Earley Parsing and Examples

Sometimes the second one is even better!

Outline

- Earley's Algorithm
  - Chart States
  - Operations
  - Example
- MyEarley.py
- PA3.jison
- Grammar “Conflicts”
  - Shift/Reduce

Administrivia

- Midterm 1 will take place Thursday in class
  - Everything including today is fair game.
- Class vote, pick one option:
  - You may bring one page of printed or hand-written notes, front-and-back (= 2 sides).
  - The test is open book: you may use any printed materials including your printed notes and/or the textbook and/or other printed readings.

- Think.

In One Slide

- Earley parsers are top-down and use dynamic programming. An Earley state records incremental information: when we started, what has been seen so far, and what we expect to see. The Earley chart holds a set of states for each input position. Shift, reduce and closure operations fill in the chart.
- You enjoy parsing. Parsing is easy and fun.

Review: Earley States

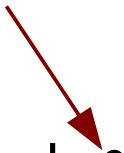
- Let X be a non-terminal
- Let a and b be (possibly-empty) sequences of terminals and non-terminals
- Let $X \rightarrow ab$ be a production in your grammar
- Let j be a position in the input
- Each **Earley State** is a tuple $< X \rightarrow a \bullet b \ , \ j >$
  - We are currently parsing an X
  - We have seen a, we expect to see b
  - We started parsing this X after seeing the first j tokens from the input.

Review: Earley Parse Table

- An **Earley parsing table** (or **chart**) is a one-dimensional array. Each array element is a set of Earley states.
  - chart[i] holds the set of valid parsing states we could be in after seeing the first *i* input tokens
- Then the string $tok_1 ... tok_n$ is in the language of a grammar with start symbol S *iff*
  - chart[n] contains < S → ab• , 0 > for some production rule S → ab in the grammar.
  - We then say the parser **accepts** the string.

Review: Filling In The Chart

- Three operations build up chart[n]
- The first is called **shift** or **scan**.
  - It corresponds to “seeing the next expected token” or “helping to confirm the current hypothesis” or “we're winning”.
- Example:
  - chart[1] contains < E → E • + E , 0 >
  - 2nd token is “+”
  - Then put < E → E + • E , 0 > in chart[2]



Review: Filling In The Chart (2)

- The second operation is the **closure** or **predictor**.
  - It corresponds to "expanding rewrite rules" or "substituting in the definitions of non-terminals"
- Suppose the grammar is:

  S → E E → E + E | E - E | int
- If chart[0] has < S → • E , 0 > then add

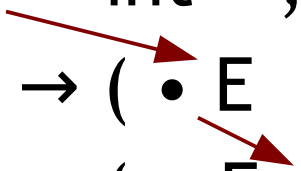
  < E → • E + E , 0 >

  < E → • E - E , 0 >

  < E → • int, 0 >

Review: Filling In The Chart (3)

- The third operation is **reduction** or **completion**.
  - It corresponds to "finishing a grammar rewrite rule" or "being done parsing a non-terminal" or "doing a rewrite rule in reverse and then shifting over the non-terminal".
- Suppose:
  - E → int | E + E | E - E | (E) , input is "(int"
  - chart[2] contains < E → int • , 1 >
  - chart[1] contains < E → (• E) , 0 >
  - Then chart[2] += < E → (E •) , 0 >



Shift Practice

- chart[3] contains

  < S → E • , 0 >

  < E → E • + E , 0>

  < E → E • - E , 2>

  < E → int • , 2 >

  < E → E • - E , 0 >

  < E → E - E • , 0 >

  < E → E • + E , 2 >

- The 4th token is "+". What does shift bring in?

Shift Practice

- chart[3] contains

  < S → E • , 0 >

  < E → E • + E , 0>

  < E → E • - E , 0>

  < E → int • , 2 >

  < E → E • - E , 0 >

  < E → E - E • , 0 >

  < E → E • + E , 2 >

- The 4th token is "+". What does shift bring in?

  < E → E + • E , 0>

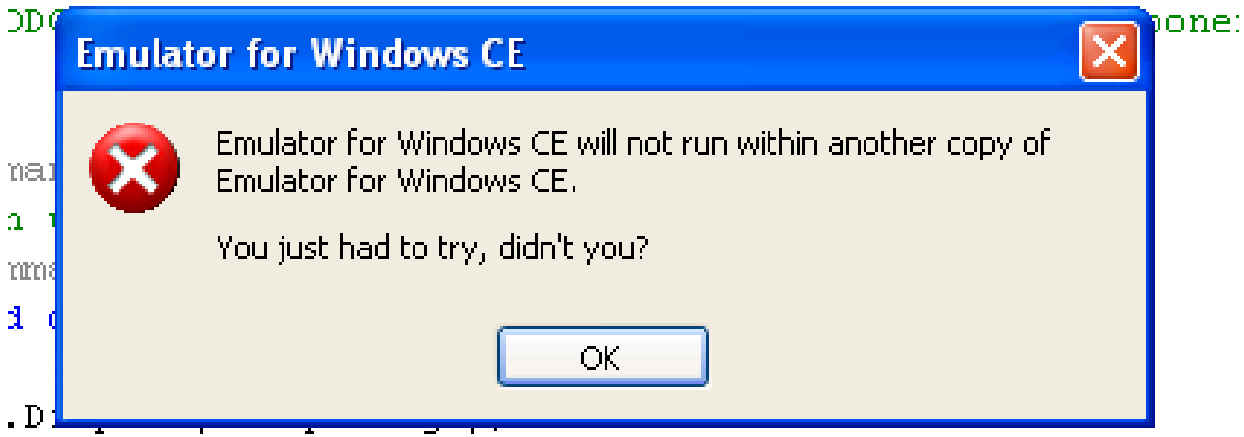
  < E → E + • E , 2 >

  … are both added to chart[4].

