Group assignment 3 – A practice exam
EECS 203
Spring 2016

Name (Print): _____

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General Instructions

This is a group assignment that is posted as a LaTeX file. You are to answer the questions using LaTeX.

The "exam" is worth 100 points. Your grade on the assignment is out of 30. For every point you lose on the exam, you lose one on the group assignment (so a 90 exam score would be a 20 group assignment score).

True/False Questions. 2 pts each.

`Question 1`

For all sets A and B, $A \cup (B \cap \emptyset) = A$.

(A) True (B) False

`Question 2`

For all sets A, $\overline{(\overline{A}) \cup A} = A$.

(A) True (B) False

`Question 3`

$\{\emptyset, \{1\}, \{2\}\}$ is the power set of some set.

(A) True (B) False

`Question 4`

Let a and b be distinct elements. Then $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ is the power set of some set.

(A) True (B) False

`Question 5`

For all sets A, B and C, if $A \cap C \subseteq B \cap C$, then $A \subseteq B$.

(A) True (B) False

`Question 6`

$(p \oplus q) \vee (\neg p \to (q \wedge p))$ is a tautology.

(A) True (B) False

Question 7

$(p \vee r) \wedge (\neg q \vee r) \wedge (\neg p \wedge \neg r)$ **is a contradiction.**

(A) True (B) False

Question 8

$(\neg q \vee p) \wedge (p \oplus q)$ **is satisfiable.**

(A) True (B) False

For problems 9, 10, and 11 below, we define the following propositions.

- s: **We will go swimming this afternoon.**
- t: **There is a thunderstorm today.**

Consider the following sentence.

> **We will not go swimming this afternoon if there is a thunderstorm today.**

`Question 9`

The sentence above is equivalent to the proposition $\neg t \to \neg s$.

(A) True (B) False

`Question 10`

The sentence above is equivalent to the proposition $s \to \neg t$.

(A) True (B) False

`Question 11`

The sentence above is equivalent to the proposition $s \vee t$.

(A) True (B) False

Question 12

Determine whether the argument given is valid or invalid:

Everyone enrolled in the university has eaten at a dining hall. Ralph has not eaten at a dining hall. Therefore, Ralph is not enrolled in the university.

(A) Valid (B) Invalid

Question 13

$\forall x \exists y (Q(x) \wedge \neg P(y))$ **is equivalent to** $\neg \exists x \forall y (Q(x) \rightarrow P(y))$**.**

(A) True (B) False

Question 14

$\exists x \forall y Q(x, y) \rightarrow \neg \exists y \forall x \neg Q(x, y)$ **is a tautology.**

(A) True (B) False

For problems 16, 17, and 18 below, we define the following predicates.

- $C(x)$: **x is a child**
- $A(x)$: **x grew up in Ann Arbor**
- $F(x)$: **x likes Michigan football**
- $B(x)$: **x likes barbecues.**

Consider the sentence

> **Every child who grew up in Ann Arbor likes Michigan football and barbecues.**

`Question 15`

The sentence above is equivalent to $\forall x(C(x) \to A(x) \wedge (F(x) \wedge B(x)))$.

(A) True (B) False

`Question 16`

The sentence above is equivalent to $\forall x(C(x) \wedge A(x) \wedge (F(x) \wedge B(x)))$.

(A) True (B) False

`Question 17`

The sentence above is equivalent to

$$\forall x(((C(x) \wedge A(x)) \to F(x)) \wedge ((C(x) \wedge A(x)) \to B(x))).$$

(A) True (B) False

Question 18

The function f from $\mathbb{Z}^+$ to $\mathbb{Z}^+$ defined by $f(x) = \lfloor 2.5x \rfloor$ is onto.

(A) True (B) False

Question 19

The function f from $\mathbb{N}$ to $\mathbb{N}$ defined by $f(x) = x^2$ is one-to-one.

(A) True (B) False

Question 20

The function f from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 4x^3 - 7$ is a bijection (one-to-one correspondence).

(A) True (B) False

Question 21

The function f from $\mathbb{R}$ to $\{-1, 1\}$ defined by

$$f(x) = \begin{cases} -1 & \textbf{if } x \in \mathbb{Q} \\ 1 & \textbf{if } x \notin \mathbb{Q} \end{cases}$$

is one-to-one.

(A) True (B) False

Question 22

The function f from $\mathbb{R} - \{0\}$ to $\mathbb{R} - \{0\}$ given by $f(x) = 1/x$ has an inverse.

(A) True (B) False

Question 23

The value of the sum

$$\sum_{k=0}^{15}(2e^{k+1} - 2e^{k})$$

is $2e^{16}$.

(A) True (B) False

Question 24

If $A \subseteq B$ and both A and B are countably infinite, then $B - A$ is finite.

(A) True (B) False

Question 25

The set $\mathbb{Z}^{-} \cup \mathbb{Q}^{+}$ is countable.

(A) True (B) False

Problem 26. 10 points.

Given the premise

$$\forall x P(x) \wedge \exists x Q(x),$$

prove

$$\exists x (P(x) \wedge Q(x))$$

using a deductive argument.

Problem 27. 10 points.

(a) Show that if $f : A \to B$ and $S, T \subseteq A$, then

$$(f(S) - f(T)) \subseteq f(S - T).$$

(b) Is it also true that $f(S - T) \subseteq (f(S) - f(T))$? Circle one:

Yes **No**

Prove your answer.

Problem 28. 10 points.

Let m and n be positive integers. Prove by contraposition that if $m \cdot n + 4$ is odd, then m is odd and n is odd.

Problem 29. 10 points.

Let $L(x, y)$ be the statement "x loves y", and let $M(x, y)$ be the statement "x is the mother of y" where the domain of discourse consists of all the people on earth. Use quantifiers to formally express the following statements.

(a) Everybody loves their mother.

(b)Nobody loves everybody.

(c) All mothers love all of their children.

(d) Everyone loves exactly one person.

(e) There is someone who loves no one except himself or herself.

Problem 30. 10 points.

Use the rules of logic to prove that $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology.