EECS 203
Homework 4 Solutions

Total Points: 50
Page 54-55:
10) Let A and B be sets. Show that:
(10 points)

a) $(A \cap B) \subseteq A$

$(A \cap B) = \{x$: x belongs to A and B$\}$

Hence every element that belongs to $(A \cap B)$ also belongs to A. Thus $(A \cap B) \subseteq A$

Using Membership Tables

A	**B**	$(A \cap B)$	$(A \cap B) \cap A$
1	**1**	**1**	**1**
1	**0**	**0**	**0**
0	**1**	**0**	**0**
0	**0**	**0**	**0**

Since $(A \cap B) = (A \cap B) \cap A$
$\therefore$ All elements of $(A \cap B)$ lie within set A
$\therefore (A \cap B) \subseteq A$

b) $A \subseteq (A \cup B)$

$(A \cup B) = \{x$: x belongs to A or x belongs to B or both$\}$

Hence, every element that belongs to A also belongs to $(A \cup B)$. Thus $A \subseteq (A \cup B)$

Using Membership Tables

A	**B**	$(A \cup B)$	$A \cap (A \cup B)$
1	**1**	**1**	**1**
1	**0**	**1**	**1**
0	**1**	**1**	**0**
0	**0**	**0**	**0**

Since $A = A \cap (A \cup B)$
$\therefore$ All elements of A lie within $(A \cup B)$
$\therefore A \subseteq (A \cup B)$

c) $A - B \subseteq A$

$(A - B) = \{x$: x belongs to A and x does not belong to B$\}$

Hence, every element that belongs to A – B has to belong to A. Thus $A - B \subseteq A$

Using Membership Tables

A	**B**	$(A-B)$	$(A-B) \cap A$
1	**1**	**0**	**0**
1	**0**	**1**	**1**
0	**1**	**0**	**0**
0	**0**	**0**	**0**

Since $(A-B) = (A-B) \cap A$
$\therefore$ All elements of $(A-B)$ lie within set A
$\therefore A - B \subseteq A$

d) A∩(B-A) = ∅

(B-A) = {x : x belongs to B and x does not belong to A}
A∩(B-A) = {x : x belongs to A and x belongs to B and x does not belong to A}.
No element can belong to A and not belong to A at the same time. Hence A∩(B-A) = ∅.

Using Membership Tables

A	**B**	(B-A)	A∩(B-A)
1	**1**	**0**	**0**
1	**0**	**0**	**0**
0	**1**	**1**	**0**
0	**0**	**0**	**0**

∴ A∩(B-A) = ∅

e) A∪(B-A) = A∪B

It can be proved that B-A = B∩Ā.
Thus A∪(B-A) = A∪(B∩Ā)
= (A∪B)∩(A∪Ā)
= (A∪B)∩(U)
= (A∪B)

where we used the Distributive Law and the fact that (A∪Ā) = U which is the universal set.

Using Membership Tables

A	**B**	B-A	A∪(B-A)	A∪B
1	**1**	**0**	**1**	**1**
1	**0**	**0**	**1**	**1**
0	**1**	**1**	**1**	**1**
0	**0**	**0**	**0**	**0**

∴ A∪(B-A) = A∪B

16)
(3 points)

This is the easiest way to show this identity.

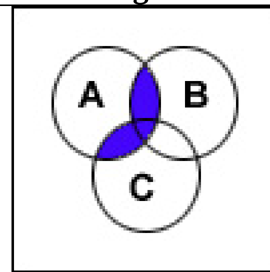
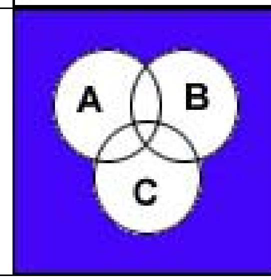
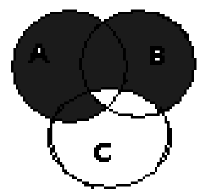
Let A, B, and C be sets. Show that (A-B)-C = (A-C)-(B-C)

Using Membership Tables

A	**B**	**C**	**A-B**	**A-C**	**B-C**	**(A-B)-C**	**(A-C)-(B-C)**
1	**1**	**1**	**0**	**0**	**0**	**0**	**0**
1	**1**	**0**	**0**	**1**	**1**	**0**	**0**
1	**0**	**1**	**1**	**0**	**0**	**0**	**0**
1	**0**	**0**	**1**	**1**	**0**	**1**	**1**
0	**1**	**1**	**0**	**0**	**0**	**0**	**0**
0	**1**	**0**	**0**	**0**	**1**	**0**	**0**
0	**0**	**1**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**

∴(A-B)-C = (A-C)-(B-C)

18) Draw the Venn diagrams for each of the following combinations of the sets A, B, and C
(9 points)

	Combination	Diagram
A)	$A \cap (B \cup C)$	
B)	$\overline{A} \cap \overline{B} \cap \overline{C}$	
C)	$(A - B) \cup (A - C) \cup (B - C)$	

28b) Show that if A and B are sets, then (A⊕B) ⊕B = A
(2 points)

Using Membership Tables

A	B	A⊕B	(A⊕B) ⊕B
1	**1**	**0**	**1**
1	**0**	**1**	**1**
0	**1**	**1**	**0**
0	**0**	**0**	**0**

∴(A⊕B) ⊕B = A

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10) (8 points) Determine whether each of the following functions from **Z** to **Z** is one-to-one.

a)	**f(n) = n-1**
	Yes. This is a strictly increasing function; i.e.; f(x) > f(y) whenever x>y, and a strictly increasing function is one-to-one (and onto).
b)	**f(n) = n^2 +1**
	No. This can proved by a counter example; f(1) = 2, and also f(-1) =2.
c)	**f(n) = n^3**
	Yes. Because this is a strictly increasing function
d)	**f(n) = $\lceil n/2 \rceil$**
	No. This can be shown by a counter example; f(1) = 1, and f(2) = 1.

12) (8 points) Give an example of a function from **N** to **N** that is
(There can be many possible answers to this question)

a) **One-to-one but not onto.**

f(n) = 2n +1. (only odd values are mapped)

b) **Onto but not one-to-one**

f(n) = $\lceil n/2 \rceil$

c) **Both onto and one-to-one (but different from the identity function)**

f(n) = n+1 when n is even (even numbers are mapped to odd numbers; take 0 as an even number)
f(n) = n-1 when n is odd (odd numbers are mapped to even numbers)

d) **Neither one-to-one nor onto**

f(n) = 10 when n is even
f(n) = 0 when n is odd

14) (2 points) Determine whether each of the following functions is a bijection from **R** to **R.**

a) f(x) = -3x + 4
Yes.

d) f(x) = x^5 +1
Yes.

20)
(8 points)

If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

Yes. If f and f *o* g are one-one functions, g is also one-one.
The proof is by contradiction. Suppose g is not one-one then we prove that either f is not one-one or f *o* g is not one-one.
The function f is not under our control but f *o* g is under our control because g is under our control.
If f is not one-one, then the proof ends there itself. But whatever be the case of f, f *o* g cannot be one-one. This can be proved by the following argument.
Let g: A $\rightarrow$ B and f: B $\rightarrow$ C.
By assumption, since g is not one-one, there exists 2 distinct elements x1 and x2 such that g(x1) = g(x2) = y where y belongs to B.
Let f(y) = z for some z belonging to C.
Thus, f *o* g (x1) = f *o* g (x2) = z. Hence f *o* g cannot be one-one.

This means that if f and f *o* g are one-one, g has to be one-one using the fact that $\boldsymbol{p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p}$