This homework is due on September 27. Also note that the solutions of the Homework 1 are in the file solhw1.ps You can access this file by logging in at
any unix station in the network and then using the change directory command
to see the files in the eecs320 directory (follow the directions exactly):

cd /afs/engin.umich.edu/u/s/i/singh/eecs320 (return)

You can get a copy of any file in the eecs320 directory into your own directory
“yourdir” (complete pathname) by typing

cp solhw1.ps yourdir (return)

To print this postscript file:

lpr prINTERNAME solhw1.ps

where prINTERNAME is the printer in the lab.

Problem 1: An electron in the silicon conduction band is subjected to an
electric field of 1 kV/cm for 0.1 ps. How far does the electron travel? The mass
of the electron is 0.98 \( m_0 \).

If the same field was applied for the same time to a GaAs conduction band
electron how far would it move?

Problem 2: Problem 8 on page 87 of the text.

Problem 3: Calculate and plot the conductivity of a Si sample as the Fermi
level moves from the valence bandedge to the conduction bandedge. Use the
following mobilities:

\[ \mu_n = 1100 \text{ cm}^2/V.s; \quad \mu_p = 250 \text{ cm}^2/V.s \]

Problem 4: A Si device has the following electron densities in the ON and
OFF states:

\[ ON: \quad 10^{18} \text{ cm}^{-3}; \quad OFF: \quad 10^{14} \text{ cm}^{-3} \]

Calculate the values of \( E_c - E_F \) for the On and OFF cases.

Also calculate the hole densities for each case.

Problem 5: Assume that an electronic device can operate upto a maximum
temperature at which the intrinsic carrier density becomes equal to 10 percent of
the dopant density. What is the maximum operation temperature if transistors
are doped at \( 10^{18} \text{ cm}^{-3} \) for:

(i) Si; (ii) GaAs and (iii) Ge.
SOME IMPORTANT ISSUES DISCUSSED THIS WEEK

- The energy-momentum relations for semiconductors:
  
  i) The electron $E-k$ relation is different in crystals. The effective momentum of the electrons is $\hbar k$ and the electrons respond to the outside world according to the equation

  \[
  \hbar \frac{dk}{dt} = F_{\text{ext}}
  \]

  where $F_{\text{ext}}$ is an external force.

  ii) The allowed energy bands start from the values $E_c$ and $E_V$ for the conduction and valence bands. Near the bandedges, it is possible to write the energy-momentum relations in a simple form which introduces the notion of the effective mass. For direct gap materials where the bottom of the conduction band and the top of the valence band occur at zero effective momentum, we have

  \[
  c - \text{band}:
  \]

  \[
  E(k) = E_c + \frac{\hbar^2 k^2}{2m^*}
  \]

  \[
  v - \text{band}:
  \]

  \[
  E(k) = E_v - \frac{\hbar^2 k^2}{2m_{h*}}
  \]

  \[
  E(k) = E_v - \frac{\hbar^2 k^2}{2m_{l*}}
  \]

  The conduction band is thus described by a single effective mass $m^*$ and the $E-k$ relation is parabolic. In the case of the valence band, there are two bands known as the heavy hole and light hole bands. Both start at $E = E_v$ and curve downwards. Each has its own effective mass. In the case of indirect bandgap semiconductors, the valence band picture is similar to one given above. However, the bottom of the conduction band does not occur at $k = 0$ but at some other value. In Si, the most important semiconductor, the conduction band bottom occurs at six points in $k - \text{space}$. These points $(k_{0x}, k_{0y}, k_{0z})$ are:

  \[
  0.85 \frac{2\pi}{a}(1,0,0)
  \]

  \[
  0.85 \frac{2\pi}{a}(-1,0,0)
  \]

  \[
  0.85 \frac{2\pi}{a}(0,1,0)
  \]

  \[
  0.85 \frac{2\pi}{a}(0,-1,0)
  \]
At each of the above points, the conduction band energy is at it’s minimum value of $E_c$ and then as we move away from these points the energy increases. Thus it appears as if we have six valleys centered at the six points. Each valley is similar to the other ones except that they are centered at different $k$-points. The energy-momentum relations for the electrons are a little more complicated for the indirect gap materials. The relations are:

- **valley along (100) direction:**
  \[
  E(k) = E_c + \frac{\hbar^2}{2} \left( \frac{k_x^2}{m_u} + \frac{k_y^2}{m_u} \right) \\
  \text{with} \quad k_i^2 = (k_x - k_{0x})^2; \quad k_i^2 = (k_y - k_{0y})^2 + (k_z - k_{0z})^2
  \]

