

## Self-Test Answers

- Are practical parsers and scanners based on deterministic or non-deterministic automata?
- Deterministic
- Half credit: "they are equivalent" w/o "convert to DFA"
- How can regular expressions be used to specify nested constructs?
- They cannot. (Scott Book 2.1.2, second sentence)
- Nested parentheses $\left({ }^{\mathrm{n}}\right)^{\mathrm{n}}$ are the canonical example.
- How is a two-dimensional transition table used in tabledriven scanning?
- new_state = table[old_state][new_character]


## Administrivia

- Number of students submitting PA- $X$ to a PA- $Y$ area: 5 / 39 = 13\%
- This is a 4000-level class.
- (We could turn off submissions after the due date: this yields a zero-tolerance late policy.)
- (We could turn off submissions before the due date: this means students cannot work ahead.)
- PA1 average: $43.8 / 50=88 \%$
- Excellent work, class!
- Real time grade-checking demo.


## Cunning Plan

- Formal Languages
- Regular Languages Revisited
- Parser Overview
- Context-Free Grammars (CFGs)
- Derivations
- Ambiguity


## One-Slide Summary

- A parser takes a sequence of tokens as input. If the input is valid, it produces a parse tree (or derivation).
- Context-free grammars are a notation for specifying formal languages. They contain terminals, non-terminals and productions (aka rewrite rules).


## Languages and Automata

- Formal languages are very important in CS
- Especially in programming languages
- Regular languages
- The weakest formal languages widely used
- Many applications
- We will also study context-free languages
- A "stronger" type of formal language


## Limitations of Regular Languages

- Intuition: A finite automaton that runs long enough must repeat states
- Pigeonhole Principle: imagine 20 states and 300 input characters
- A finite automaton can't remember how often it has visited a particular state
- Only enough to store in which state it is in
- Cannot count, except up to a finite limit
- Language of balanced parentheses is not regular: $\left\{\left(^{i}\right)^{i} \mid i \geq 0\right\}$
- http://en.wikipedia.org/wiki/Pumping_lemma_for_regular_languages


## The Functionality of the Parser

- Input: sequence of tokens from lexer
- e.g., the .cl-lex files you make in PA2
- Output: parse tree of the program
- Also called an abstract syntax tree
- Output: error if the input is not valid
- e.g., "parse error on line 3"


## Example

- Cool program text

$$
\text { if } x=y \text { then } 1 \text { else } 2 \text { fi }
$$

- Parser input (tokens)

IF ID = ID THEN INT ELSE INT FI

- Parser output (tree)

IF-THEN-ELSE


## Comparison: Lexical Analysis

| Phase | Input | Output |
| :--- | :--- | :--- |
| Lexer | Sequence of characters | Sequence of tokens |
| Parser | Sequence of tokens | Parse tree |

Weather: Five Day Forecast
Plan your week with this forecast from the Jersey Meteorological Department.
See also: • Today's Weather • UV Index


## The Role of the Parser

- Not all sequences of tokens are programs
- then x * + 3 while x ; y z then
- The parser must distinguish between valid and invalid sequences of tokens
- We need
- A language or formalism to describe valid sequences of tokens
- A method (an algorithm) for distinguishing valid from invalid sequences of tokens


## Programming Language Structure

- Programming languages have recursive structure
- Consider the language of arithmetic expressions with integers, +, *, and ( )
- An expression is either:
- an integer
- an expression followed by "+" followed by expression
- an expression followed by "*" followed by expression
- a '(' followed by an expression followed by ')'
- int , int + int , ( int + int) * int are expressions


## Notation for

## Programming Languages

- An alternative notation:

$$
\begin{aligned}
& \mathrm{E} \rightarrow \text { int } \\
& \mathrm{E} \rightarrow \mathrm{E}+\mathrm{E} \\
& \mathrm{E} \rightarrow \mathrm{E} \text { E } \\
& \mathrm{E} \rightarrow(\mathrm{E})
\end{aligned}
$$

- We can view these rules as rewrite rules
- We start with E and replace occurrences of E with some right-hand side
- $\mathrm{E} \rightarrow \mathrm{E}$ * $\mathrm{E} \rightarrow(\mathrm{E})^{*} \mathrm{E} \rightarrow(\mathrm{E}+\mathrm{E})^{*} \mathrm{E}$ $\rightarrow \ldots \rightarrow(\text { int }+ \text { int })^{*}$ int


## Observation

- All arithmetic expressions can be obtained by a sequence of replacements
- Any sequence of replacements forms a valid arithmetic expression
- This means that we cannot obtain ( int ) )
by any sequence of replacements. Why?
- This notation is a context-free grammar


## Context Free Grammars

- A context-free grammar consists of
- A set of non-terminals $N$
- Written in uppercase in these notes
- A set of terminals T
- Lowercase or punctuation in these notes
- A start symbol S (a non-terminal)
- A set of productions (rewrite rules)
- Assuming $\mathrm{E} \in \mathrm{N}$

$$
\begin{array}{ll}
E \rightarrow \varepsilon & \text {, or } \\
E \rightarrow Y_{1} Y_{2} \ldots Y_{n} & \text { where } \quad Y_{i} \subseteq N \cup T
\end{array}
$$

## Examples of CFGs

Simple arithmetic expressions:

$$
\begin{aligned}
& E \rightarrow \text { int } \\
& E \rightarrow E+E \\
& E \rightarrow E * E \\
& E \rightarrow(E)
\end{aligned}
$$

- One non-terminal: E
- Several terminals: int + * ( )
- Called terminals because they are never replaced
- By convention the non-terminal for the first production is the start symbol


## The Language of a CFG

Read productions as replacement rules:
$X \rightarrow Y_{1} \ldots Y_{n}$
Means $X$ can be replaced by $Y_{1} \ldots Y_{n}$
$X \rightarrow \varepsilon$
Means $X$ can be erased or eaten (replaced with empty string)


## Key Idea

To construct a valid sequence of terminals:

- Begin with a string consisting of the start symbol " $S$ "
- Replace any non-terminal $X$ in the string by a right-hand side of some production

$$
X \rightarrow Y_{1} \ldots Y_{n}
$$

- Continue replacing until there are only terminals in the string


## The Language of a CFG: $\rightarrow$

More formally, write

$$
\begin{gathered}
X_{1} \ldots X_{i-1} X_{i} X_{i+1} \ldots X_{n} \\
\xrightarrow{\rightarrow} \\
X_{1} \ldots X_{i-1} Y_{1} \ldots Y_{m} X_{i+1} \ldots X_{n}
\end{gathered}
$$

if there is a production

$$
X_{i} \rightarrow Y_{1} \ldots Y_{m}
$$

## The Language of a CFG: $\rightarrow^{*}$

## Write

$$
X_{1} \ldots X_{n} \rightarrow Y_{1} \ldots Y_{m}
$$

if

$$
X_{1} \ldots X_{n} \rightarrow \ldots \rightarrow \ldots \rightarrow Y_{1} \ldots Y_{m}
$$

## in 0 or more steps.



You've been warned 3 times that this file does not exist. Now you've made us catch this worthless exception and we're upset. Do not do this again.

> OK

## The Language of a CFG

Let $G$ be a context-free grammar with start symbol $S$. Then the language of $G$ is:

$$
\begin{aligned}
L(G)=\left\{a_{1} \ldots a_{n} \mid\right. & S \rightarrow{ }^{*} a_{1} \ldots a_{n} \text { and } \\
& \text { every } \left.a_{i} \text { is a terminal }\right\}
\end{aligned}
$$

$\mathrm{L}(\mathrm{G})$ is a set of strings over the alphabet of terminals.

## CFG Language Examples

- $S \rightarrow 0$ also written as $S \rightarrow 0 \mid 1$
$S \rightarrow 1$
Generates the language \{ " 0 ", " 1 " $\}$
- What about $\mathrm{S} \rightarrow 1 \mathrm{~A}$

$$
\mathrm{A} \rightarrow 0 \mid 1
$$

- What about $\mathrm{S} \rightarrow 1 \mathrm{~A}$

- What about $S \rightarrow \varepsilon \mid(S)$


## Arithmetic Example

Simple arithmetic expressions:

$$
\mathrm{E} \rightarrow \mathrm{E}+\mathrm{E}|\mathrm{E} * \mathrm{E}|(\mathrm{E}) \mid \mathrm{id}
$$

Some elements of the language:

$$
\begin{array}{l|l}
\text { id } & \text { id + id } \\
\text { (id) } & \text { id } * \text { id } \\
\text { (id) * id } & \text { id } *(i d)
\end{array}
$$



## Cool Example

A fragment of COOL:
EXPR $\rightarrow$ if EXPR then EXPR else EXPR fi while EXPR loop EXPR pool | id

## Cool Example (Cont.)

Some elements of the language
id
if id then id else id fi
while id loop id pool
if while id loop id pool then id else id
if if id then id else id fi then id else id fi

## Notes

The idea of a CFG is a big step. But:

- Membership in a language is "yes" or "no"
- we also need parse tree of the input
- We must handle errors gracefully
- Need an implementation of CFGs
- bison, yacc, ocamlyacc, ply, etc.


## More Notes

- Form of the grammar is important
- Many grammars generate the same language
- Give me an example.
- Automatic tools are sensitive to the grammar
- Note: Tools for regular languages (e.g., flex) are also sensitive to the form of the regular expression, but this is rarely a problem in practice


## Derivations and Parse Trees

A derivation is a sequence of productions $\mathrm{S} \rightarrow \ldots \rightarrow \ldots$

A derivation can be drawn as a tree

- Start symbol is the tree's root
- For a production $X \rightarrow Y_{1} \ldots Y_{n}$ add children $Y_{1}, \ldots, Y_{n}$ to node $X$


## Derivation Example

- Grammar $\quad \mathrm{E} \rightarrow \mathrm{E}+\mathrm{E}|\mathrm{E} * \mathrm{E}|(\mathrm{E}) \mid$ id
- String id * id + id
- We're going to build a derivation!


## Derivation Example (Cont.)

E
$\rightarrow \quad \mathrm{E}+\mathrm{E}$
$\rightarrow$ E*E+E
$\rightarrow \quad \mathrm{id} * \mathrm{E}+\mathrm{E}$
$\rightarrow \quad \mathrm{id} * \mathrm{id}+\mathrm{E}$
$\rightarrow \quad \mathrm{id} * \mathrm{id}+\mathrm{id}$

## Derivation in Detail (1)

E

E

## Derivation in Detail (2)



## E

$\rightarrow \quad \mathrm{E}+\mathrm{E}$

## Derivation in Detail (3)



## Derivation in Detail (4)



## Derivation in Detail (5)



## Derivation in Detail (6)



## Notes on Derivations

- A parse tree has
- Terminals at the leaves
- Non-terminals at the interior nodes
- A left-right traversal of the leaves is the original input
- The parse tree shows the association of operations, the input string does not!
Q: Radio (117 / 842)
- This NPR radio show features Tom and Ray Magliozzi as Click and Clack the Tappet Brothers. It includes Boston accents, a weekly "Puzzler", and is brought to you in part by "Paul Murky of Murky Research" and the law firm of "Dewey, Cheetham and Howe".


## Q: Games (548 / 842)

- In the card game Hearts, how many points would you accumulate if you ended up with all of the Queens and all of the Jacks? (Standard scoring.)


## Q: Music (158 / 842)

-Name the "shocking" 1976 line dance created by Ric Silver and sung and choreographed by Marcia Griffiths.

## Q: Games (582 / 842)

- Minty, Snuzzle, Butterscotch, Bluebelle, Cotton Candy, and Blossom were the first six characters in the 1982 edition of this series of Hasbro-produced brushable hoofed dolls with designs on their hips.


## Left-most and Right-most Derivations

- The example here is a leftmost derivation
- At each step, replace the leftmost non-terminal

$$
\begin{array}{ll} 
& \mathrm{E} \\
\rightarrow & \mathrm{E}+\mathrm{E} \\
\rightarrow & \mathrm{E}+\mathrm{id} \\
\rightarrow & \mathrm{E} * \mathrm{E}+\mathrm{id} \\
\rightarrow & \mathrm{E} * \mathrm{id}+\mathrm{id} \\
\rightarrow & \mathrm{id} * \mathrm{id}+\mathrm{id}
\end{array}
$$

- There is an equivalent notion of a right-most derivation
- Not shown on this slide.
- But coming up next!


## Right-most Derivation in Detail <br> (1)

$E$

E

## Right-most Derivation in Detail (2)



E
$\rightarrow \quad \mathrm{E}+\mathrm{E}$

## Right-most Derivation in Detail (3)



## Right-most Derivation in Detail <br> (4)


$\rightarrow \mathrm{E} * \mathrm{E}+\mathrm{id}$

## Right-most Derivation in Detail (5)

E<br>$\rightarrow \quad \mathrm{E}+\mathrm{E}$<br>$\rightarrow \mathrm{E}+\mathrm{id}$<br>$\rightarrow \mathrm{E} * \mathrm{E}+\mathrm{id}$<br>$\rightarrow$ E*id +id



