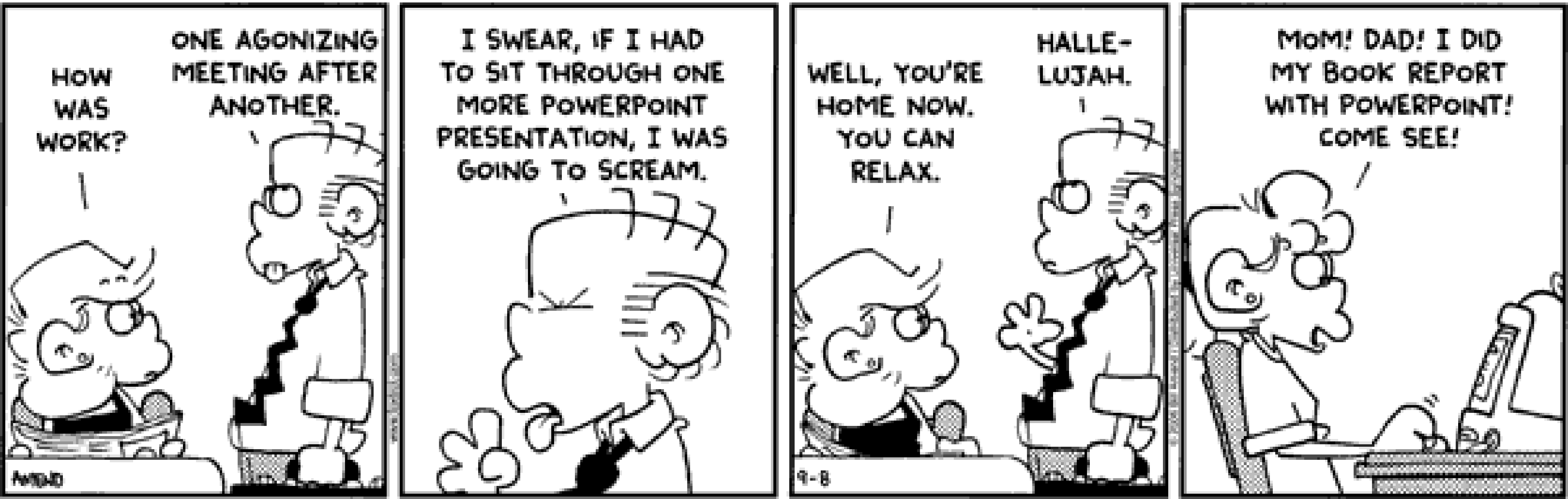
Lexical Analysis

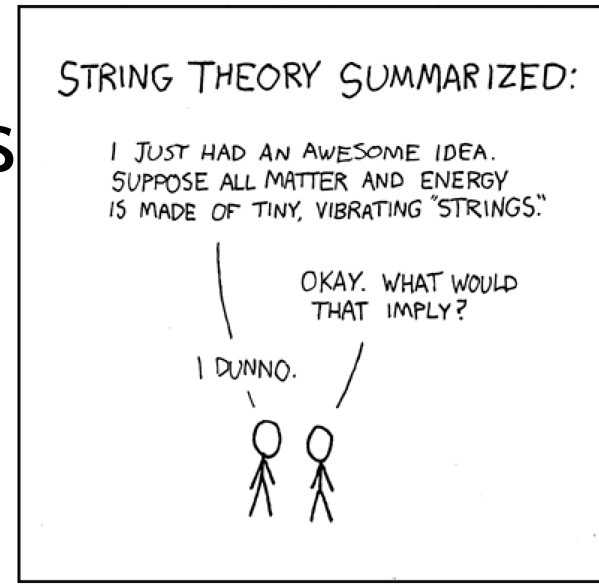
Finite Automata

(Part 2 of 2)



Cunning Plan

- Regular expressions provide a concise notation for **string patterns**
- Use in lexical analysis requires extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)



One-Slide Summary

- **Finite automata** are formal models of computation that can accept regular languages corresponding to regular expressions.
- **Nondeterministic** finite automata (NFA) feature epsilon transitions and multiple outgoing edges for the same input symbol.
- Regular expressions can be **converted** to NFAs.
- Tools will **generate** DFA-based lexer code for you from regular expressions.

Finite Automata

- Regular expressions = specification
- Finite automata = implementation

- A finite automaton consists of
  - An input alphabet Σ
  - A set of states **S**
  - A start state **n**
  - A set of accepting states $F \subseteq S$
  - A set of transitions **state** $\rightarrow^{input}$ **state**

Finite Automata

- Transition

$$s_1 \rightarrow^a s_2$$

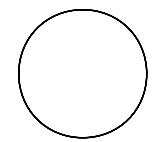
- Is read

  In state s_1 on input "a" go to state s_2

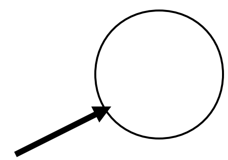
- If end of input
  - If in accepting state $\Rightarrow$ accept
  - Otherwise $\Rightarrow$ reject
- If still input, no transitions possible $\Rightarrow$ reject

Finite Automata State Graphs

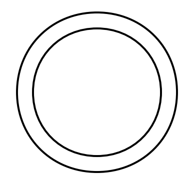
- A state



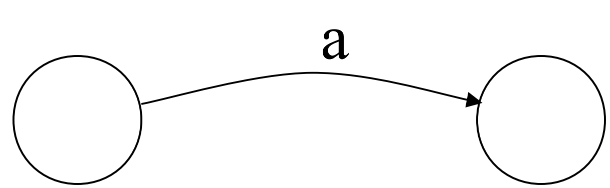
- The start state



- An accepting state



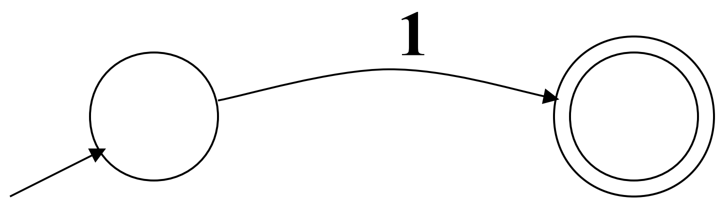
- A transition



You can hand-write on any Exam or RS.

