

Parking  
For  
Drive-Thru  
Service  
Only

More  
Static  
Semantics

2

Thank You



# One-Slide Summary

- **Typing rules** formalize the semantics checks necessary to validate a program. Well-typed programs do not go wrong.
- **Subtyping** relations ( $\leq$ ) and **least-upper-bounds (lub)** are powerful tools for type-checking **dynamic dispatch**.
- We will use  **$\text{SELF\_TYPE}_C$**  for “C or any subtype of C”. It will show off the **subtlety** of type systems and allow us to check methods that **return self objects**.

# Lecture Outline

- Typing Rules
- Dispatch Rules
  - Static
  - Dynamic
- SELF\_TYPE

# Assignment

What is this thing? What's  $\vdash$ ?  $\mathcal{O}$ ?  $\leq$ ?

$$\mathcal{O}(\text{id}) = T_0$$

$$\mathcal{O} \vdash e_1 : T_1$$

$$T_1 \leq T_0$$

---

$$\mathcal{O} \vdash \text{id} \leftarrow e_1 : T_1 \quad [\text{Assign}]$$

# Initialized Attributes

- Let  $O_c(x) = T$  for all attributes  $x:T$  in class  $C$ 
  - $O_c$  represents the class-wide scope
    - we “preload” the environment  $O$  with all attributes
- Attribute initialization is similar to **let**, except for the scope of names

$$O_c(\text{id}) = T_0$$

$$O_c \vdash e_1 : T_1$$

$$T_1 \leq T_0$$

$$\frac{}{O_c \vdash \text{id} : T_0 \leftarrow e_1 ;} \text{ [Attr-Init]}$$

# If-Then-Else

- Consider:  $\text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi}$
- The result can be either  $e_1$  or  $e_2$
- The dynamic type is either  $e_1$ 's or  $e_2$ 's type
- The best we can do statically is the **smallest supertype** larger than the type of  $e_1$  and  $e_2$

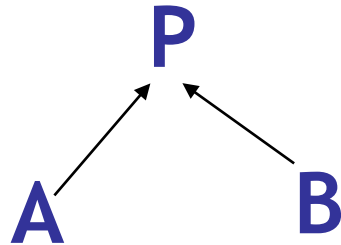
## Wayne Hart's 5-day forecast



Watch NEWS 25 for weather changes throughout the day

# If-Then-Else example

- Consider the class hierarchy



- ... and the expression  
    if ... then new A else new B fi
- Its type should allow for the dynamic type to be both A or B
  - Smallest supertype is P

# Least Upper Bounds

- Define:  $\text{lub}(X, Y)$  to be the **least upper bound** of  $X$  and  $Y$ .  $\text{lub}(X, Y)$  is  $Z$  if
  - $X \leq Z \wedge Y \leq Z$   
 $Z$  is an upper bound
  - $X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$   
 $Z$  is least among upper bounds
- In Cool, the least upper bound of two types is their **least common ancestor** in the **inheritance tree**



# If-Then-Else Revisited

$$O \vdash e_0 : \text{Bool}$$
$$O \vdash e_1 : T_1$$
$$O \vdash e_2 : T_2$$

---

$$O \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \mathbf{lub}(T_1, T_2)$$

[If-Then-Else]

# Case

- The rule for **case** expressions takes a lub over all branches

$$O \vdash e_0 : T_0$$

$$O[T_1/x_1] \vdash e_1 : T_1'$$

...

$$O[T_n/x_n] \vdash e_n : T_n'$$

[Case]

---

$$O \vdash \text{case } e_0 \text{ of } x_1:T_1 \Rightarrow e_1;$$
$$\dots; x_n : T_n \Rightarrow e_n; \text{ esac} : \text{lub}(T_1', \dots, T_n')$$

# Method Dispatch

- There is a problem with type checking method calls:

$$\frac{\begin{array}{c} \mathbf{O} \vdash \mathbf{e}_0 : \mathbf{T}_0 \\ \mathbf{O} \vdash \mathbf{e}_1 : \mathbf{T}_1 \\ \dots \\ \mathbf{O} \vdash \mathbf{e}_n : \mathbf{T}_n \end{array}}{\mathbf{O} \vdash \mathbf{e}_0.\mathbf{f}(\mathbf{e}_1, \dots, \mathbf{e}_n) : ?} \text{ [Dispatch]}$$