- **valley along (010) direction:**
  \[
  E(k) = E_c + \frac{\hbar^2}{2} \left( \frac{k_y^2}{m_u} + \frac{k_z^2}{m_u} \right) \\
  \text{with} \quad k_i^2 = (k_y - k_{0y})^2; \quad k_i^2 = (k_x - k_{0x})^2 + (k_z - k_{0z})^2
  \]

- **valley along (001) direction:**
  \[
  E(k) = E_c + \frac{\hbar^2}{2} \left( \frac{k_z^2}{m_u} + \frac{k_x^2}{m_u} \right) \\
  \text{with} \quad k_i^2 = (k_z - k_{0z})^2; \quad k_i^2 = (k_x - k_{0x})^2 + (k_y - k_{0y})^2
  \]

Note that there is a transverse effective mass and a longitudinal effective mass describing the conduction band edge electrons in Si.

- **Density of states:** Since near the band edges, it is possible to have an energy-momentum relation that looks very similar to the one in free space we can rewrite the density of states expression we obtained for the free electrons to get the relations for the semiconductors. The key differences are that the density of states starts at the band edges and the effective mass replaces the free electron mass. For the direct gap electrons the mass that enters the density of states expression is

  \[
  m_{dos}^* = m^* \\
  (m_{dos}^*)^{3/2} = \left( m_{hh}^{3/2} + m_{lh}^{3/2} \right)
  \]
Note that in the valence band we have two bands and each of them contributes to the density of states a term proportional to $m^{3/2}$. For an indirect gap material the valence band density of states mass is given by the expression above. However, the conduction band density of states mass is

$$m_{\text{dos}}^* = (m_1 m_2 m_3)^{1/3} = (m_1 m_2^2)^{1/3}$$

- Holes in Semiconductors: We have seen that at 0 K there are no electrons in the conduction band of a semiconductor. Also there are no empty states in the valence band. As a result there can be no current flow in the material. If we can get electrons out of the valence band, we can create empty states in the band. This can allow other electrons to move when a field is applied to the material. The negatively charged electrons move in a direction opposite to the applied field, while the hole moves in the direction of the field. From the outside world point of view, we represent the current flow in the valence band by the movement of the empty state. The empty state is called a “hole”. The hole behaves as if it is a positively charged particle.

In semiconductor physics the valence band properties are represented by holes. The conduction band properties are represented by electrons.

- Carrier density in Semiconductors: The mobile carriers that can carry current in semiconductors are:
  i) electrons in the conduction band with a density $n$.
  ii) holes in the valence band with a density $p$.

Of course there are a lot more electrons in the semiconductor ($\sim 10^{23} \text{cm}^{-3}$) but these are the only ones that carry current.

The density of these carriers is given by the position of the Fermi level, the density of states, the density of states mass, the temperature and the positions of the bandedges. In the Boltzmann approximation, it is possible to write the expressions for the carrier densities as simple analytical expressions.

- Intrinsic Semiconductors: Pure semiconductors are called intrinsic or undoped semiconductors. The free carrier density $n + p = n_i + p_i$ in these pure materials is very low unless the temperature is high. Also these carrier densities, known as the intrinsic carrier densities, are not controllable by external fields. In other words the minimum density of mobile carriers is

$$n_i + p_i = 2n_i$$

This means that we cannot use the semiconductor as a switch that can be shut off unless the intrinsic carrier density is very small.

- Use of Dopants in Semiconductors: To obtain mobile carrier densities that can be manipulated and controlled by applied fields, we use dopants. The dopants are able to either contribute an electron to the conduction band (such dopants are called donors) or take an electron from the valence band which is equivalent to giving a hole (such dopants are called acceptors).
It is important to note that within the Boltzmann approximation we have the following relations

\[ np = n_i p_i = \text{constant at given temperature} \]

\[ n = N_e e^{-\frac{E_F - E_F}{k_B T}} \]
\[ p = N_e e^{-\frac{E_F - E_F}{k_B T}} \]
\[ \frac{n}{n_i} = \exp\left(\frac{E_F - E_F}{k_B T}\right) \]

These equations will be very useful in our discussions on devices.

**TOPICS TO BE COVERED NEXT WEEK**

Next week we will discuss the following topics:

- We will start our discussion of how electrons and holes respond to external fields (sections 3.2 and 3.3).
- We will discuss the concepts of carrier transport, velocity-electric field relations, transport by diffusion (Sections 3.3 — 3.6)
- We will also discuss carrier recombination and the continuity equation (Sections 3.7—3.9)