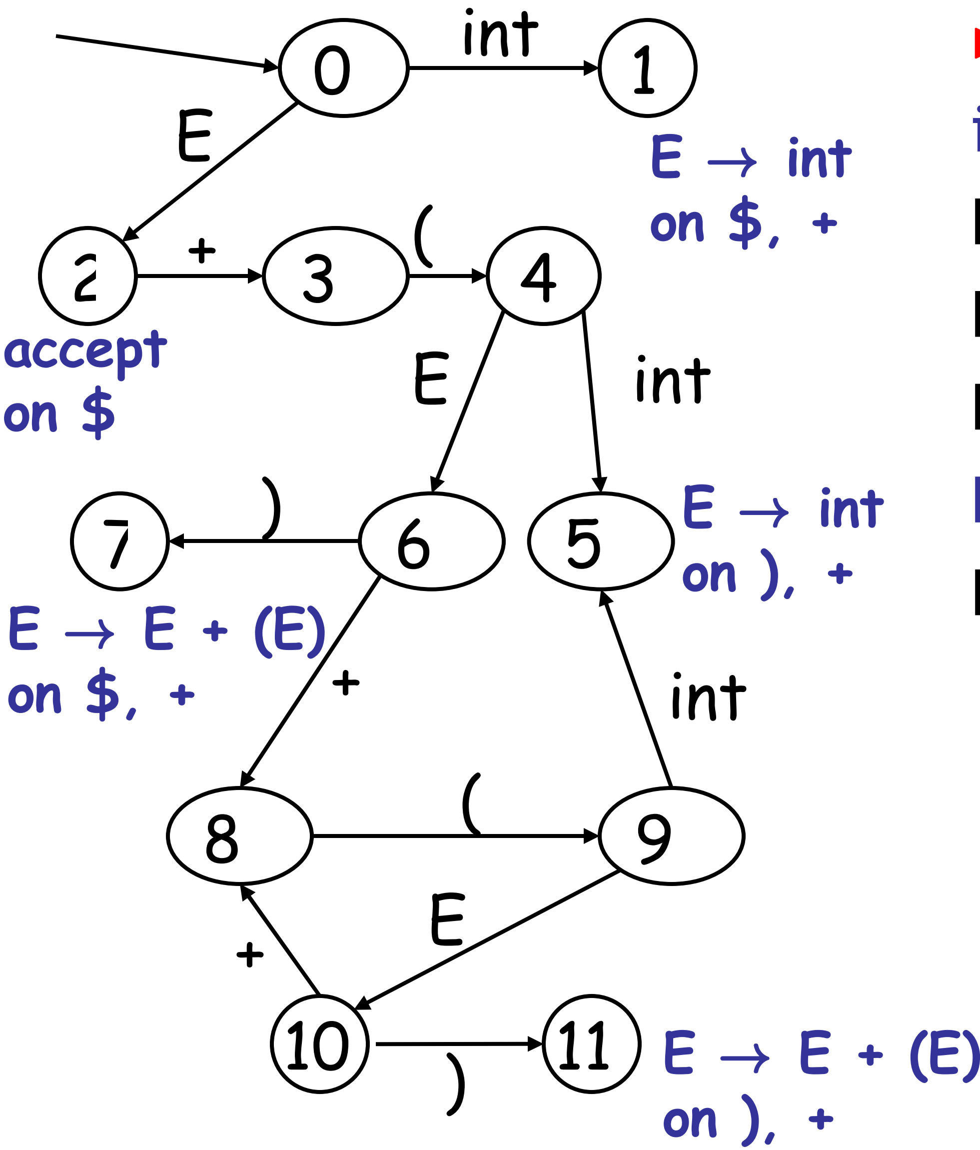
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	