Closure Practice

- Grammar is
  - S → E E → E + E | E - E | (E) | int
- chart[4] contains:

  < E → E + • E , 0 > < E → E + • E , 2 >
- What does the closure operation bring in?



Closure Practice

- Grammar is
  - S → E E → E + E | E - E | (E) | int
- chart[4] contains:

  < E → E + • E , 0 > < E → E + • E , 2 >
- What does the closure operation bring in?

  < E → • E + E , 4 > < E → • E - E , 4 >

  < E → • (E) , 4 > < E → • int , 4 >

  … are all added to chart[4].

Reduction Practice

- chart[4] contains:
  - < E → E + • E , 0 >
  - < E → E + • E , 2 >
  - < E → • E + E , 4 >
  - < E → • E - E , 4 >
  - < E → • (E) , 4 >
  - < E → • int , 4 >
- chart[5] contains:
  - < E → int • , 4 >
- What does the reduce operator bring in?

Reduction Practice

- chart[4] contains:
  - < E → E + • E , 0 >
  - < E → E + • E , 2 >
  - < E → • E + E , 4 >
  - < E → • E - E , 4 >
  - < E → • (E) , 4 >
  - < E → • int , 4 >
- chart[5] contains:
  - < E → int • , 4 >
- What does the reduce operator bring in?
  - < E → E + E • , 0 >
  - < E → E + E • , 2 >
  - < E → E • + E , 4 >
  - < E → E • - E , 4 >
  - … are all added to chart[5]. (Plus more in a bit!)

Earley Parsing Algorithm

- Input: CFG G, Tokens $tok_1 ... tok_n$
- Work:

