Problem 1 An optical power density of 1W/cm² is incident on a GaAs sample. The photon energy is 2.0 eV and there is no reflection from the surface. Calculate the excess electron-hole carrier densities at the surface and 0.5 μm from the surface. The e-h recombination time is 10⁻⁸ s.

Solution

The optical absorption coefficient at $\hbar\omega = 2.0$ eV is (you may use Fig. 3.11 also)

$$\alpha(\hbar\omega = 2.0 \text{ eV}) = \frac{5.7 \times 10^4 (2.0 - 1.43)^{1/2}}{2.0} = 2.15 \times 10^4 \text{ cm}^{-1}$$

The generation rate at the surface is

$$G_L = \frac{\alpha P_{op}(0)}{\hbar\omega} = \frac{(2.15 \times 10^4 \text{ cm}^{-1})(1 \text{ W cm}^{-2})}{(2.0 \times 1.6 \times 10^{-19} \text{ J})} = 6.72 \times 10^{22} \text{ cm}^{-3} \text{ s}^{-1}$$

The excess carrier density is

$$\delta n = \delta p = G_L \tau_n = G_L \tau_p = (6.72 \times 10^{22} \text{ cm}^{-3} \text{ s}^{-1})(10^{-8}) = 6.72 \times 10^{14} \text{ cm}^{-3}$$

The optical power density at a depth of 0.5 μm is

$$P_{op}(x = 0.5 \mu m) = P_{op}(0) \exp (-\alpha x) = 0.34 \text{ W cm}^{-2}$$

The e-h pair generation rate is now

$$G_L = 2.28 \times 10^{22} \text{ cm}^{-3} \text{ s}^{-1}$$

and the excess charge density is

$$\delta n = \delta p = 2.28 \times 10^{14} \text{ cm}^{-3}$$

Problem 2 Consider a long Si p-n junction with a reverse bias of 1 V at
300 K. The diode has the following parameters:

- **Diode area**, \( A = 1 \text{ cm}^2 \)
- **\( p \)-side doping**, \( N_a = 3 \times 10^{17} \text{ cm}^{-3} \)
- **\( n \)-side doping**, \( N_d = 10^{17} \text{ cm}^{-3} \)
- **Electron diffusion coefficient**, \( D_n = 12 \text{ cm}^2/\text{s} \)
- **Hole diffusion coefficient**, \( D_p = 8 \text{ cm}^2/\text{s} \)
- **Electron minority carrier lifetime**, \( \tau_n = 10^{-7} \text{ s} \)
- **Hole minority carrier lifetime**, \( \tau_p = 10^{-7} \text{ s} \)
- **Optical absorption coefficient**, \( \alpha = 10^3 \text{ cm}^{-1} \)
- **Optical power density**, \( P_{op} = 10 \text{ W/cm}^2 \)
- **Photon energy**, \( \hbar \omega = 1.7 \text{ eV} \)

Calculate the photocurrent in the diode.

**Solution**

We will assume that the carrier generation rate is constant. This is a good approximation for Si where \( \alpha \) is quite small. We need to find the values of \( G_L, W, L_p \) and \( L_n \) to find the photocurrent.

The carrier generation rate is

\[
G_L = \frac{\alpha P_{op}}{\hbar \omega} = \frac{(10^3 \text{ cm}^{-1})(10 \text{ W cm}^{-2})}{(1.7 \times 1.6 \times 10^{-19} \text{J})} = 3.68 \times 10^{22} \text{ cm}^{-3}\text{s}^{-1}
\]

The built-in voltage in the diode is

\[
V_{bi} = k_B T \ln \frac{n_n}{n_p} = 0.85 \text{ eV}
\]

The depletion width is

\[
W = \left[ \frac{2e(V_{bi} + V_r)}{e} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} = 1.8 \times 10^{-5} \text{ cm}
\]

The diffusion lengths are

\[
L_p = (D_p \tau_p)^{1/2} = 8.94 \times 10^{-4} \text{ cm}
\]
\[
L_n = (D_n \tau_n)^{1/2} = 10.95 \times 10^{-4} \text{ cm}
\]

The photocurrent is now

\[
I_L = eG_L (L_p + L_n + W) A
\]
\[
= (1.6 \times 10^{-19} \text{C})(3.68 \times 10^{22} \text{ cm}^{-3}\text{s}^{-1}) [8.94 \times 10^{-4} + 10.95 \times 10^{-4} + 1.8 \times 10^{-5} \text{ cm}] \text{ (1 cm}^2\text{)}
\]
\[
= 11.8 \text{ A}
\]
**Problem 3** Consider a long Si p-n junction solar cell with an area of 4 cm\(^2\) at 300 K. The solar cell has the following parameters:

- **n-type doping**, \(N_d = 10^{18} \text{ cm}^{-3}\)
- **p-type doping**, \(N_a = 3 \times 10^{17} \text{ cm}^{-3}\)
- **Electron diffusion coefficient**, \(D_n = 15 \text{ cm}^2/\text{s}\)
- **Hole diffusion coefficient**, \(D_p = 7.5 \text{ cm}^2/\text{s}\)
- **Electron minority carrier lifetime**, \(\tau_n = 10^{-7} \text{ s}\)
- **Hole minority carrier lifetime**, \(\tau_p = 10^{-7} \text{ s}\)
- **Electron current**, \(I_L = 1.0 \text{ A}\)
- **Diode ideality factor**, \(m = 1.25\)

Calculate the open circuit voltage of the diode. If the fill factor is 0.75, calculate the maximum power output.