- We need information about the **formal parameters and return type** of **f**

# Notes on Dispatch

- In Cool, method and object identifiers live in different **name spaces**
  - A method **foo** and an object **foo** can coexist in the same scope
- In the type rules, this is reflected by a **separate** mapping **M** for method signatures:

$$M(C, f) = (T_1, \dots, T_n, T_{ret})$$

means in class **C** there is a method **f**

$$f(x_1:T_1, \dots, x_n:T_n): T_{ret}$$

# An Extended Typing Judgment

- Now we have *two* environments:  $O$  and  $M$
- The form of the typing judgment is

$$O, M \vdash e : T$$

read as: “with the assumption that the object identifiers have types as given by  $O$  and the method identifiers have signatures as given by  $M$ , the expression  $e$  has type  $T$ ”

# The Method Environment

- The method environment must be added to all rules
- In most cases,  $M$  is passed down but not actually used
  - Example of a rule that does **not** use  $M$ :

$$\frac{O, M \vdash e_1 : T_1 \quad O, M \vdash e_2 : T_2}{O, M \vdash e_1 + e_2 : \text{Int}} \text{ [Add]}$$

- Only the dispatch rules uses  $M$

# The Dispatch Rule Revisited

$O, M \vdash e_0 : T_0$

*Check receiver  
object  $e_0$*

$O, M \vdash e_1 : T_1$

...

$O, M \vdash e_n : T_n$

*Check actual  
arguments*

$M(T_0, f) = (T_1', \dots, T_n', T_{n+1}')$

*Look up formal  
argument types  $T_i'$*

$T_i \leq T_i' \quad (\text{for } 1 \leq i \leq n)$

[Dispatch]

---

$O, M \vdash e_0.f(e_1, \dots, e_n) : T_{n+1}'$

# Static Dispatch

- **Static dispatch** is a variation on normal dispatch
- The method is found in the class **explicitly named** by the programmer (not via  $e_0$ )
- The inferred type of the dispatch expression must **conform to the specified type**



# Static Dispatch (Cont.)

$$O, M \vdash e_0 : T_0$$
$$O, M \vdash e_1 : T_1$$

...

$$O, M \vdash e_n : T_n$$
$$T_0 \leq T$$
$$M(T, f) = (T_1', \dots, T_n', T_{n+1}')$$
$$T_i \leq T_i' \quad (\text{for } 1 \leq i \leq n)$$

---

$$O, M \vdash e_0 @ T.f(e_1, \dots, e_n) : T_{n+1}'$$

[Static Dispatch]

How should  
we handle  
**SELF\_TYPE** ?



# Flexibility vs. Soundness

- Recall that type systems have two conflicting goals:
  - Give **flexibility** to the programmer
  - Prevent valid programs from “going **wrong**”
    - Milner, 1981: “Well-typed programs do not go wrong”
- An active line of research is in the area of inventing more flexible type systems while preserving soundness

# Dynamic And Static Types

- The **dynamic type** of an object is ?
- The **static type** of an expression is ?
- You tell me!



# Dynamic And Static Types

- The **dynamic type** of an object is the class **C** that is used in the “**new C**” expression that created it
  - A run-time notion
  - Even languages that are not statically typed have the notion of dynamic type
- The **static type** of an expression is a notation that captures all possible dynamic types the expression could take
  - A compile-time notion

# Recall: Soundness

Soundness theorem for the Cool type system:

$$\forall E. \text{dynamic\_type}(E) \leq \text{static\_type}(E)$$

Why is this OK?

- All operations that can be used on an object of type  $C$  can also be used on an object of type  $C' \leq C$ 
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can **only add** attributes or methods
- Methods can be redefined but with same type!

# An Example

```
class Count {  
  i : int ← 0;  
  inc () : Count {  
    {  
      i ← i + 1;  
      self;  
    }  
  };  
};
```

But there is **disaster lurking** in the type system!