  chart[0] = { < S → •ab , 0 > }

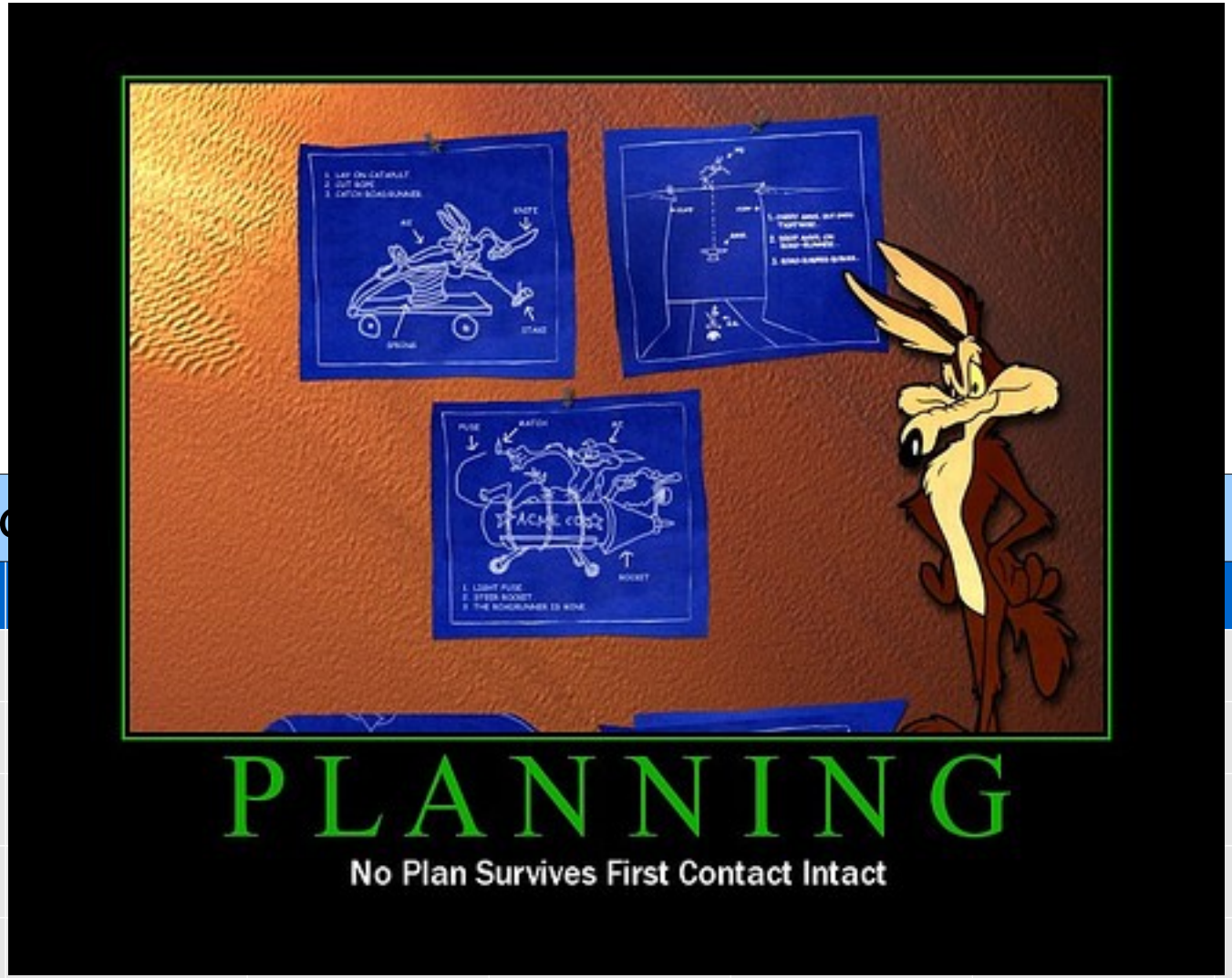
  for i = 0 to n

    repeat

      use shift, reduce and closure on chart[i]

    until no new states are added
- Output:
  - true iff < S → ab• , 0 > in chart[n]

Massive Earley Example



chart[0]						chart[6]
S→•F ,0						

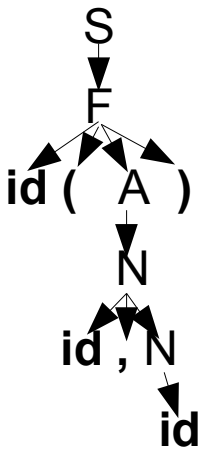
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



	id	(	id	,	id	)
chart[0]	**chart[1]**	**chart[2]**	**chart[3]**	**chart[4]**	**chart[5]**	**chart[6]**
S→•F ,0						

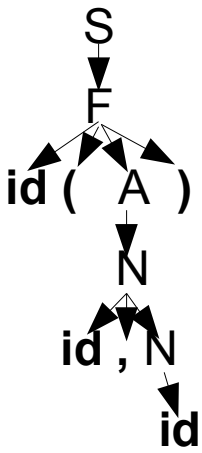
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



	id	(	id	,	id	)
chart[0]	**chart[1]**	**chart[2]**	**chart[3]**	**chart[4]**	**chart[5]**	**chart[6]**
S→•F ,0						
F→•id(A) ,0						

Closure on F

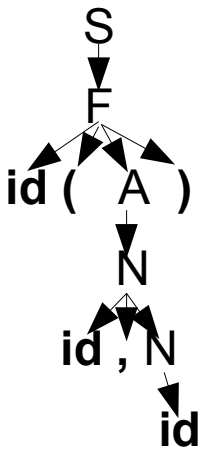
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



	id	(	id	,	id	)
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0					
F→•id(A) ,0						

Shift on “id”

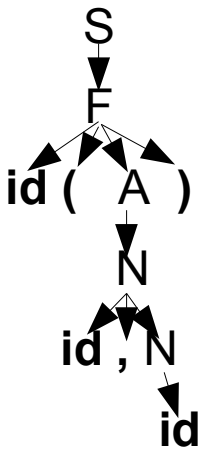
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0				
F→•id(A) ,0						

Shift on “(”

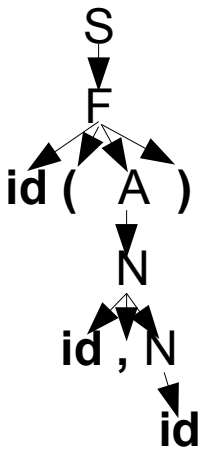
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



	id	(	id	,	id	)
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0				
F→•id(A) ,0		A→•N ,2				
		A→• ,2				

Closure on A

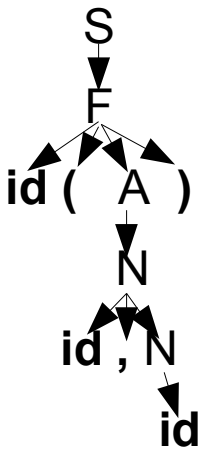
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



	id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0				
F→•id(A) ,0		A→•N ,2				
		A→• ,2				
		N→•id ,2				
		N→•id,N ,2				

Closure on N

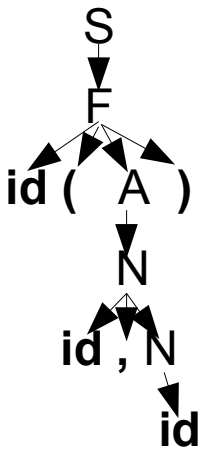
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)

chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0				
F→•id(A) ,0		A→•N ,2				
		A→• ,2				
		N→•id ,2				
		N→•id,N ,2				
		F→id(A•) ,0				