A Simple Example

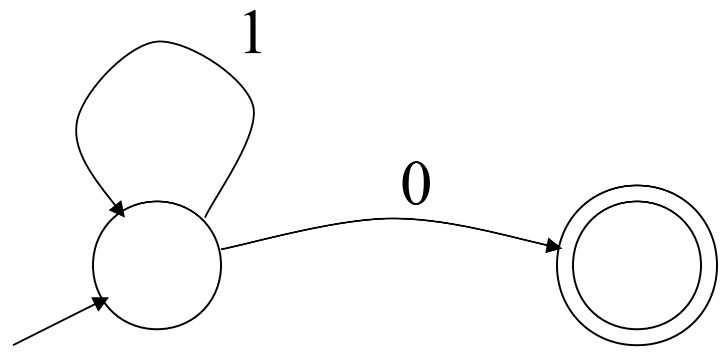
- A finite automaton that accepts only "1"



- A finite automaton accepts a string if we can follow transitions labeled with the characters in the string from the start to some accepting state

Another Simple Example

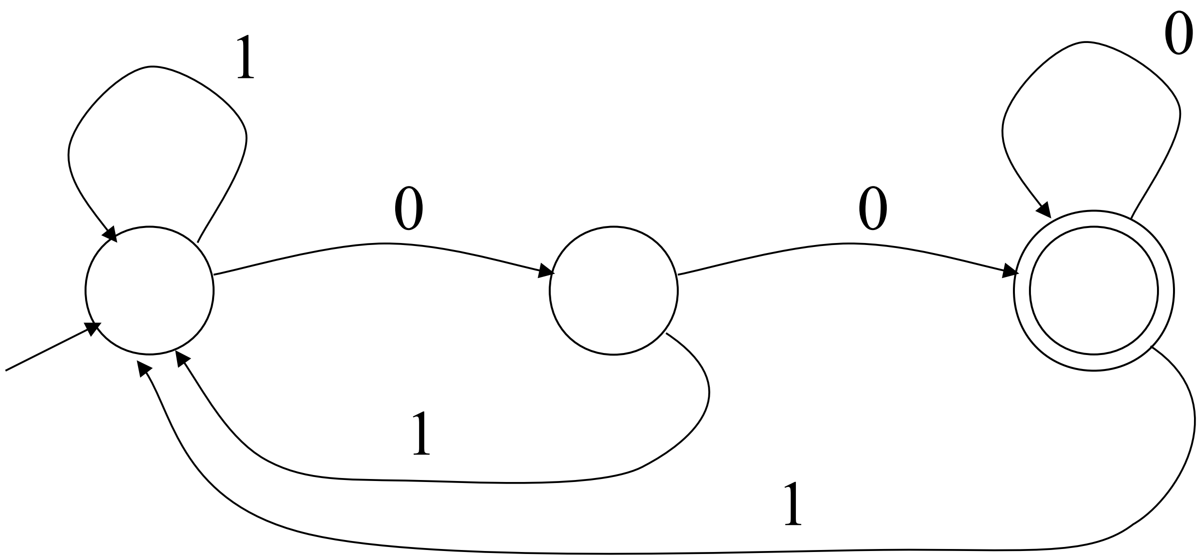
- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet $\Sigma = \{0,1\}$

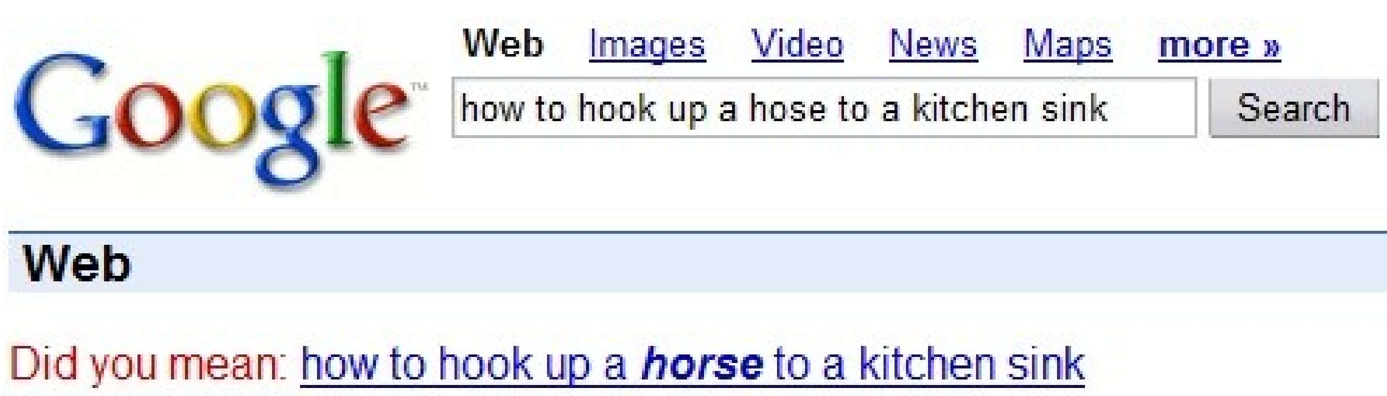


- Check that "1110" is accepted but "110…" is not

And Another Example

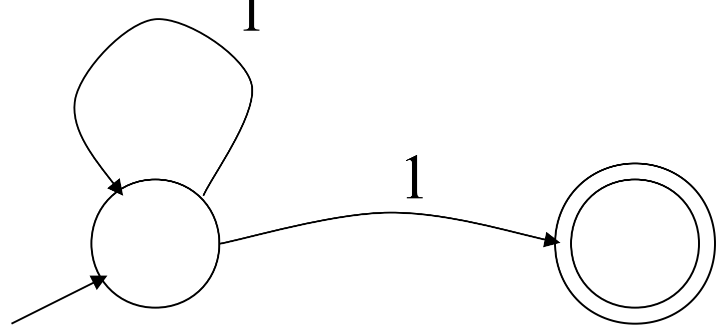
- Alphabet $\Sigma = \{0,1\}$
- What language does this recognize?





And A Fourth Example

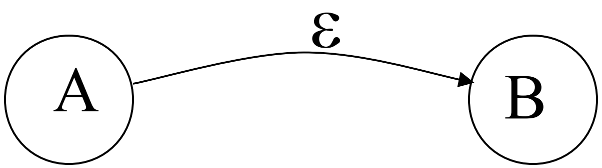
- Alphabet still $\Sigma = \{ 0, 1 \}$



- The operation of the automaton is not completely defined by the input
  - On input “11” the automaton could be in either state

Epsilon Moves

- Another kind of transition: ε-moves



  - Machine can move from state A to state B *without reading input*

Deterministic and Nondeterministic Automata

- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Finite automata have finite memory
  - Need only to encode the current state