- Class **Count** incorporates a counter
- The **inc** method works for any subclass



# Continuing Example

- Consider a subclass **Stock** of **Count**

```
class Stock inherits Count {  
    name() : String { ...}; -- name of item  
};
```

- And the following use of **Stock**:

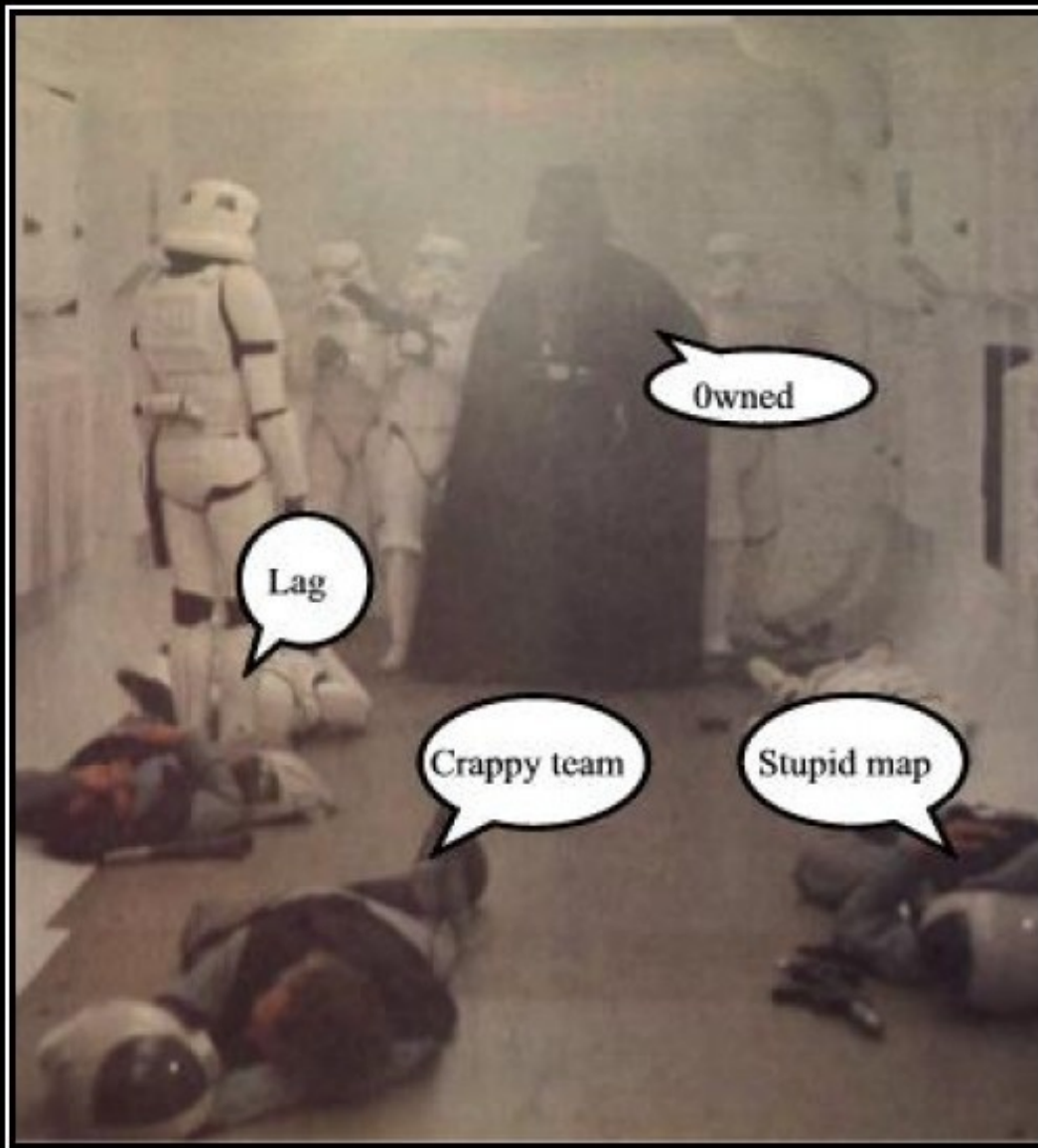
```
class Main {  
    a : Stock ← (new Stock).inc ();  
    ... a.name() ...  
};
```