Reduce on A

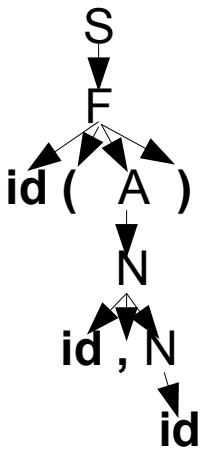
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



	id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2			
F→•id(A) ,0		A→•N ,2	N→id•,N ,2			
		A→• ,2				
		N→•id ,2				
		N→•id,N ,2				
		F→id(A•) ,0				

Shift on “id”

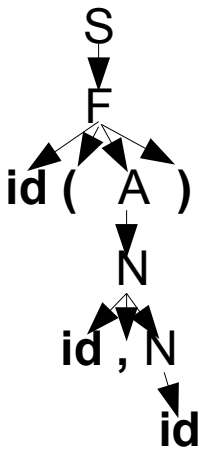
Massive Earley Example

Grammar

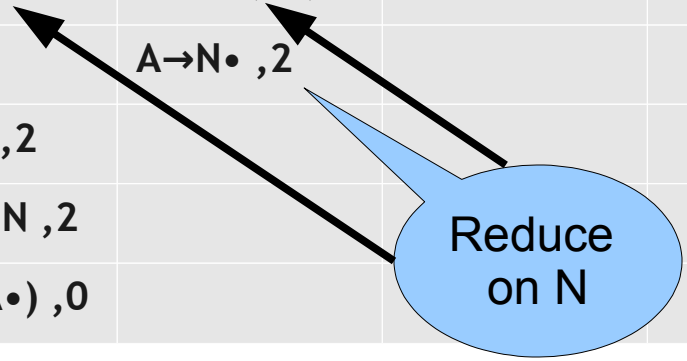
S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2			
F→•id(A) ,0		A→•N ,2	N→id•,N ,2			
		A→• ,2	A→N• ,2			
		N→•id ,2				
		N→•id,N ,2				
		F→id(A•) ,0				



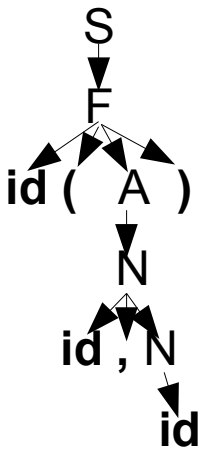
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2			
F→•id(A) ,0		A→•N ,2	N→id•,N ,2			
		A→• ,2	A→N• ,2			
		N→•id ,2	F→id(A•) ,0			
		N→•id,N ,2				
		F→id(A•) ,0				

Reduce on A

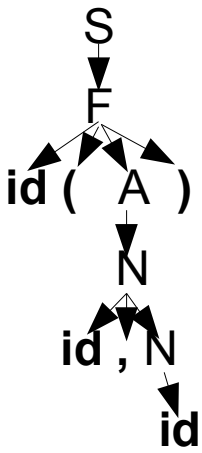
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2		
F→•id(A) ,0		A→•N ,2	N→id•,N ,2			
		A→• ,2	A→N• ,2			
		N→•id ,2	F→id(A•) ,0			
		N→•id,N ,2				
		F→id(A•) ,0				

Shift on “,”

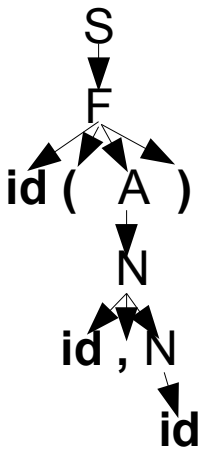
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2		
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4		
		A→• ,2	A→N• ,2	N→•id,N ,4		
		N→•id ,2	F→id(A•) ,0			
		N→•id,N ,2				
		F→id(A•) ,0				

Closure on N

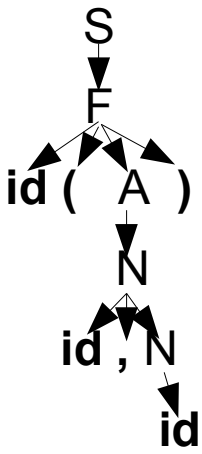
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	
		A→• ,2	A→N• ,2	N→•id,N ,4		
		N→•id ,2	F→id(A•) ,0			
		N→•id,N ,2				
		F→id(A•) ,0				

Shift on “id”

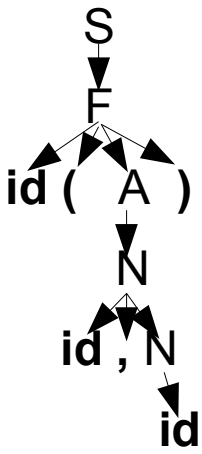
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	
		A→• ,2	A→N• ,2	N→•id,N ,4	N→id,N• ,2	
		N→•id ,2	F→id(A•) ,0			
		N→•id,N ,2				
		F→id(A•) ,0				

Reduce on N

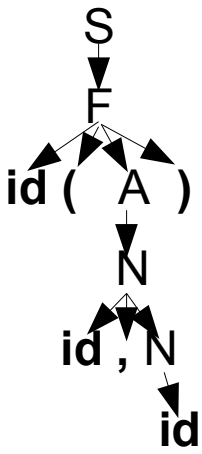
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	
		A→• ,2	A→N• ,2	N→•id,N ,4	N→id,N• ,2	
		N→•id ,2	F→id(A•) ,0		A→N• ,2	
		N→•id,N ,2				
		F→id(A•) ,0				