LR(1) Parsing Example



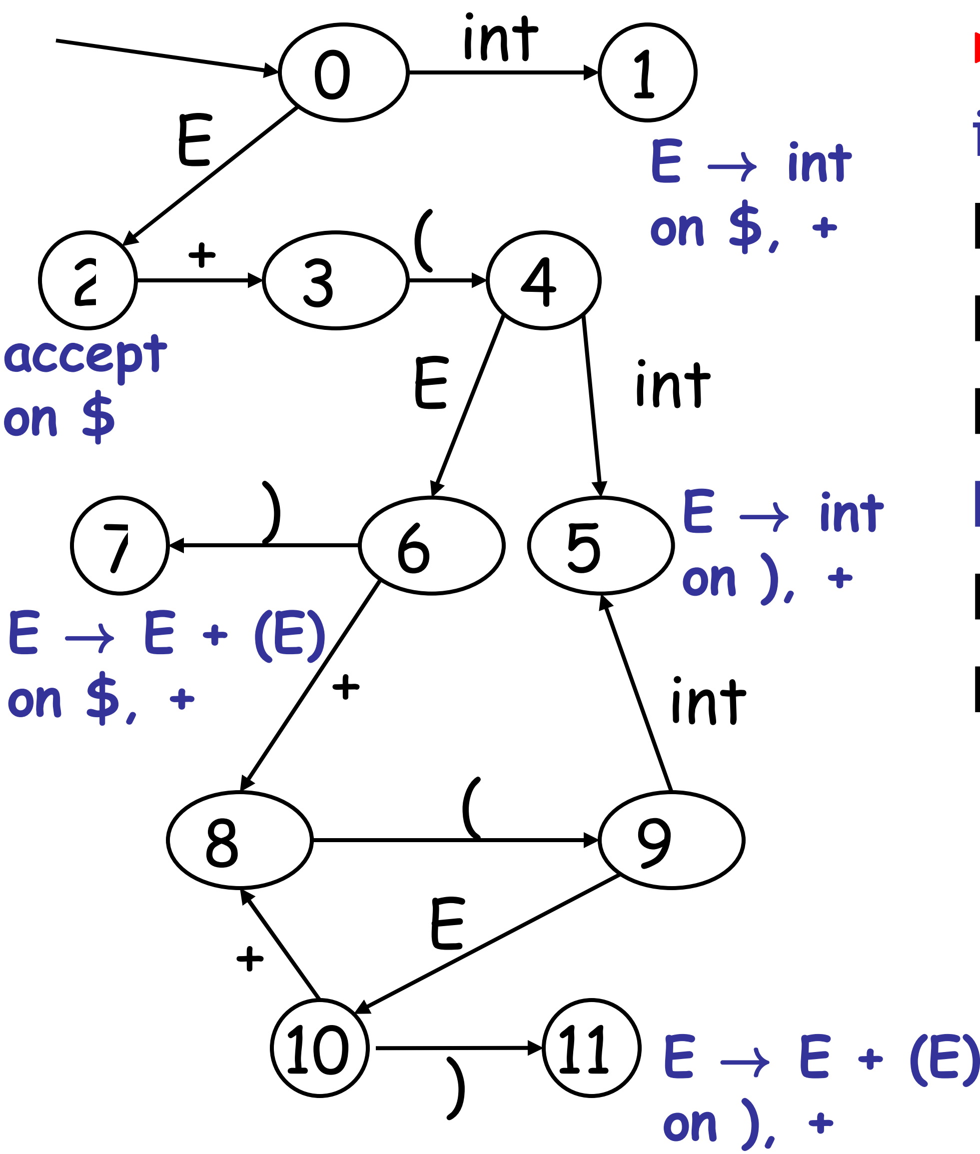
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	