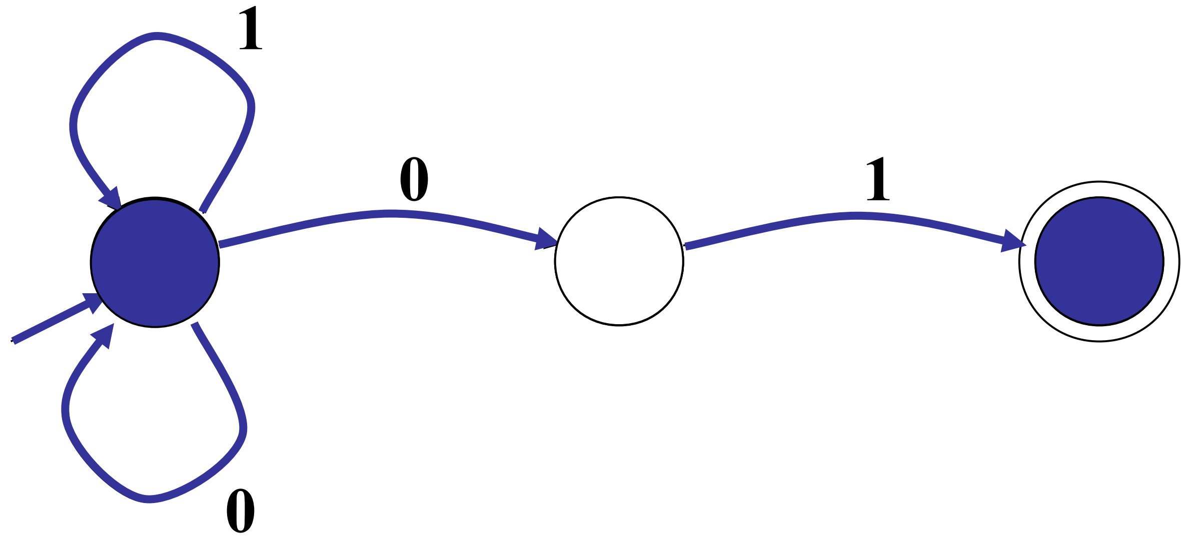
Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input

- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: **1 0 1**
- Rule: NFA accepts if it can get in a final state

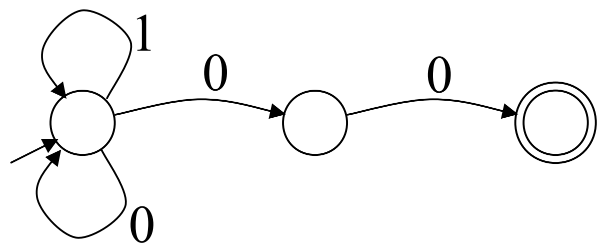
NFA vs. DFA (1)

- NFAs and DFAs recognize the *same* set of languages (regular languages)
  - They have the same expressive power
- DFAs are easier to implement
  - There are no choices to consider

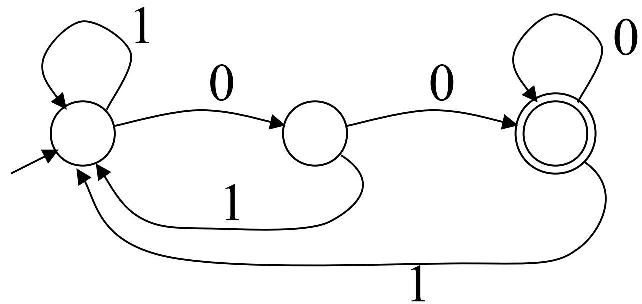
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



DFA



- DFA can be *exponentially* larger than NFA

Natural Languages

- This North Germanic language is generally mutually intelligible with Norwegian and Danish, and descends from Old Norse of the Viking Era to a modern speaking population of about 10 million people. The language contains two genders, nouns that are rarely inflected, and a typical subject-verb-object ordering. Its home country is one of the largest music exporters of the modern world, often targeting English-speaking audiences. Bands such as Ace of Base, ABBA and Roxette are examples, with over 420m combined album sales.

Unnatural Languages

- This stack-based structured computer programming language appeared in the 1970's and went on to influence PostScript and RPL. It is typeless and is often used in bootloaders and embedded applications. Example:

  ```
  25 10 * 50 +
  ```

- Simple C Program:

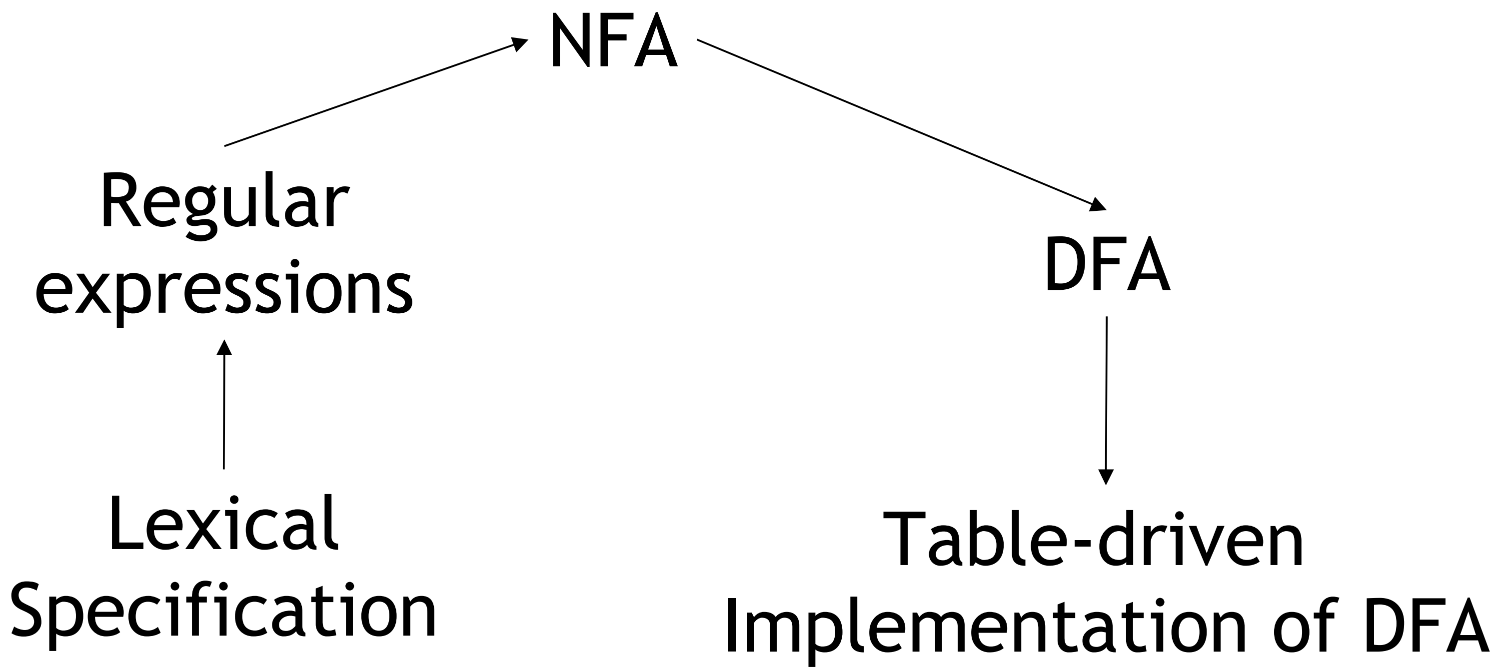
  ```
  int floor5(int v) { return (v < 6) ? 5 : (v – 1); }
  ```

- Same program in *this* Language:

  ```
  : FLOOR5 ( n -- n' ) DUP 6 < IF DROP 5 ELSE 1 – THEN ;
  ```