Reduce on N

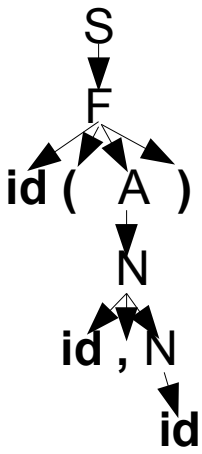
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	**chart[1]**	**chart[2]**	**chart[3]**	**chart[4]**	**chart[5]**	**chart[6]**
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	
		A→• ,2	A→N• ,2	N→•id,N ,4	N→id,N• ,2	
		N→•id ,2	F→id(A•) ,0		A→N• ,2	
		N→•id,N ,2			F→id(A•) ,0	
		F→id(A•) ,0				

Reduce on A

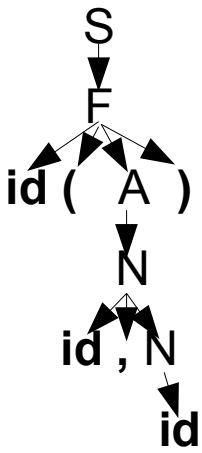
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	**chart[1]**	**chart[2]**	**chart[3]**	**chart[4]**	**chart[5]**	**chart[6]**
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	F→id(A)• ,0
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	
		A→• ,2	A→N• ,2	N→•id,N ,4	N→id,N• ,2	
		N→•id ,2	F→id(A•) ,0		A→N• ,2	
		N→•id,N ,2			F→id(A•) ,0	
		F→id(A•) ,0				

Shift on ")"

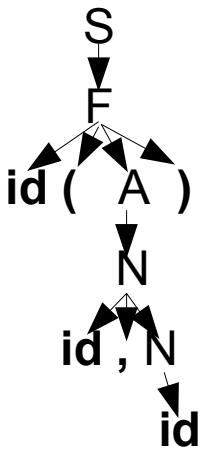
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	**chart[1]**	**chart[2]**	**chart[3]**	**chart[4]**	**chart[5]**	**chart[6]**
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	F→id(A)• ,0
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	S→F• ,0
		A→• ,2	A→N• ,2	N→•id,N ,4	N→id,N• ,2	
		N→•id ,2	F→id(A•) ,0		A→N• ,2	
		N→•id,N ,2			F→id(A•) ,0	
		F→id(A•) ,0				

Reduce on F

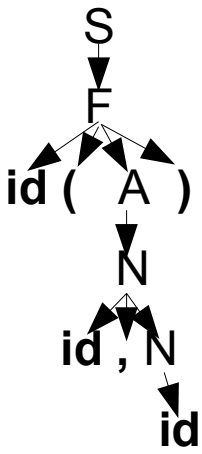
Massive Earley Example

Grammar

S → F
F → id (A)
A → N
A → ε
N → id
N → id , N

Input

id (id , id)



id	(	id	,	id	)	
chart[0]	chart[1]	chart[2]	chart[3]	chart[4]	chart[5]	chart[6]
S→•F ,0	F→id•(A) ,0	F→id(•A) ,0	N→id• ,2	N→id,•N ,2	N→id• ,4	F→id(A)• ,0
F→•id(A) ,0		A→•N ,2	N→id•,N ,2	N→•id ,4	N→id•,N ,4	S→F• ,0
		A→• ,2	A→N• ,2	N→•id,N ,4	N→id,N• ,2	
		N→•id ,2	F→id(A•) ,0		A→N• ,2	
		N→•id,N ,2			F→id(A•) ,0	
		F→id(A•) ,0				

Accept!