LR(1) Parsing Example



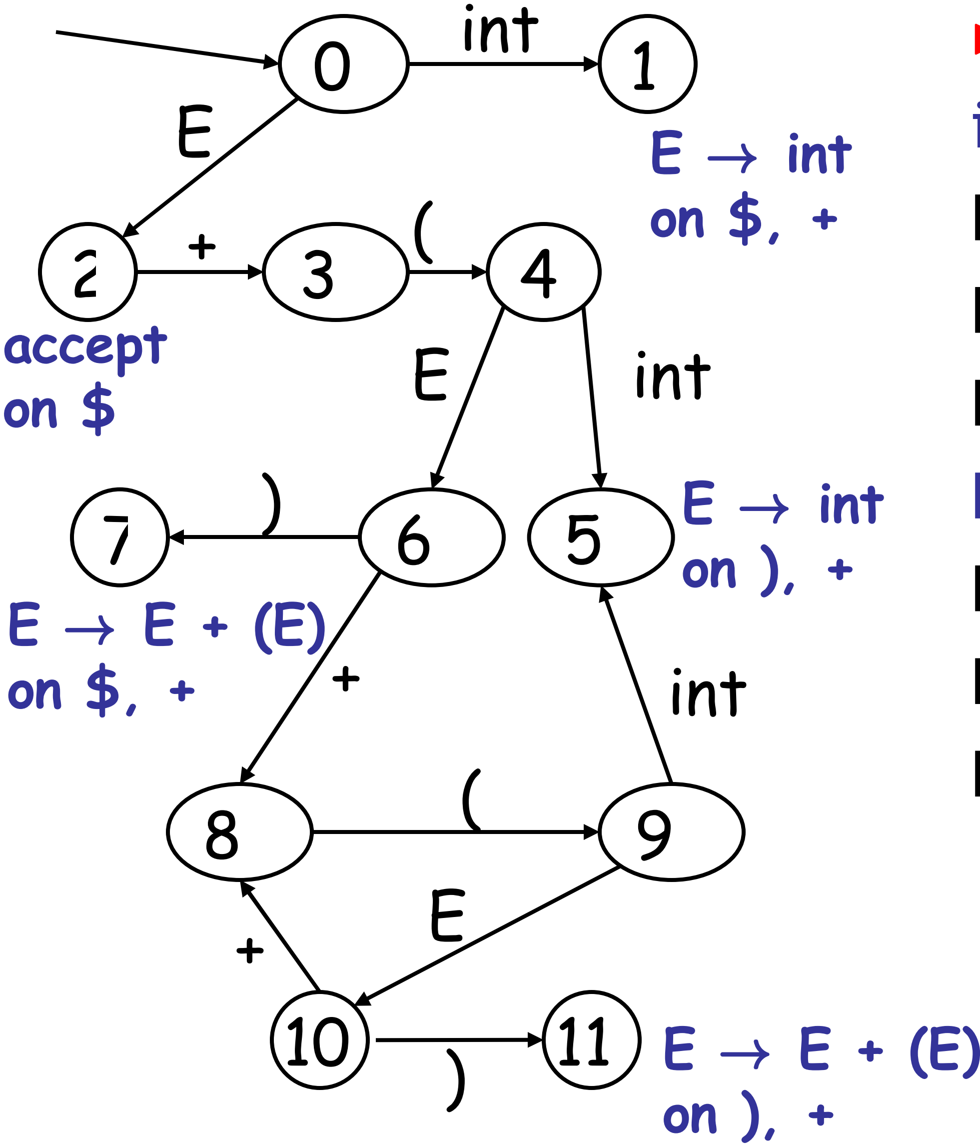
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	E → E+(E)
E ▶ + (int)$	

LR(1) Parsing Example



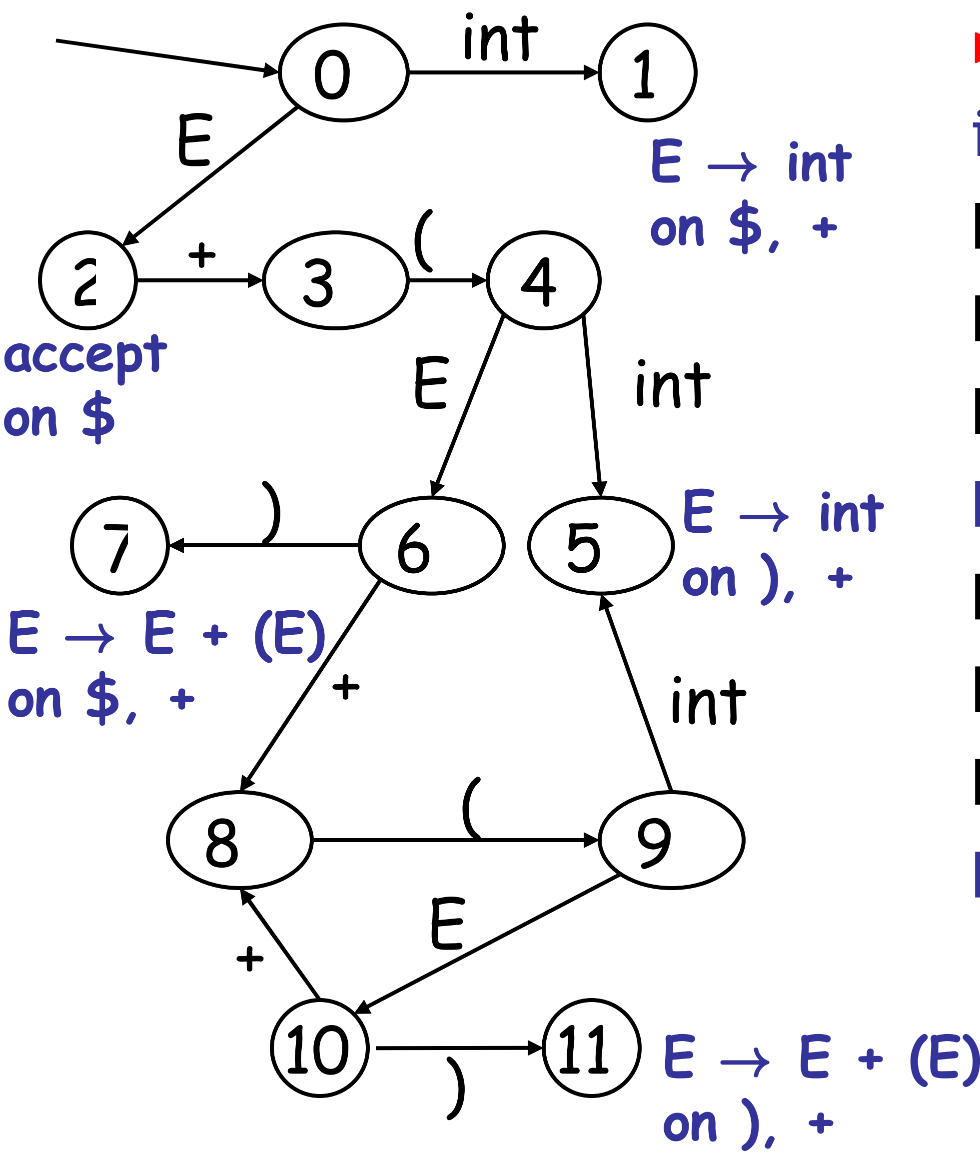
► int + (int) + (int)$	shift
int ► + (int) + (int)$	E → int
E ► + (int) + (int)$	shift(x3)
E + (int ►) + (int)$	E → int
E + (E ►) + (int)$	shift
E + (E) ► + (int)$	E → E+(E)
E ► + (int)$	shift (x3)
E + (int ►)$	