**Type checking  
error !**



# Post-Mortem

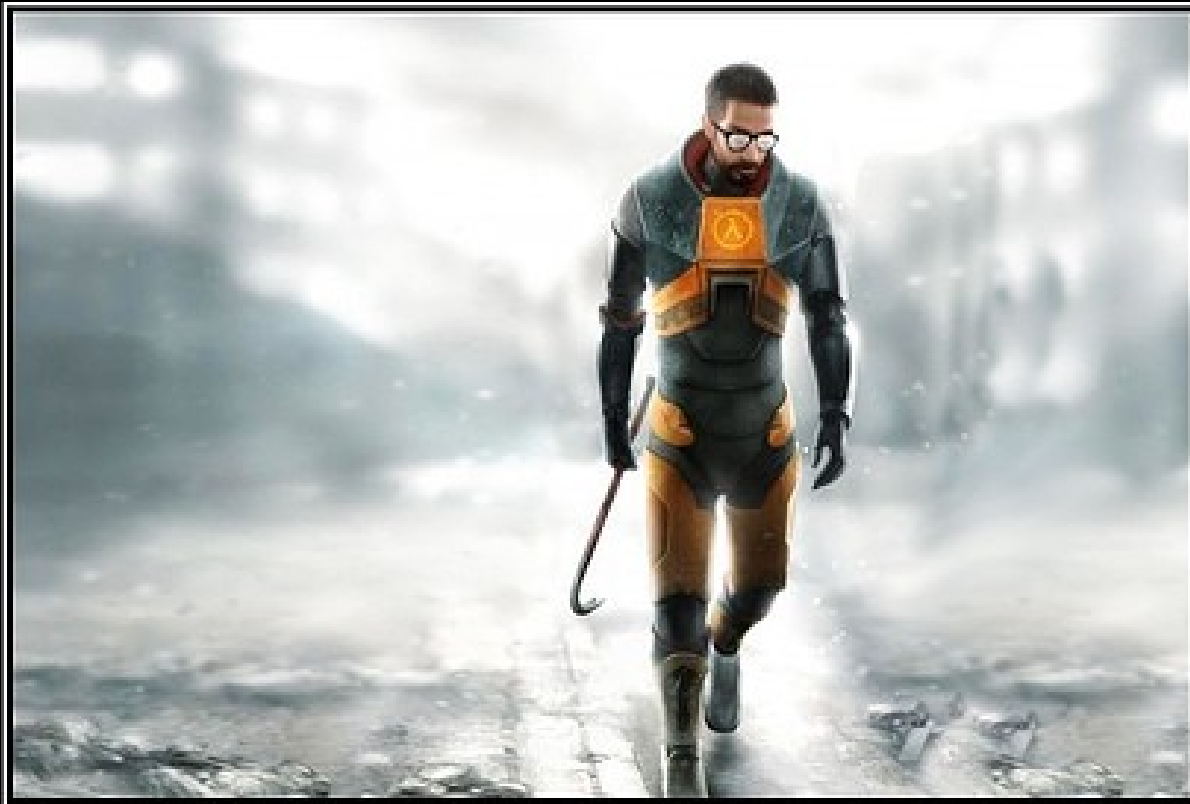
- **(new Stock).inc()** has **dynamic** type **Stock**
- So it is legitimate to write
  - a : Stock ← (new Stock).inc ()
- But this is not well-typed
  - (new Stock).inc() has **static** type **Count**
- The type checker “loses” type information
- This makes inheriting **inc** *useless*
  - So, we must redefine **inc** for each of the subclasses, with a specialized return type



# ONLINE GAMING

Get your excuses ready beforehand.  
You're going to need them.

# I Need A Hero!



## Type Systems

One tool. One million uses.

# SELF\_TYPE to the Rescue

- We will **extend the type system**
- Insight:
  - `inc` returns “`self`”
  - Therefore the return value has same type as “`self`”
  - Which could be `Count` or *any subtype of Count!*
  - In the case of `(new Stock).inc()` the type is `Stock`
- We introduce the keyword `SELF_TYPE` to use for the return value of such functions
  - We will also modify the typing rules to handle `SELF_TYPE`

# SELF\_TYPE to the Rescue (2)

- **SELF\_TYPE** allows the return type of **inc** to change when **inc** is inherited
- Modify the declaration of **inc** to read  
$$\text{inc}() : \text{SELF\_TYPE} \{ \dots \}$$
- The type checker can now prove:  
$$O, M \vdash (\text{new Count}).\text{inc}() : \text{Count}$$
$$O, M \vdash (\text{new Stock}).\text{inc}() : \text{Stock}$$
- The program from before is now well typed

# SELF\_TYPE as a Tool



- SELF\_TYPE is **not** a dynamic type
- SELF\_TYPE is a static type
- It helps the type checker to keep better track of types
- It enables the type checker to accept more correct programs
- In short, having SELF\_TYPE increases the expressive power of the type system