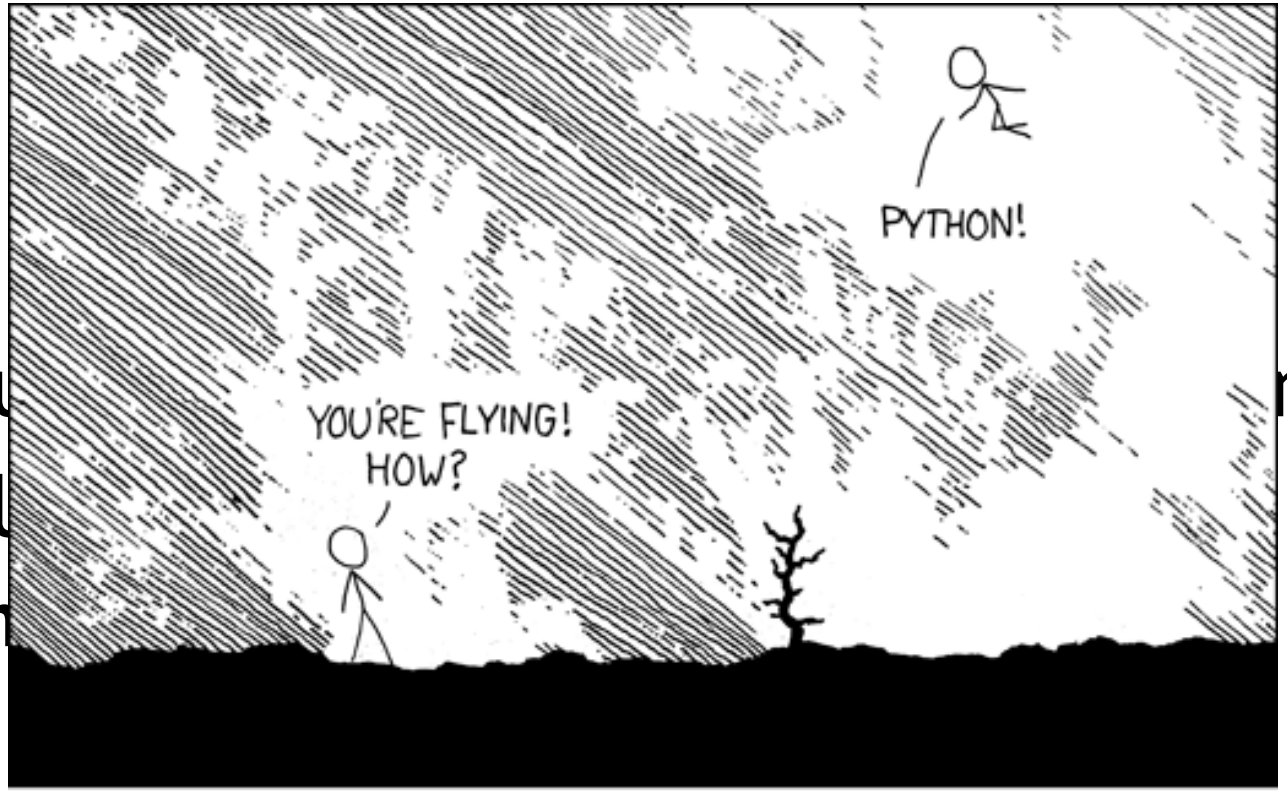
Let's Implement It

- We'll use Python and Functional Programming
- Recall: **List Comprehensions**

```
>>> range(10)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> [ x*x for x in range(10) ]
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
>>> [ x for x in range(10) if x > 5 ]
[6, 7, 8, 9]
>>> [ x*x for x in range(10) if x > 5 ]
[36, 49, 64, 81]
```

- We'll u mming
- Recall

```
>>> ran
[0, 1,
>>> [ x
[0, 1,                                    ]
>>> [ x                                   ]
[6, 7,
>>> [ x                                   5 ]
[36, 49
```



Data Structure Decisions

- For brevity, we'll use Lists and Tuples.
  - Named Tuples in Python 3, Classes, etc

```
grammar = [
  ("S", ["F"]),
  ("F", ["id", "(", "A", ")"]),
  ("A", [ ] ),
  ("A", ["N"] ),
  ("N", ["id", ]),
  ("N", ["id", ",", "N" ]),
]
tokens = [ "id" , "(" , "id", "," , "id", ")" ]
# X→ab.cd, i == (“X”,[“a”,”b”],[“c”,”d”],i)
```

Initialization

```
# By convention, the starting rule is
# the first rule in the grammar.
start_rule = grammar[0]

# The starting parse state is "S -> . abcd , from 0"
start_state = (start_rule[0], [], start_rule[1], 0)

# The parsing chart is a one-dimensional array,
# initially empty.
chart = {}
for i in range(len(tokens)+1):  chart[i] = [ ]

# Start by placing the starting state in chart[0].
chart[0] = [ start_state ]
```

Shift

```
# If chart[i] contains "X -> ab.cd , from j"
# and c is token[i] then add:
# "X -> abc.d , from j" to chart[i+1]
def shift(tokens, i, x, ab, cd, j):
  if cd <> [] and tokens[i] == cd[0]:
    c = cd[0]
    d = cd[1:]
    abc = ab + [c]
    new_chart_state = (x, abc, d, j)
    new_chart_index = i + 1
    return [(new_chart_index, new_chart_state)]
  else:
    return []
```

Closure

```
# If chart[i] contains "X -> ab.cd , from j":
#       and cd is not empty
#       and c is a non-terminal
#       and there is a grammar rule "c -> pqr"
# Then add:
#       "c -> . pqr , from i"
#       to chart[i]
def closure(grammar,i,x,ab,cd,j):
  return [ (i , (rule[0],[],rule[1],i)) \
         for rule in grammar \
         if cd <> [] and cd[0] == rule[0] ]
```

Reduction

```
# If chart[i] contains "X -> ab. , from j"
#  (that is: cd is empty)
#  and chart[j] contains "Y -> pq.Xr , from k"
# Then add
#  "Y -> pqX.r , from k" to chart[i]
def reduction(chart,i,x,ab,cd,j):
  return [ (i, (jstate[0], jstate[1] + [x],
                (jstate[2])[1:], jstate[3] ))
    for jstate in chart[j]
    if cd == [] and jstate[2] <> []
                and (jstate[2])[0] == x ]
```