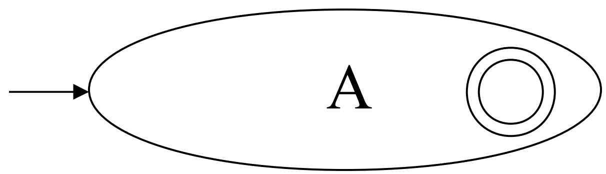
Regular Expressions to Finite Automata

- High-level sketch

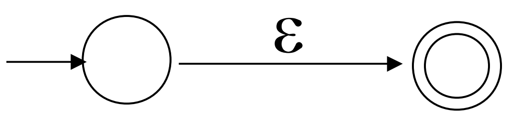


Regular Expressions to NFA (1)

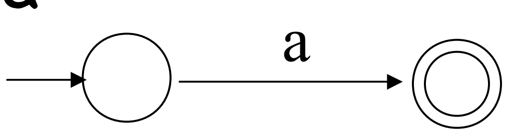
- For each kind of rexp, define an NFA
  - Notation: NFA for rexp A



- For ε

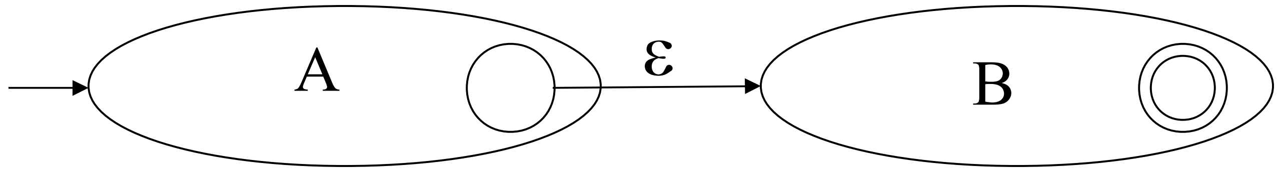


- For input a

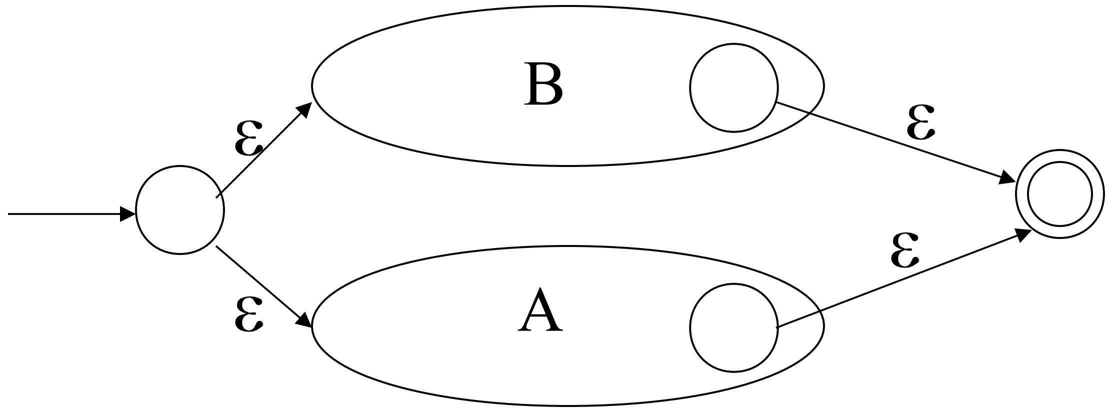


Regular Expressions to NFA (2)

- For AB

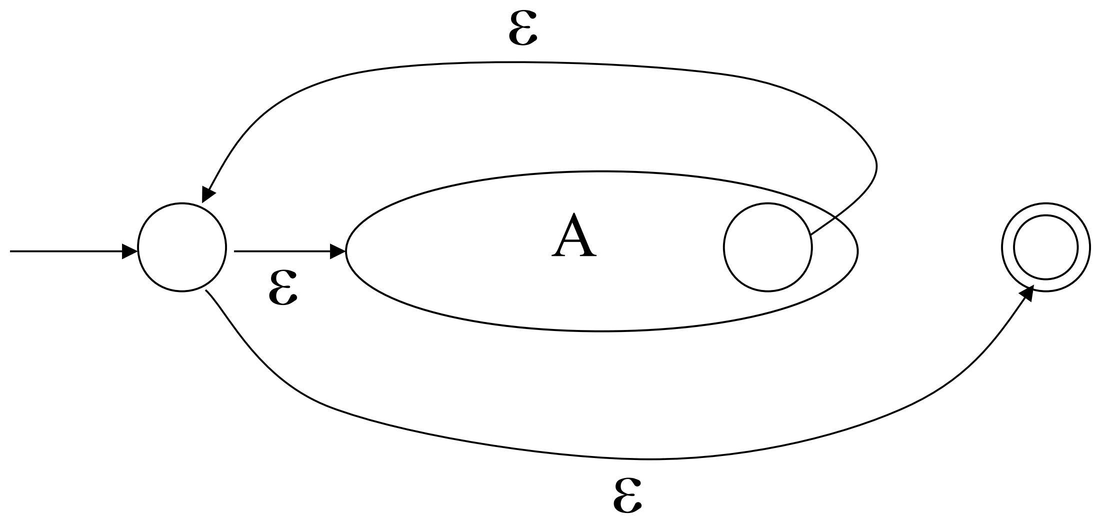


- For A | B



Regular Expressions to NFA (3)