LR(1) Parsing Example



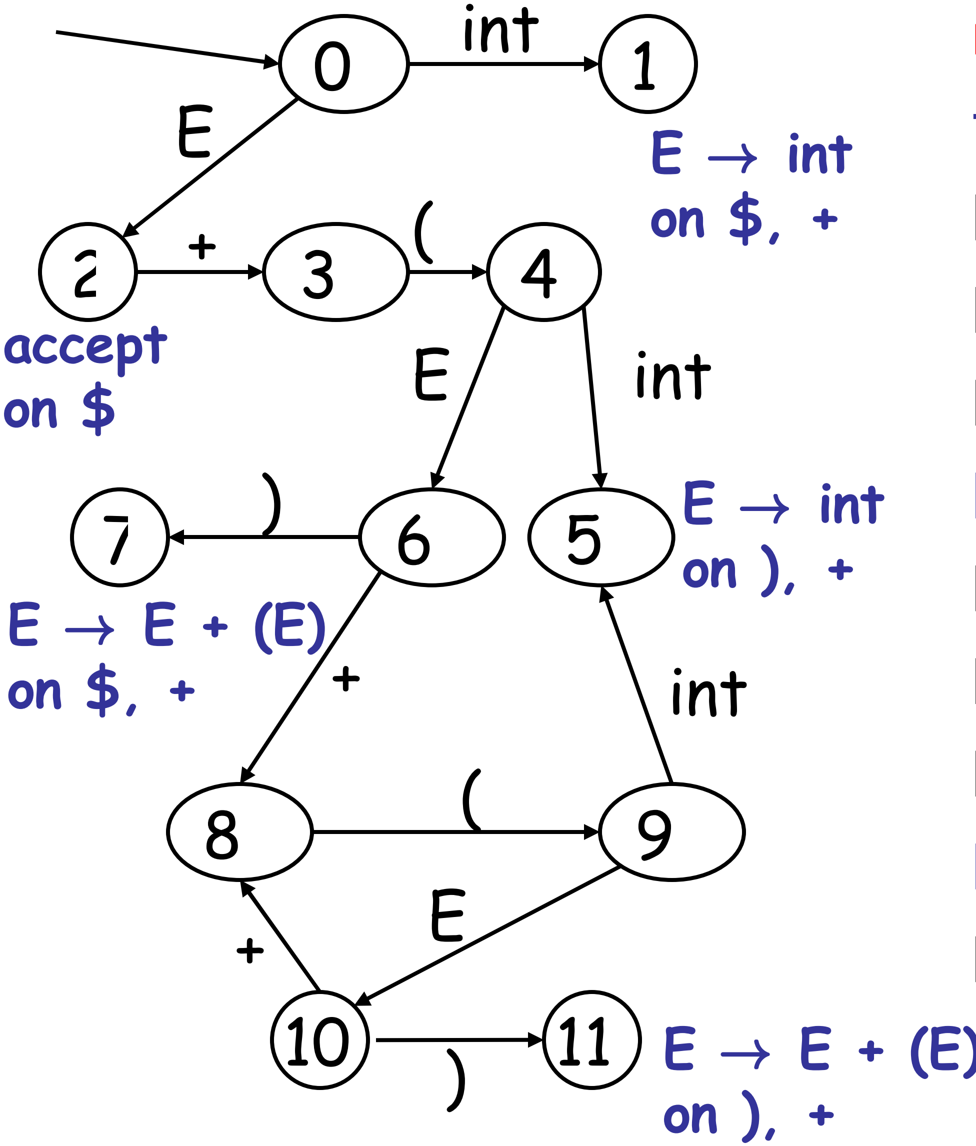
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	E → E+(E)
E ▶ + (int)$	shift (x3)
E + (int ▶)$	E → int
E + (E ▶)$	