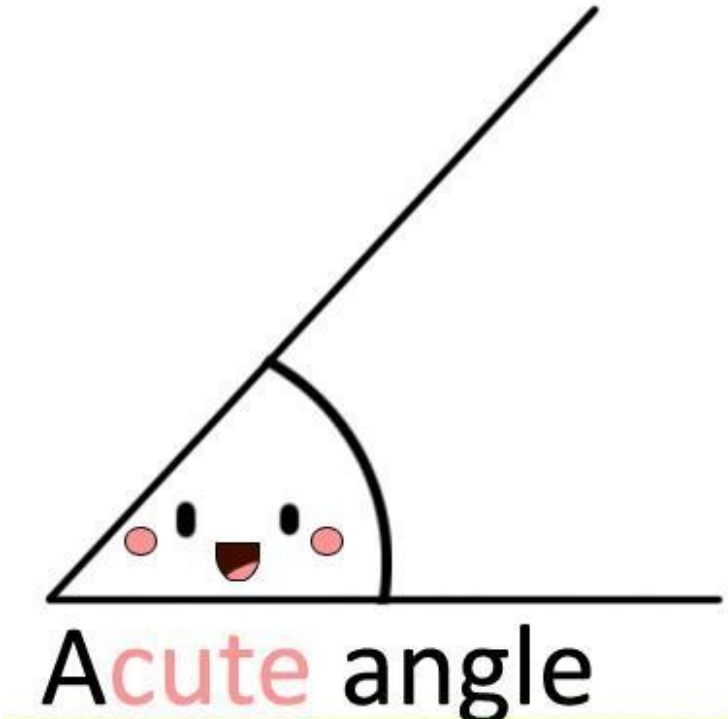
Main Loop

```
# Step 2: Dynamic Programming
for i in range(len(tokens)):
  # Apply shift, closure and reduction until
  # no new parsing states are added to the chart.
  def apply_shift_closure_reduction():
    if any([add_to_chart(chart,
                shift(tokens,i,x,ab,cd,j) +
                closure(grammar,i,x,ab,cd,j) +
                reduction(chart,i,x,ab,cd,j))
              for x, ab, cd, j in chart[i] ]):
      apply_shift_closure_reduction()
      # do it again if any changes

  apply_shift_closure_reduction()
```

Example

```
grammar3 = [
  ("S", ["E"]),
  ("E", ["E", "-", "E" ]),
  ("E", ["E", "+", "E" ]),
  ("E", ["(", "E", ")" ]),
  ("E", ["int"]),
]
tokens3 = [ "int", "-", "int" ]
chart[0]
  S ->  . E              , from 0
  E ->  . E - E          , from 0
  E ->  . E + E          , from 0
  E ->  . ( E )          , from 0
  E ->  . int            , from 0
chart[1]
  E -> int .             , from 0
  S -> E .               , from 0
...
String Accepted: True
```



PA3 in JavaScript: parser.jison

```
%token PLUS MINUS INT
%left PLUS MINUS
%start program

%%
program: exp EOF { return $1; }
       ;

exp: exp PLUS exp  { $$ = ["plus_node", $1, $3]; }
   | exp MINUS exp { $$ = ["minus_node", $1,$3]; }
   | INT           { $$ = ["int_node",
                           Number(yytext) ]; }
   ;
```

PA3 in JavaScript: main.js

```
var cl_lex = [
  ['INT', "11"] ,
  ['PLUS' ] ,
  ['INT', "22"] ,
  ['MINUS' ] ,
  ['INT', "33"] ,
  ['EOF' ] ,
]
var token_count = 0

var parser =
    require("./parser").parser;
```

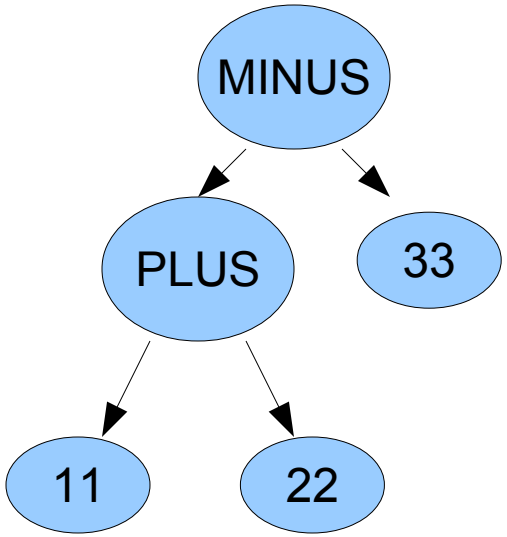
```
parser.lexer = {
  lex : function() {
    var cl_lex_entry =
        cl_lex[token_count++] ;
    var token = cl_lex_entry[0] ;
    var lexeme = cl_lex_entry[1] ;
    parser.lexer.yytext = lexeme ;
    return token;
  },
  setInput : function(str) { }
}

var final_ast = parser.parse("");

console.log(final_ast);
```

PA3 in JavaScript Output:

```
$ node main.js
[ 'minus_node',
    [ 'plus_node',
        [ 'int_node', 11 ],
        [ 'int_node', 22 ]
    ],
    [ 'int_node', 33 ]
]
```



PA3 Not Shown Here

- Reading in the .cl-lex file
- Handling line number information
- Printing out the AST in the desired format
- Adding parsing rules for whole classes and not just simple expressions
- Massive testing effort
  - diff vs. “cool --parse” requires “almost done”
- Dealing with ambiguity (“conflicts”)
  - Let's do this one now.