- For A*

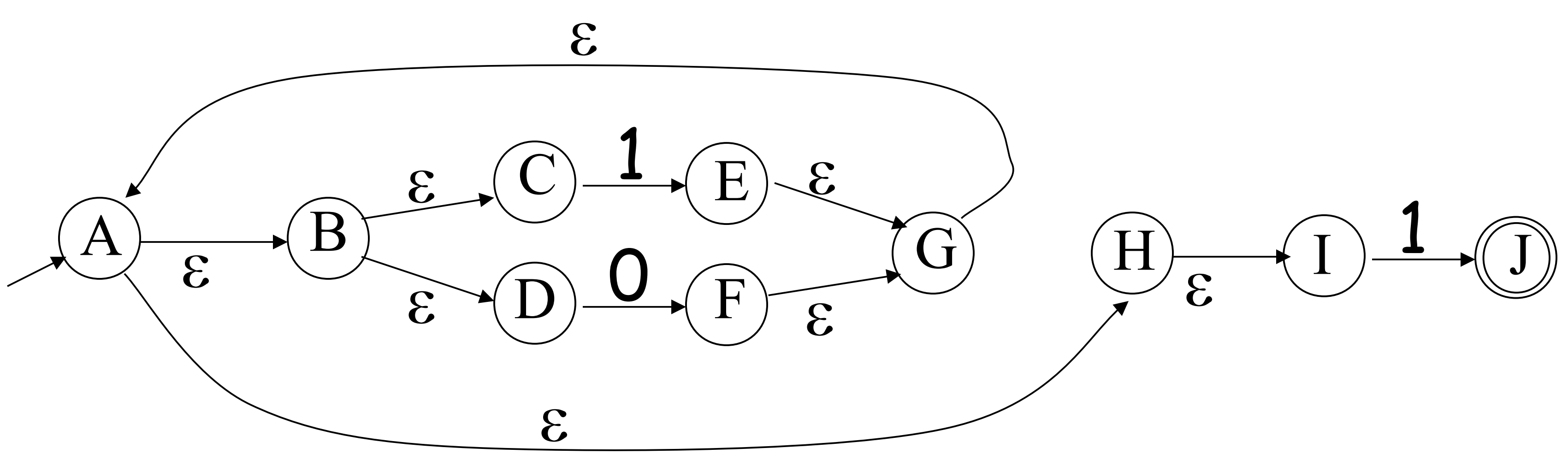


Example of RegExp -> NFA Conversion

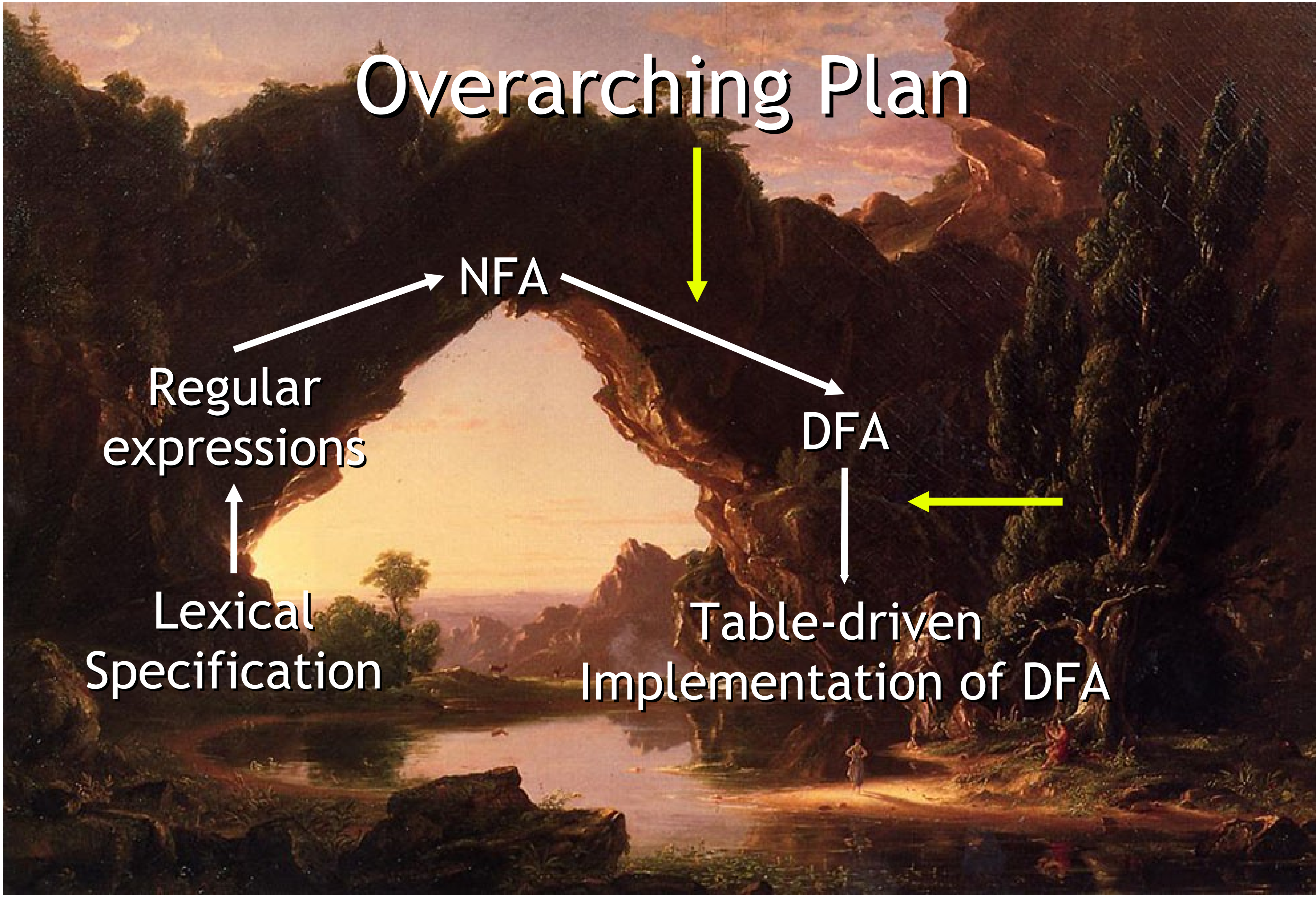
- Consider the regular expression

  (1 | 0)* 1

- The NFA is



Overarching Plan

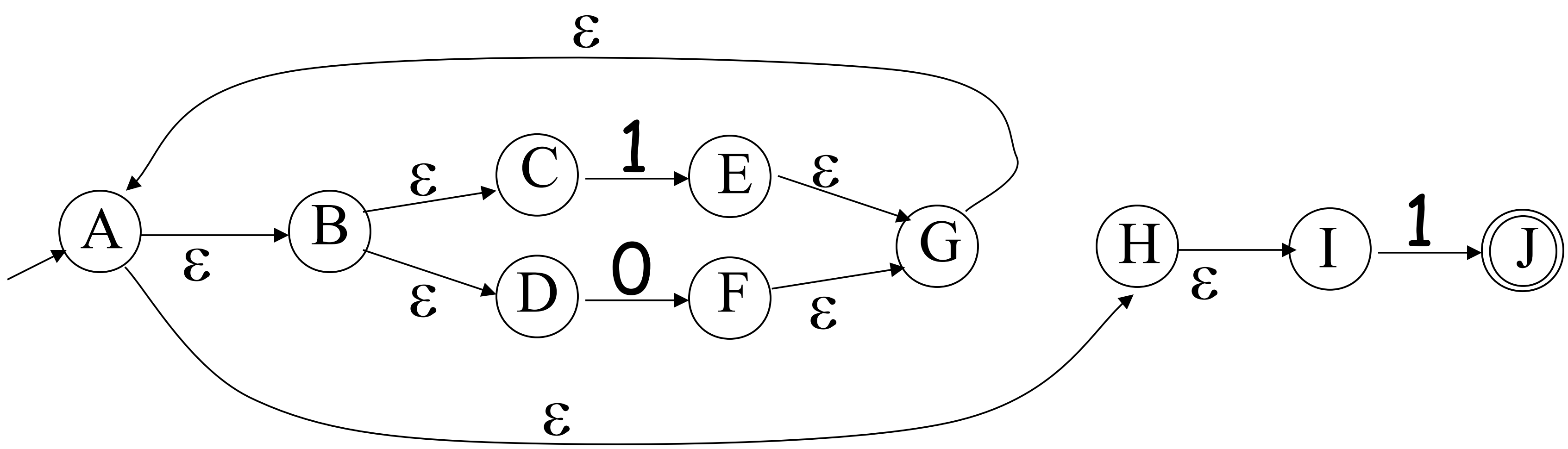


Thomas Cole – Evening in Arcady (1843)