# SELF\_TYPE and Dynamic Types (Example)

- What can be the dynamic type of the object returned by `inc`?
  - Answer: whatever could be the type of “`self`”

```
class A inherits Count { } ;
```

```
class B inherits Count { } ;
```

```
class C inherits Count { } ;
```

(`inc` could be invoked through any of these classes)

- Answer: ***Count or any subtype of Count***

# SELF\_TYPE and Dynamic Types (Example)

- In general, if **SELF\_TYPE** appears textually in the class **C** as the declared type of **E** then it denotes the dynamic type of the “**self**” expression:

$$\text{dynamic\_type}(E) = \text{dynamic\_type}(\text{self}) \leq C$$

- Note: The meaning of **SELF\_TYPE** depends on where it appears
  - We write **SELF\_TYPE<sub>C</sub>** to refer to an occurrence of **SELF\_TYPE** in the body of **C**



# Type Checking

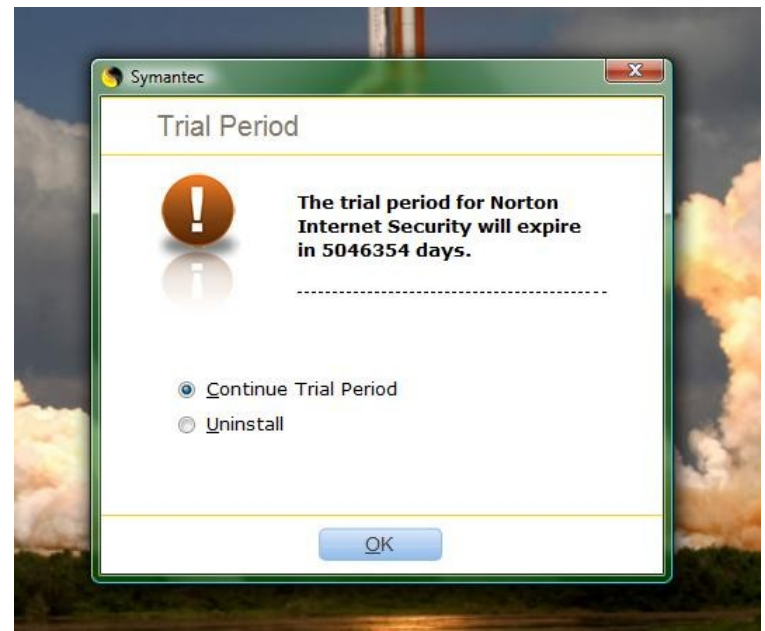
- This suggests a typing rule:

$$\mathit{SELF\_TYPE}_c \leq C$$

- This rule has an important consequence:
  - In type checking it is always safe to replace  $\mathit{SELF\_TYPE}_c$  by  $C$
- This suggests one way to handle  $\mathit{SELF\_TYPE}$  :
  - Replace all occurrences of  $\mathit{SELF\_TYPE}_c$  by  $C$
- This would be correct but it is like not having  $\mathit{SELF\_TYPE}$  at all (whoops!)

# Operations on SELF\_TYPE

- Recall the operations on types
  - $T_1 \leq T_2$       $T_1$  is a subtype of  $T_2$
  - $\text{lub}(T_1, T_2)$      the least-upper bound of  $T_1$  and  $T_2$
- We must extend these operations to handle SELF\_TYPE
- Might take some time ...