Conflicts

- Add “%token NEG” and “exp: NEG exp”.
- Oh noes:

```
Conflict in grammar: multiple actions possible when lookahead token is
PLUS in state 8
- reduce by rule: exp -> NEG exp
- shift token (then go to state 6)
Conflict in grammar: multiple actions possible when lookahead token is
MINUS in state 8
- reduce by rule: exp -> NEG exp
- shift token (then go to state 7)

States with conflicts:
State 8
  exp -> NEG exp .          #lookaheads= EOF PLUS MINUS
  exp -> exp . PLUS exp
  exp -> exp . MINUS exp
```

Cor

- Add “%token NEG” a
- Oh noes:

```
Conflict in grammar: multiple acti
PLUS in state 8
- reduce by rule: exp -> NEG exp
- shift token (then go to state 6)
Conflict in grammar: multiple acti
MINUS in state 8
- reduce by rule: exp -> NEG exp
- shift token (then go to state 7)

States with conflicts:
State 8
  exp -> NEG exp .          #lookah
  exp -> exp . PLUS exp
  exp -> exp . MINUS exp
```

Conflict Interpretation

- So some table entry has all three:
  - exp → NEG exp .
  - exp → exp . PLUS exp
  - exp → exp . MINUS exp
- What would the input have to look like to get to that table entry?

Conflict Interpretation

- So some table entry has all three:
  - exp → NEG exp .
  - exp → exp . PLUS exp
  - exp → exp . MINUS exp
- What would the input have to look like to get to that table entry?
  - NEG INT . PLUS INT

Conflict Interpretation

- So some table entry has all three:
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  - exp → exp . MINUS exp
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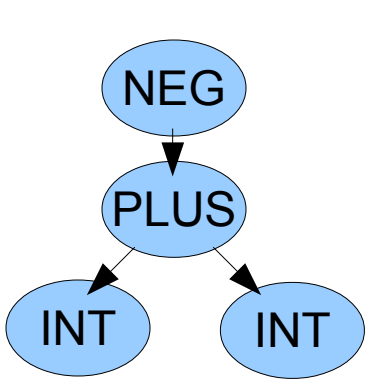
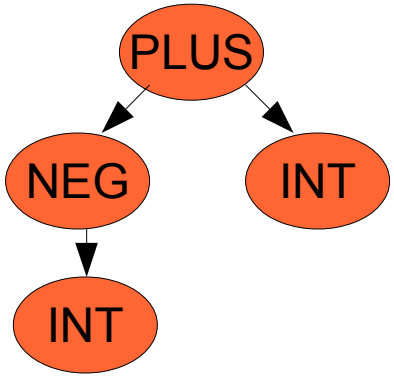
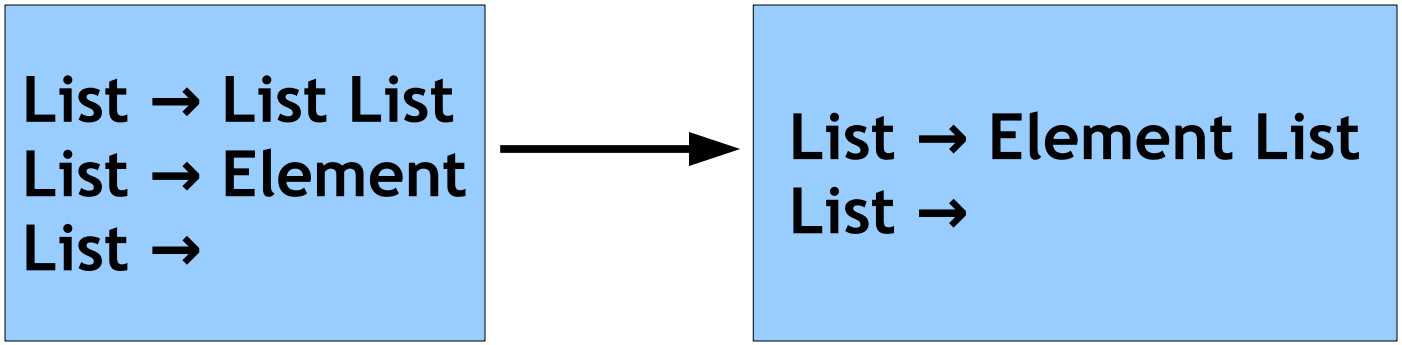
  - NEG INT . PLUS INT





Conflict Solution

- Shift/Reduce
  - Carefully specify precedence and associativity of operators (and sometimes of random tokens).
    - In last example, NEG has higher precedence than PLUS or MINUS.
- Reduce/Reduce
  - Rewrite grammar to avoid gross ambiguity:



Homework

- Midterm 1 Next Class

- PS3 recommended for next Tuesday
- PA3 due next Tuesday
- CA2 due next Tuesday