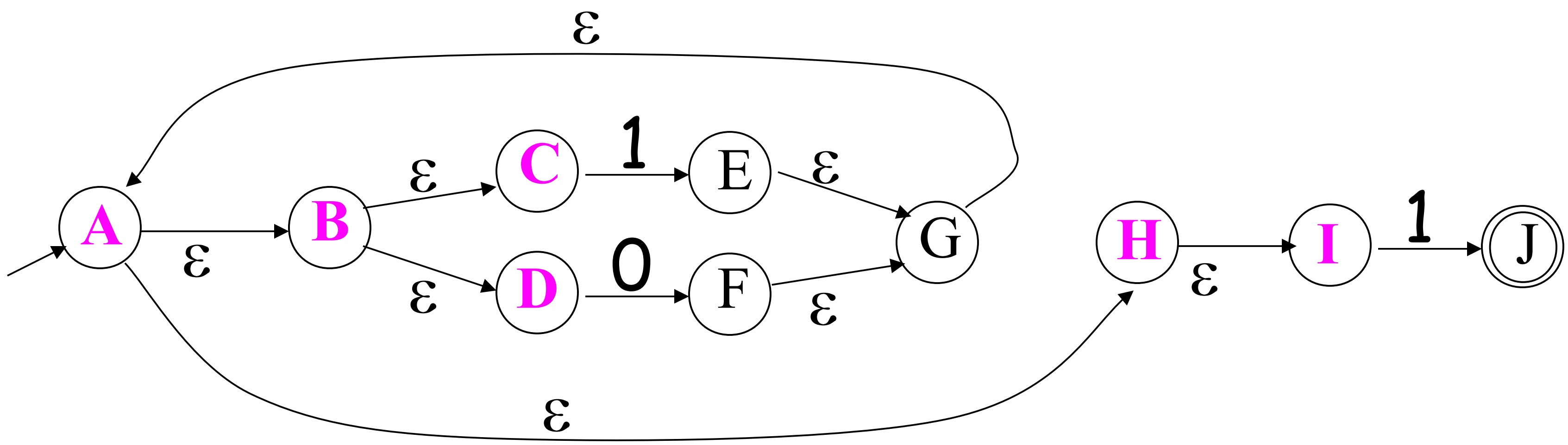
NFA to DFA: The Trick

- Simulate the NFA
- Each state of DFA
  = a non-empty *subset of states* of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \rightarrow^a S'$ to DFA iff
  - S' is the set of NFA states reachable from the states in S after seeing the input a
    - considering ε-moves as well