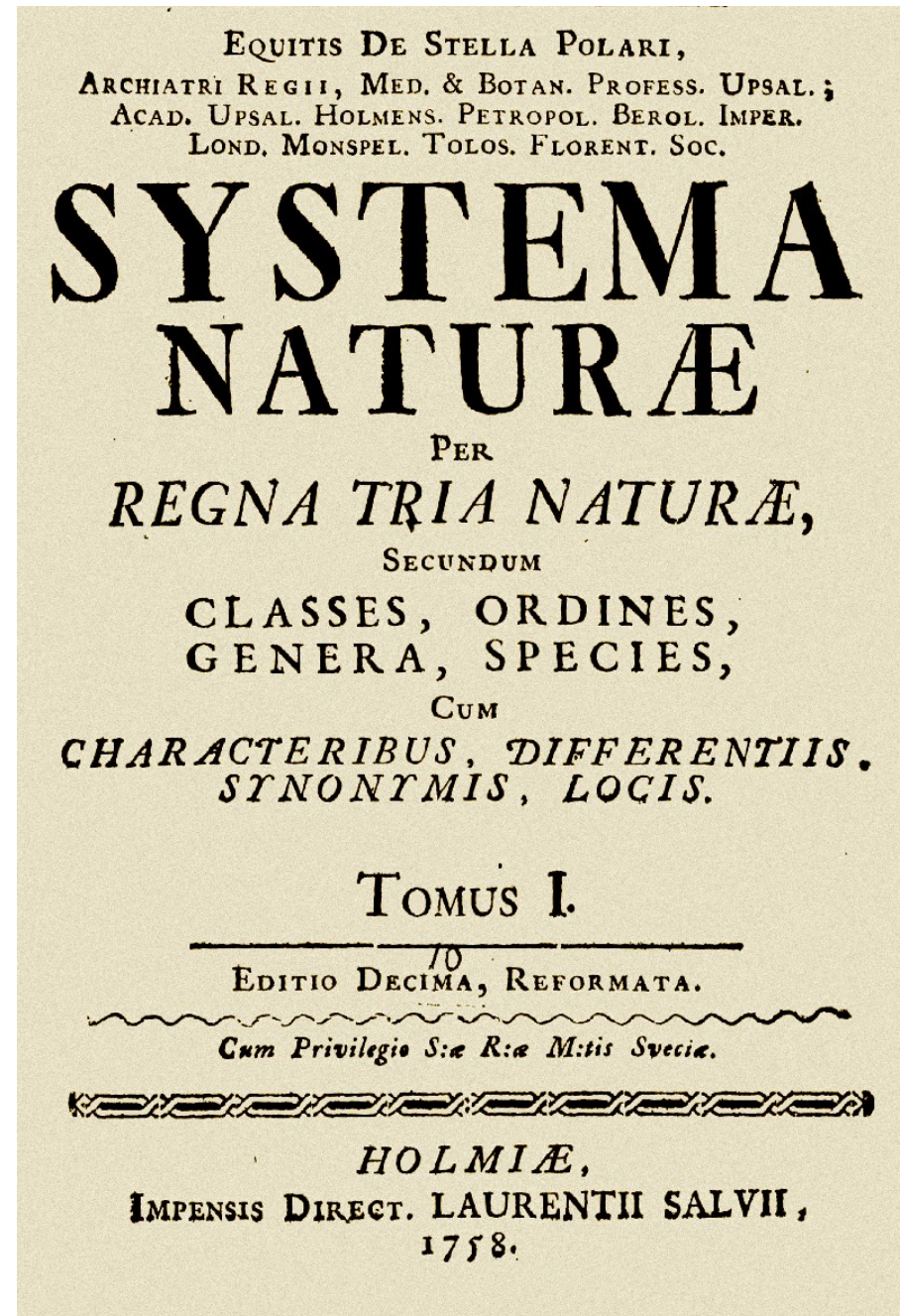


# Medieval History

- This collection of verse and prose tales by Geoffrey Chaucer describes the stories told by a group of travelers. The stories present an oblique critique of society and the church. It was influential in promoting the English vernacular (as opposed to the more stylish French or Latin) as a vehicle for literature.

# Medical History

- This 18<sup>th</sup> century Swedish botanist introduced the modern taxonomy used to classify plants and animals. His influential *Systema Naturae* spearheaded and popularized the use of “two word” descriptors: a generic name (genus) and a specific name (species).



## Q: Games (503 / 842)

- This 1983 adventure game designed by Roberta Williams described Sir Graham's attempts to recover the three magical treasures of Daventry and become the next king. It featured a parser for simple textual commands (e.g., "get carrot") and spawned numerous sequels.

# Real-World Languages

- This is the second-largest Slavic language (after Russian but ahead of Ukrainian). It features an extended Latin alphabet, high inflection, no articles, free word order, and mostly S-V-O sentences. Stanisław Lem is the most famous science fiction and fantasy writer in this language.