## Right-most Derivation in Detail <br> (6)



## Derivations and Parse Trees

- Note that for each parse tree there is a left-most and a right-most derivation
- The difference is the order in which branches are added
- We will start with a parsing technique that yields left-most derivations
- Later we'll move on to right-most derivations


## Summary of Derivations

- We are not just interested in whether

$$
s \in L(G)
$$

- We also need a parse tree for $s$
- A derivation defines a parse tree
- But one parse tree may have many derivations
- Left-most and right-most derivations are important in parser implementation


## Review

- A parser consumes a sequence of tokens $s$ and produces a parse tree
- Issues:
- How do we recognize that $s \in L(G)$ ?
- A parse tree of $s$ describes how $s \in L(G)$
- Ambiguity: more than one parse tree (interpretation) for some string s
- Error: no parse tree for some string s
- How do we construct the parse tree?


## Ambiguity

- (Ambiguous) Grammar G

$$
E \rightarrow E+E|E * E|(E) \mid \text { int }
$$

- Some strings in $L(G)$

$$
\begin{aligned}
& \text { int + int + int } \\
& \text { int * int + int }
\end{aligned}
$$



## Ambiguity. Example

The string int + int + int has two parse trees


+ is left-associative


## Ambiguity. Example

The string int * int + int has two parse trees


* has higher precedence than +


## Ambiguity (Cont.)

- A grammar is ambiguous if it has more than one parse tree for some string
- Equivalently, there is more than one rightmost or left-most derivation for some string
- Ambiguity is bad
- Leaves meaning of some programs ill-defined
- Ambiguity is common in programming languages
- Arithmetic expressions
- IF-THEN-ELSE


## Dealing with Ambiguity

- There are several ways to handle ambiguity
- Most direct method is to rewrite the grammar unambiguously

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{E}+\mathrm{T} \mid \mathrm{T} \\
& \left.\mathrm{~T} \rightarrow \mathrm{~T}^{*} \text { int | int \| ( } \mathrm{E}\right)
\end{aligned}
$$

- Enforces precedence of * over +
- Enforces left-associativity of + and *


## Ambiguity. Example

The int * int + int has ony one parse tree



## Ambiguity: The Dangling Else

- Consider this new grammar

$$
\begin{aligned}
E \rightarrow & \text { if } E \text { then } E \\
& \text { | if } E \text { then } E \text { else } E \\
& \text { OTHER }
\end{aligned}
$$

- This grammar is also ambiguous


## The Dangling Else: Example

- The string if $E_{1}$ then if $E_{2}$ then $E_{3}$ else $E_{4}$ has two parse trees

- Typically we want the second form


## The Dangling Else: A Fix

- else matches the closest unmatched then
- We can describe this in the grammar (distinguish between matched and unmatched "then")
$\begin{array}{rlr}\mathrm{E} \rightarrow & \text { MIF } & \text { /* all } \\ \text { | UIF } & \text { /* som } \\ \text { MIF } \rightarrow \text { if } \mathrm{E} \text { then MIF else MIF }\end{array}$
| OTHER
UIF $\rightarrow$ if E then E
| if E then MIF else UIF
- Describes the same set of strings


## The Dangling Else: Example Revisited

- The expression if $E_{1}$ then if $E_{2}$ then $E_{3}$ else $\mathrm{E}_{4}$

- A valid parse tree (for a UIF)

- Not valid because the then expression is not a MIF


## Ambiguity

- No general techniques for handling ambiguity
- Impossible to convert automatically every ambiguous grammar to an unambiguous one
- Used with care, ambiguity can simplify the grammar
- Sometimes allows more natural definitions
- So we need disambiguation mechanisms
- As shown next ...


# Precedence and Associativity Declarations 

- Instead of rewriting the grammar
- Use the more natural (ambiguous) grammar
- Along with disambiguating declarations
- Most tools allow precedence and associativity declarations to disambiguate grammars
- Examples ...


## Associativity Declarations

- Consider the grammar $\mathrm{E} \rightarrow \mathrm{E}+\mathrm{E} \mid$ int
- And the string: int + int + int

- Left-associativity declaration: \%left + \%left *


## Review

- We can specify language syntax using CFG
- A parser will answer whether $s \in L(G)$
- ... and will build a parse tree
- ... and pass on to the rest of the compiler
- Next:
- How do we answer $s \in L(G)$ and build a parse tree?


## Homework

- WA1 (written homework) due
- You must work alone.
- Write or print out your answers.
- Turn in before class or in drop-box (423 Rice).
- PA2 (Lexer) Due
- You may work in pairs.

