Mathematical Theory of Rational Behavior: A Probabilistic Approach

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Abstract

This dissertation proposal develops a probabilistic approach to the theory of rational behavior. In the current literature, rational behavior is modeled using automata or retardable elements. However, these modeling approaches involve not only issues of rationality but also complex dynamic behavior, which, on one hand, complicate the analysis and, on the other hand, obscure the issues of rational behavior. In this dissertation proposal, rational behavior is modeled in a purely probabilistic way, which does not give rise to the complicated dynamics specific to automata and retardable elements. In this framework, the behavior of the rational element is characterized by its level of sensibility and the penalty function of the environment.

Within this approach, rational behavior is analyzed utilizing Markov chain techniques. In particular, we analyze the individual rational behavior in stationary and symmetric Markov switch environments and the collective rational behavior in 2×2 zero sum matrix games and under fractional interaction rules. The preliminary results obtained are summarized as follows. In the stationary environment, a rational element exhibits either individual wavering or individual rational behavior depending on the level of sensibility and the penalty function. In the symmetric Markov switch environment, when the environment is switching slowly enough, there is a nontrivial optimal level of sensibility for the rational element to be least penalized. Rational elements as players of 2×2 zero sum matrix games having pure optimal strategies are able to find their optimal strategies with high probability when the level of sensibility is sufficiently large. However, if the 2×2 zero sum matrix game does not have a pure optimal strategy, the rational elements are not capable of finding the mixed optimal strategy. A collective of rational elements with

homogeneous fractional interactions behaves optimally as a whole if the level of sensibility of each individual grows at least as fast as the size of the collective. A collective of rational elements with non-homogeneous fractional interactions behaves optimally as a whole even if the size of the collective tends to infinity as long as the level of sensibility of each individual is sufficiently large.

Two applications of rational elements are discussed in this dissertation proposal. First, rational elements are used as controllers for power-efficiency of a wireless personal-area network (WPAN) with two devices. A collaborative control structure that achieves energy savings of 38% compared to a selfish control structure is obtained. Future research of this application includes using rational elements as controllers for power-efficiency of WPANs with more than two devices and various job loads. Second, rational elements are used to model individuals in an Edgeworth exchange economy, which intends to model individuals with bounded rationality. Future research of this application includes examining the validity of fundamental theorems of economics under the assumption of bounded rationality.

To complete the dissertation, we plan to carry out the following: (1) extend the current results of rational behavior to rational elements with more than two states, (2) introduce the notion of learning into the model of rational elements, and (3) investigate the application of the theory developed in this dissertation proposal to pay and incentive systems.