# Extending $\leq$

Let  $T$  and  $T'$  be any types except `SELF_TYPE`

There are four cases in the definition of  $\leq$

- $\text{SELF\_TYPE}_C \leq T$  if  $C \leq T$ 
  - $\text{SELF\_TYPE}_C$  can be any subtype of  $C$
  - This includes  $C$  itself
  - Thus this is the most flexible rule we can allow
- $\text{SELF\_TYPE}_C \leq \text{SELF\_TYPE}_C$ 
  - $\text{SELF\_TYPE}_C$  is the type of the “self” expression
  - In Cool we **never** need to compare `SELF_TYPE`s coming from different classes

# Extending $\leq$ (Cont.)

- $T \leq \text{SELF\_TYPE}_C$  always false  
Note:  $\text{SELF\_TYPE}_C$  can denote any subtype of  $C$ .
- $T \leq T'$  (according to the rules from before)

Based on these rules we can extend **lub** ...



# Extending $\text{lub}(T, T')$

Let  $T$  and  $T'$  be any types except  $\text{SELF\_TYPE}$

Again there are four cases:

- $\text{lub}(\text{SELF\_TYPE}_C, \text{SELF\_TYPE}_C) = \text{SELF\_TYPE}_C$
- $\text{lub}(\text{SELF\_TYPE}_C, T) = \text{lub}(C, T)$

This is the best we can do because  $\text{SELF\_TYPE}_C \leq C$

- $\text{lub}(T, \text{SELF\_TYPE}_C) = \text{lub}(C, T)$
- $\text{lub}(T, T')$  defined as before

# Where Can SELF\_TYPE Appear in COOL?

- The parser checks that SELF\_TYPE appears only where a type is expected
- But SELF\_TYPE is not allowed everywhere a type can appear:
- `class T inherits T' {...}`
  - T, T' **cannot** be SELF\_TYPE
  - Because SELF\_TYPE is never a dynamic type
- `x : T`
  - T can be SELF\_TYPE
  - An attribute whose type is SELF\_TYPE<sub>c</sub>

# Where Can SELF\_TYPE Appear in COOL?

## 1. let $x : T$ in $E$

- $T$  can be SELF\_TYPE
- $x$  has type SELF\_TYPE<sub>C</sub>

## 2. new $T$

- $T$  can be SELF\_TYPE
- Creates an object of the same type as `self`
- $m@T(E_1, \dots, E_n)$ 
  - $T$  **cannot** be SELF\_TYPE

# Typing Rules for SELF\_TYPE

- Since occurrences of **SELF\_TYPE** depend on the enclosing class we need to carry more context during type checking
- New form of the typing judgment:

$$\mathbf{O, M, C} \vdash \mathbf{e} : \mathbf{T}$$

(An expression **e** occurring in the body of **C** has static type **T** given a variable type environment **O** and method signatures **M**)

OMC = “Oh My Cool!” ?

# Type Checking Rules

- The next step is to design type rules using **SELF\_TYPE** for each language construct
- Most of the rules remain the same except that  $\leq$  and **lub** are the new ones
- Example:

$$O(\text{id}) = T_0$$

$$O, M, C \vdash e_1 : T_1$$

$$T_1 \leq T_0$$

---

$$O, M, C \vdash \text{id} \leftarrow e_1 : T_1$$

# What's Different?

- Recall the old rule for dispatch

$$\mathbf{O, M, C} \vdash \mathbf{e_0} : \mathbf{T_0}$$

...

$$\mathbf{O, M, C} \vdash \mathbf{e_n} : \mathbf{T_n}$$

$$\mathbf{M(T_0, f)} = (\mathbf{T_1', \dots, T_n', T_{n+1}'})$$

$$\mathbf{T_{n+1}'} \neq \mathbf{SELF\_TYPE}$$

$$\mathbf{T_i} \leq \mathbf{T_i'} \quad \mathbf{1 \leq i \leq n}$$

---

$$\mathbf{O, M, C} \vdash \mathbf{e_0.f(e_1, \dots, e_n)} : \mathbf{T_{n+1}'}$$

# The Big Rule for SELF\_TYPE

- If the return type of the method is **SELF\_TYPE** then the type of the dispatch is the type of the dispatch expression:

$$\mathbf{O, M, C \vdash e_0 : T_0}$$

...

$$\mathbf{O, M, C \vdash e_n : T_n}$$

$$\mathbf{M(T_0, f) = (T_1', \dots, T_n', \mathbf{SELF\_TYPE})}$$

$$\mathbf{T_i \leq T_i' \quad 1 \leq i \leq n}$$

---

$$\mathbf{O, M, C \vdash e_0.f(e_1, \dots, e_n) : T_0}$$

# What's Different?

- Note this rule handles the `Stock` example
- Formal parameters **cannot** be `SELF_TYPE`
- Actual arguments can be `SELF_TYPE`
  - The extended  $\leq$  relation handles this case
- The type  $T_0$  of the dispatch expression could be `SELF_TYPE`
  - Which class is used to find the declaration of `f`?
  - Answer: it is safe to use the class where the dispatch appears