LR(1) Parsing Example



▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	E → E+(E)
E ▶ + (int)$	shift (x3)
E + (int ▶)$	E → int
E + (E ▶)$	shift
E + (E) ▶ $	

LR(1) Parsing Example



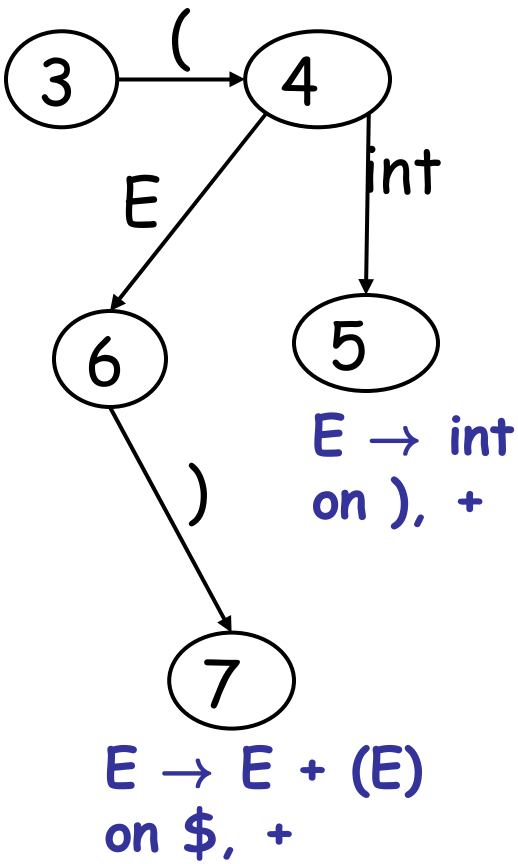
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	E → E+(E)
E ▶ + (int)$	shift (x3)
E + (int ▶)$	E → int
E + (E ▶)$	shift
E + (E) ▶ $	E → E+(E)
E ▶ $	accept

Representing the DFA

- Parsers represent the DFA as a 2D table
  - Recall table-driven lexical analysis
- Lines (rows) correspond to DFA states
- Columns correspond to terminals and non-terminals
- Typically columns are split into:
  - Those for terminals: **action table**
  - Those for non-terminals: **goto table**

Representing the DFA. Example

- The table for a fragment of our DFA:



	int	+	(	)	$	E
…						
3			s4			
4	s5					g6
5		$r_{E\to\ int}$		$r_{E\to\ int}$		
6	s8		s7			
7		$r_{E\to\ E+(E)}$			$r_{E\to\ E+(E)}$	
…						

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated
- Optimization: remember for each stack element to which state it brings the DFA
- LR parser maintains a stack

$$\langle\ \mathbf{sym}_1,\ \mathbf{state}_1\ \rangle\ .\ .\ .\ \langle\ \mathbf{sym}_n,\ \mathbf{state}_n\ \rangle$$

state_k is the final state of the DFA on $\text{sym}_1 \ldots \text{sym}_k$

The LR Parsing Algorithm

Let S = w$ be initial input

Let j = 0

Let DFA state 0 be the start state

Let stack = < dummy, 0 >

repeat

match action[top_state(stack), S[j]] with

| shift k: push < S[j++], k >

| reduce $X \to \alpha$:

pop $|\alpha|$ pairs,

push < X, Goto[top_state(stack), X] >

| accept: halt normally

| error: halt and report error

LR Parsing Notes

- Can be used to parse more grammars than LL
- Most PL grammars are LR
- Can be described as a simple table
- There are tools for building the table
  - Often called “yacc” or “bison”

- How is the table constructed? Next time!

Homework

- Tuesday: WA2 due
- Tuesday: Read 2.3.4-2.3.5, 2.4.2-2.4.3
- **Wednesday Feb 27: PA3 due**
  - Parsing!
  - When should I start? Early or late?