**Solution**

The open circuit voltage is given by

\[
V_{oc} = \frac{m k_B T}{e} \ln \left( \frac{I_L}{I_o} \right)
\]

The prefactor \(I_o\) is given by

\[
I_o = e A \left( \frac{D_p n_p}{L_p} + \frac{D_n n_p}{L_n} \right)
\]

\[
= (1.6 \times 10^{-19} \text{ C})(4 \text{ cm}^2) \left( \frac{(7.5 \text{ cm}^2/\text{s})(2.25 \times 10^2 \text{ cm}^{-3})}{(2.73 \times 10^{-3} \text{ cm})} + \frac{(15 \text{ cm}^2/\text{s})(7.5 \times 10^2 \text{ cm}^{-3})}{(1.22 \times 10^{-3} \text{ cm})} \right)
\]

\[
= 6.3 \times 10^{-12} \text{ A}
\]

The open circuit voltage is now

\[
V_{oc} = (1.25)(0.026 \text{ V}) \ln \left( \frac{1.0}{6.3 \times 10^{-12}} \right)
\]

\[
= 0.84 \text{ V}
\]

The maximum power output is

\[
P_{out} = I_L V_{oc} F = (1.0 \text{ A})(0.84 \text{ V})(0.75)
\]

\[
= 0.63 \text{ W}
\]

**Problem 4** Consider a GaAs p-n\(^+\) junction LED with the following parameters at 300 K:

- **Electron diffusion coefficient**, \(D_n = 25 \text{ cm}^2/\text{s}\)
- **Hole diffusion coefficient**, \(D_p = 12 \text{ cm}^2/\text{s}\)
- **n-side doping**, \(N_d = 5 \times 10^{17} \text{ cm}^{-3}\)
- **p-side doping**, \(N_a = 10^{16} \text{ cm}^{-3}\)
- **Electron minority carrier lifetime**, \(\tau_n = 10 \text{ ns}\)
- **Hole minority carrier lifetime**, \(\tau_p = 10 \text{ ns}\)
Calculate the injection efficiency of the LED assuming no trap-related recombination.

**Solution**

The injection efficiency for the GaAs $p^n$ LED is given by

$$\gamma_{in,j} = \frac{D_n n_p}{L_n + D_p n_n}$$

The diffusion lengths are

$$L_n = \sqrt{D_n \tau_n} = 1.12 \times 10^{-3} \text{ cm}$$
$$L_p = \sqrt{D_p \tau_p} = 1.1 \times 10^{-3} \text{ cm}$$

Also

$$n_p = \frac{n_i^2}{N_n} = \frac{(1.84 \times 10^6 \text{ cm}^{-3})^2}{(10^{16} \text{ cm}^{-3})} = 3.39 \times 10^{-4} \text{ cm}^{-3}$$
$$p_n = \frac{n_i^2}{N_d} = \frac{(1.84 \times 10^6 \text{ cm}^{-3})^2}{(5 \times 10^{17} \text{ cm}^{-3})} = 6.77 \times 10^{-6}$$

$$\gamma_{in,j} = 0.9903$$

**Problem 5** The diode in Problem 4 is to be used to generate an optical power of 1 mW. The diode area is 1 mm$^2$ and the external radiative efficiency is 20%. Calculate the forward bias voltage required.

**Solution**

The current in the diode is (relevant for photon emission)

$$I_n = \frac{A e D_n n_p}{L_n} \left[ \exp \left( \frac{e V}{k_B T} \right) - 1 \right]$$

and the photons generated per second are

$$I_{ph} = \frac{I_n}{e \eta_{ext}} = \frac{A D_n n_p \eta_{ext}}{L_n} \exp \left( \frac{e V}{k_B T} \right)$$

The optical power is

$$(P_{op} \cdot A) = I_{ph} \cdot \hbar \omega = \frac{A D_n n_p \eta_{ext} \hbar \omega}{L_n} \exp \left( \frac{e V}{k_B T} \right)$$

If this power is 1 mW, the external voltage is

$$V = \frac{k_B T}{e} \ln \left[ \frac{(P_{op} \cdot A)L_n}{A D_n n_p \eta_{ext} \hbar \omega} \right]$$
$$= 0.026 \ln \left[ \frac{(10^{-3} \text{ W})(1.12 \times 10^{-3} \text{ cm})}{(10^{-4} \text{ cm}^2)(25 \text{ cm}^2/\text{s})(3.39 \times 10^{-4} \text{ cm}^{-3})(0.2)(1.43 \times 1.6 \times 10^{-19} \text{ J})} \right]$$
$$= 1.165 \text{ V}$$