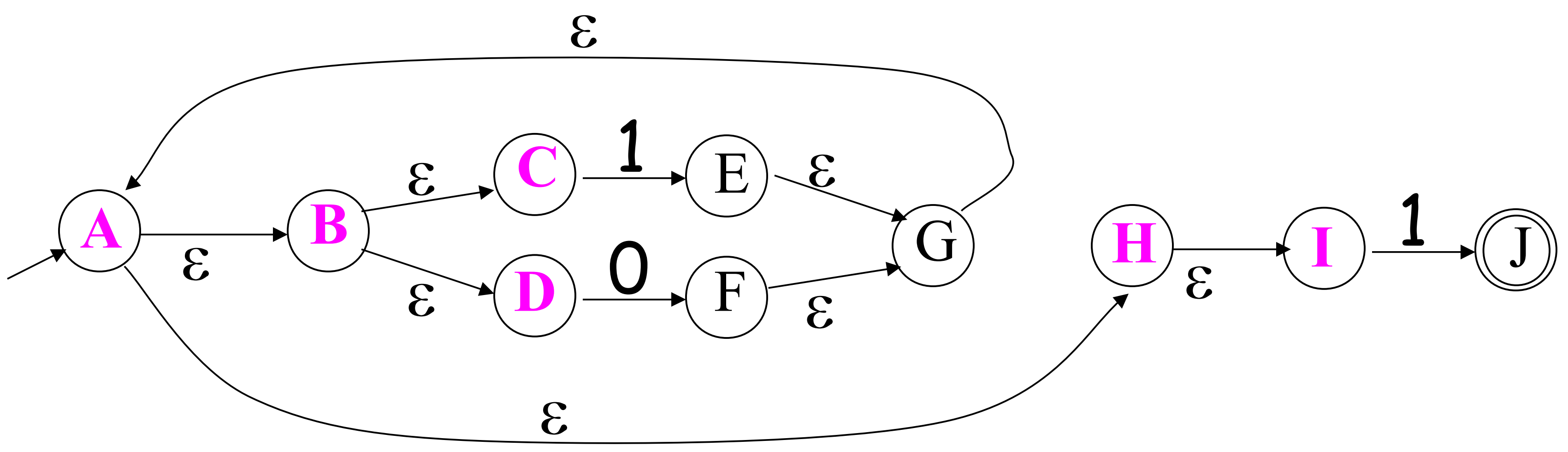
NFA → DFA Example

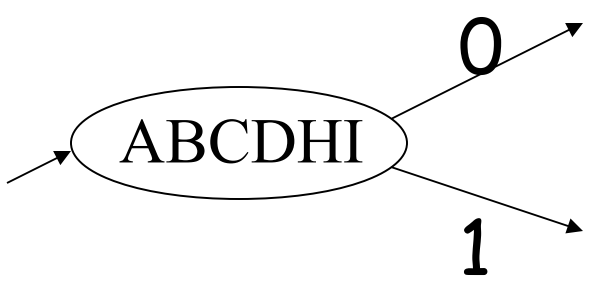


NFA → DFA Example

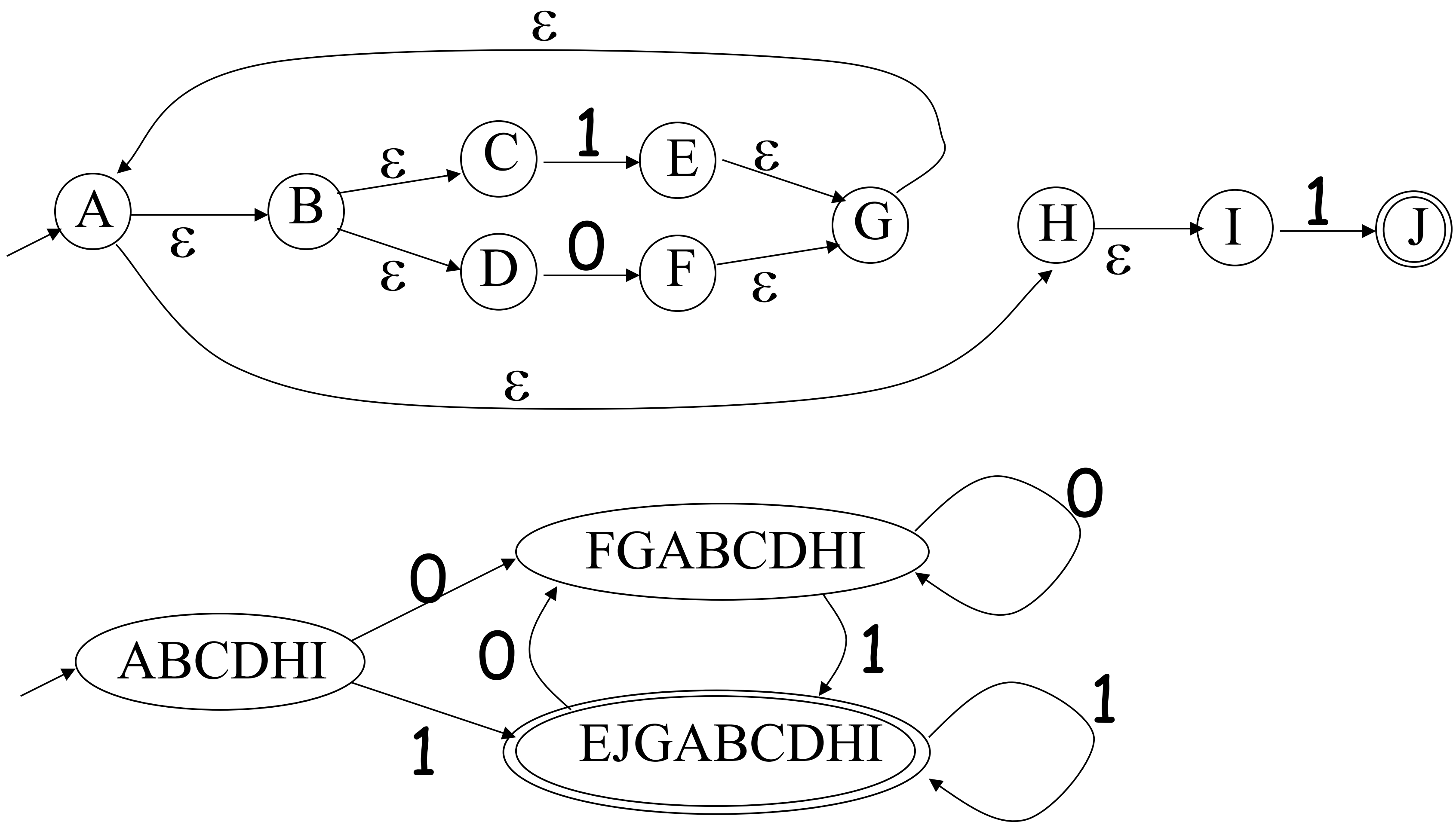


NFA → DFA Example





NFA → DFA Example



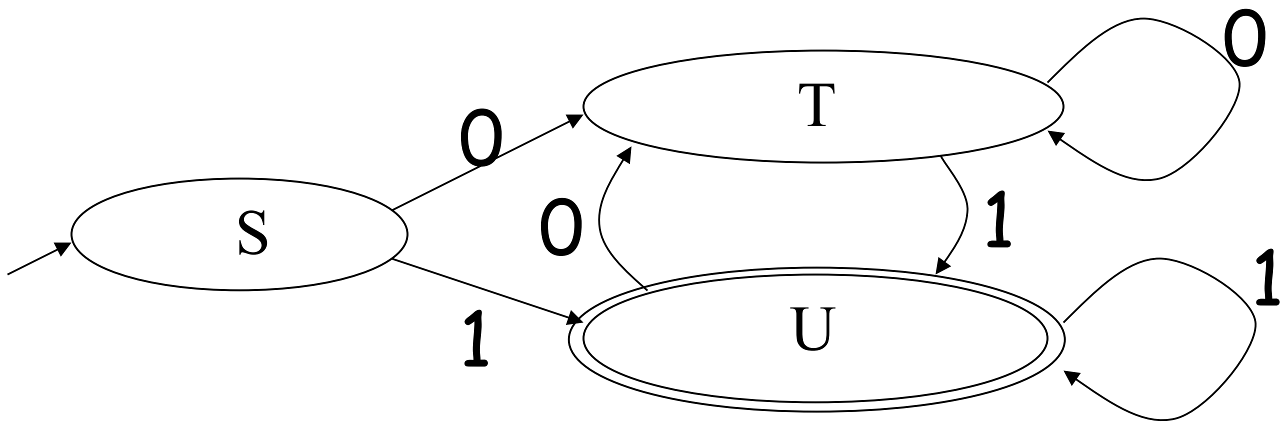
NFA $\rightarrow$ DFA: Remark

- An NFA may be in many states at any time
- How many different states?
- If there are N states, the NFA must be in some subset of those N states
- How many non-empty subsets are there?
  - $2^N - 1$ = finitely many

Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbols”
  - For every transition $S_i \to^a S_k$ define T[i,a] = k
- DFA “execution”
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



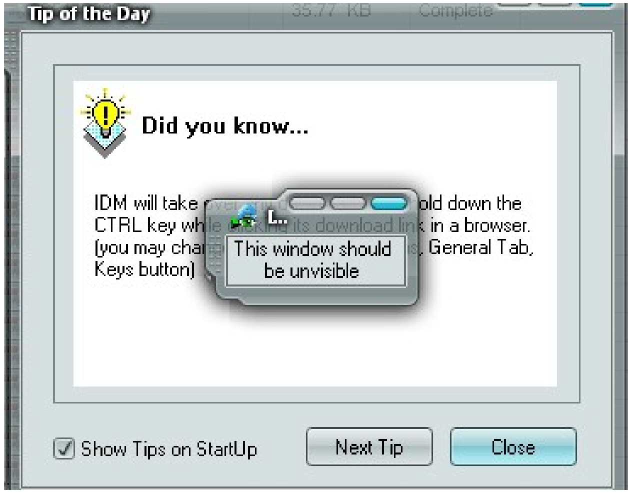
	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA $\rightarrow$ DFA conversion is at the heart of tools such as flex or ocamllex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations

PA2: Lexical Analysis

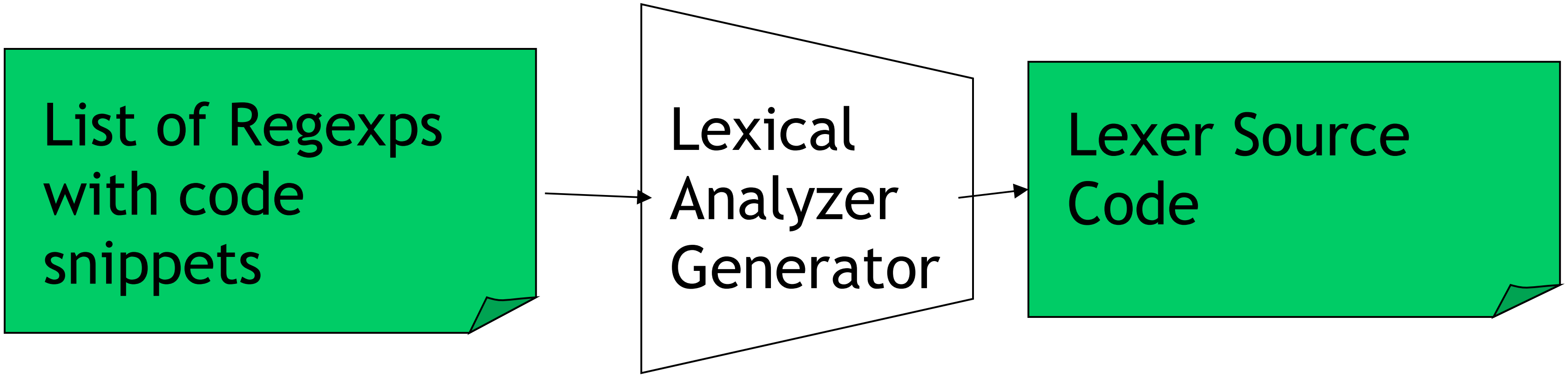
- **Correctness is job #1.**
  - And job #2 and #3!



- Tips on building large systems:
  - Keep it simple
  - Design systems that can be tested
  - Don’t optimize prematurely
  - It is easier to modify a working system than to get a system working

Lexical Analyzer Generator

- Tools like *lex* and *flex* and *ocamllex* will build lexers for you!
- You must use such a tool for PA2



- I'll explain ocamllex; others are similar
  - See PA2 documentation

Ocamllex "lexer.mll" file

```
{
  (* raw preamble code
      type declarations, utility functions, etc. *)
}
let re_name_i = re_i
rule normal_tokens = parse
  re_1          { token_1 }
| re_2          { token_2 }
and special_tokens = parse
| re_n          { token_n }
```

Example “lexer.mll”

```
{
   type token = Tok_Integer of int       (* 123 *)
              | Tok_Divide               (*  /  *)
}
let digit = [‘0’ - ‘9’]
rule initial = parse
   ‘/’            { Tok_Divide }
| digit digit* { let token_string = Lexing.lexeme lexbuf in
                 let token_val = int_of_string token_string in
                 Tok_Integer(token_val) }
| _             { Printf.printf “Error!\n”; exit 1 }
```

Adding Winged Comments

```
{
   type token = Tok_Integer of int       (* 123 *)
               | Tok_Divide              (*  /  *)
}
let digit = [‘0’ - ‘9’]
rule initial = parse
   “//”           { eol_comment }
|  ‘/’            { Tok_Divide }
| digit digit*    { let token_string = Lexing.lexeme lexbuf in
                    let token_val = int_of_string token_string in
                    Tok_Integer(token_val) }
| _               { Printf.printf “Error!\n”; exit 1 }

and eol_comment = parse
  ‘\n’     { initial lexbuf }
| _        { eol_comment lexbuf }
```

Using Lexical Analyzer Generators

```
$ ocamllex lexer.mll
45 states, 1083 transitions, table size 4602 bytes

(* your main.ml file … *)
let file_input = open_in “file.cl” in
let lexbuf = Lexing.from_channel file_input in
let token = Lexer.initial lexbuf in
match token with
| Tok_Divide -> printf “Divide Token!\n”
| Tok_Integer(x) -> printf “Integer Token = %d\n” x
```

How Big Is PA2?

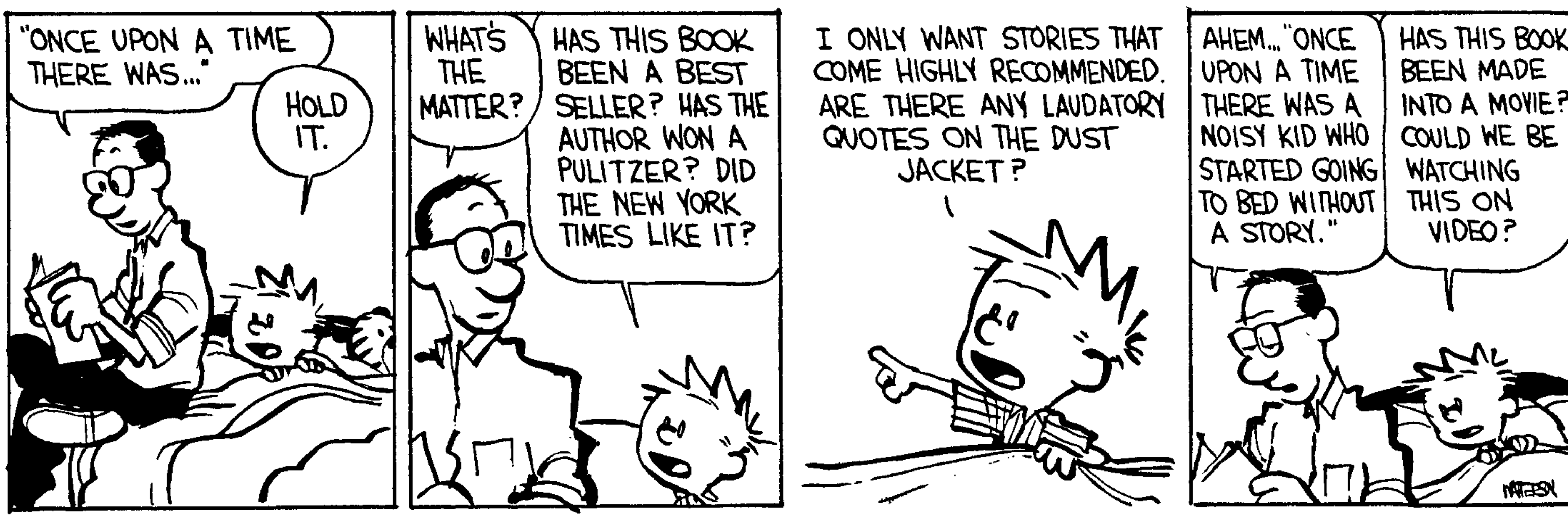
- The reference "lexer.mll" file is 88 lines
  - Perhaps another 20 lines to keep track of input line numbers
  - Perhaps another 20 lines to open the file and get a list of tokens
  - Then 65 lines to serialize the output
  - I'm sure it's possible to be smaller!
- Conclusion:
  - This isn't a code slog, it's about careful forethought and precision.

Legacy Warning!

- Legacy students may be tempted to use OCaml for PA2.
- However, Legacy students should save OCaml for one of the harder assignments later.
- Normal LDI: OCaml is a great choice.

Test Yourself! Exam Practice.

- Are practical parsers and scanners based on deterministic or non-deterministic automata?
- How can regular expressions be used to specify nested constructs?
- How is a two-dimensional *transition table* used in table-driven scanning?



Homework

- Textbook Reading, CD Reading – 2.4
- On-Line: Udacity 262 Lesson 2

- PA2 due Tuesday
- RS1 recommended Tuesday