# Static Dispatch

- Recall the original rule for static dispatch

$$\mathbf{O, M, C} \vdash \mathbf{e_0} : \mathbf{T_0}$$

...

$$\mathbf{O, M, C} \vdash \mathbf{e_n} : \mathbf{T_n}$$

$$\mathbf{T_0} \leq \mathbf{T}$$

$$\mathbf{M(T, f)} = (\mathbf{T_1'}, \dots, \mathbf{T_n'}, \mathbf{T_{n+1}'})$$

$$\mathbf{T_{n+1}'} \neq \mathbf{SELF\_TYPE}$$

$$\mathbf{T_i} \leq \mathbf{T_i'} \quad \mathbf{1} \leq \mathbf{i} \leq \mathbf{n}$$

---

$$\mathbf{O, M, C} \vdash \mathbf{e_0@T.f(e_1, \dots, e_n)} : \mathbf{T_{n+1}'}$$

# Static Dispatch

- If the return type of the method is **SELF\_TYPE** we have:

$$\mathbf{O, M, C} \vdash \mathbf{e_0} : \mathbf{T_0}$$

...

$$\mathbf{O, M, C} \vdash \mathbf{e_n} : \mathbf{T_n}$$

$$\mathbf{T_0} \leq \mathbf{T}$$

$$\mathbf{M(T, f)} = (\mathbf{T_1'}, \dots, \mathbf{T_n'}, \mathbf{SELF\_TYPE})$$

$$\mathbf{T_i} \leq \mathbf{T_i'} \quad \mathbf{1} \leq \mathbf{i} \leq \mathbf{n}$$

---

$$\mathbf{O, M, C} \vdash \mathbf{e_0@T.f(e_1, \dots, e_n)} : \mathbf{T_0}$$

# Static Dispatch

- Why is this rule correct?
- If we dispatch a method returning `SELF_TYPE` in class `T`, don't we get back a `T`?
- No. `SELF_TYPE` is the type of the self parameter, which may be a subtype of the class in which the method body appears
  - *Not* the class in which the call **appears!**
- The static dispatch class cannot be `SELF_TYPE`

# New Rules

- There are two new rules using **SELF\_TYPE**

---

**$O, M, C \vdash \text{self} : \text{SELF\_TYPE}_c$**

---

**$O, M, C \vdash \text{new SELF\_TYPE} : \text{SELF\_TYPE}_c$**

- There are a number of other places where **SELF\_TYPE** is used

# Where is SELF\_TYPE Illegal in COOL?

$m(x : T) : T' \{ \dots \}$

- Only  $T'$  can be SELF\_TYPE !

What could go wrong if  $T$  were SELF\_TYPE?

```
class A { comp(x : SELF_TYPE) : Bool {...}; };
class B inherits A {
  b() : int { ... };
  comp(y : SELF_TYPE) : Bool { ... y.b() ...}; };
...
let x : A ← new B in ... x.comp(new A); ...
...
```

# Summary of SELF\_TYPE

- The extended  $\leq$  and **lub** operations can do a lot of the work. Implement them to handle **SELF\_TYPE**
- **SELF\_TYPE** can be used only in a few places. **Be sure it isn't used anywhere else.**
- A use of **SELF\_TYPE** always refers to any subtype in the current class
  - The exception is the type checking of dispatch.
  - **SELF\_TYPE** as the return type in an invoked method might have nothing to do with the current class

# Why Cover SELF\_TYPE ?

- SELF\_TYPE is a research idea
  - It adds more expressiveness to the type system
  - Without allowing in any “bad” programs
- SELF\_TYPE is itself not so important
  - except for the project
- Rather, SELF\_TYPE is meant to illustrate that type checking can be quite subtle
- In practice, there should be a balance between the complexity of the type system and its expressiveness

# Type Systems

- The rules in these lecture were Cool-specific
  - Other languages have very different rules
  - We'll survey a few more type systems later
- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment
- Types are a play between flexibility and safety



# Homework

- PA4t Testing Due Today
- PA4c Checkpoint Due Wednesday
- WA4 Due Next Monday