Group assignment 3 – A practice exam
EECS 203
Spring 2016

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General Instructions

This is a group assignment that is posted as a LaTeX file. You are to answer the questions using LaTeX.

The "exam" is worth 100 points. Your grade on the assignment is out of 30. For every point you lose on the exam, you lose one on the group assignment (so a 90 exam score would be a 20 group assignment score).

True/False Questions. 2 pts each.

Question 1

For all sets A and B, $A \cup (B \cap \emptyset) = A$.

[(A) True] **(B) False**

Question 2

For all sets A, $\overline{(\overline{A}) \cup A} = A$.

(A) True **[(B) False]**

Question 3

$\{\emptyset, \{1\}, \{2\}\}$ is the power set of some set.

(A) True **[(B) False]**

Question 4

Let a and b be distinct elements. Then $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ is the power set of some set.

[(A) True] **(B) False**

Question 5

For all sets A, B and C, if $A \cap C \subseteq B \cap C$, then $A \subseteq B$.

(A) True **[(B) False]**

Question 6

$(p \oplus q) \vee (\neg p \to (q \wedge p))$ is a tautology.

(A) True **[(B) False]**

Question 7

$(p \vee r) \wedge (\neg q \vee r) \wedge (\neg p \wedge \neg r)$ **is a contradiction.**

(A) True **(B) False**

Question 8

$(\neg q \vee p) \wedge (p \oplus q)$ **is satisfiable.**

(A) True **(B) False**

For problems 9, 10, and 11 below, we define the following propositions.

- s: **We will go swimming this afternoon.**
- t: **There is a thunderstorm today.**

Consider the following sentence.

> **We will not go swimming this afternoon if there is a thunderstorm today.**

`Question 9`

The sentence above is equivalent to the proposition $\neg t \to \neg s$.

(A) True **[(B) False]**

`Question 10`

The sentence above is equivalent to the proposition $s \to \neg t$.

[(A) True] **(B) False**

`Question 11`

The sentence above is equivalent to the proposition $s \vee t$.

(A) True **[(B) False]**

Question 12

Determine whether the argument given is valid or invalid:

Everyone enrolled in the university has eaten at a dining hall. Ralph has not eaten at a dining hall. Therefore, Ralph is not enrolled in the university.

(A) Valid **(B) Invalid**

Question 13

$\forall x \exists y (Q(x) \wedge \neg P(y))$ **is equivalent to** $\neg \exists x \forall y (Q(x) \rightarrow P(y))$**.**

(A) True **(B) False**

Question 14

$\exists x \forall y Q(x, y) \rightarrow \neg \exists y \forall x \neg Q(x, y)$ **is a tautology.**

(A) True **(B) False**

For problems 16, 17, and 18 below, we define the following predicates.

- $C(x)$: **x is a child**
- $A(x)$: **x grew up in Ann Arbor**
- $F(x)$: **x likes Michigan football**
- $B(x)$: **x likes barbecues.**

Consider the sentence

> **Every child who grew up in Ann Arbor likes Michigan football and barbecues.**

`Question 15`

The sentence above is equivalent to $\forall x(C(x) \to A(x) \wedge (F(x) \wedge B(x)))$.

(A) True **(B) False**

`Question 16`

The sentence above is equivalent to $\forall x(C(x) \wedge A(x) \wedge (F(x) \wedge B(x)))$.

(A) True **(B) False**

`Question 17`

The sentence above is equivalent to

$$\forall x(((C(x) \wedge A(x)) \to F(x)) \wedge ((C(x) \wedge A(x)) \to B(x))).$$

(A) True **(B) False**

Question 18

The function f from $\mathbb{Z}^+$ to $\mathbb{Z}^+$ defined by $f(x) = \lfloor 2.5x \rfloor$ is onto.

(A) True **(B) False**

Question 19

The function f from $\mathbb{N}$ to $\mathbb{N}$ defined by $f(x) = x^2$ is one-to-one.

(A) True **(B) False**

Question 20

The function f from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 4x^3 - 7$ is a bijection (one-to-one correspondence).

(A) True **(B) False**

Question 21

The function f from $\mathbb{R}$ to $\{-1, 1\}$ defined by

$$f(x) = \begin{cases} -1 & \text{if } x \in \mathbb{Q} \\ 1 & \text{if } x \notin \mathbb{Q} \end{cases}$$

is one-to-one.

(A) True **(B) False**

Question 22

The function f from $\mathbb{R} - \{0\}$ to $\mathbb{R} - \{0\}$ given by $f(x) = 1/x$ has an inverse.

(A) True **(B) False**

Question 23

The value of the sum

$$\sum_{k=0}^{15}(2e^{k+1} - 2e^k)$$

is $2e^{16}$.

(A) True **(B) False**

Question 24

If $A \subseteq B$ and both A and B are countably infinite, then $B - A$ is finite.

(A) True **(B) False**

Question 25

The set $\mathbb{Z}^- \cup \mathbb{Q}^+$ is countable.

(A) True **(B) False**

Problem 26. 10 points.

Given the premise

$$\forall x P(x) \wedge \exists x Q(x),$$

prove

$$\exists x (P(x) \wedge Q(x))$$

using a deductive argument.

Solution:

1) $\forall x P(x) \wedge \exists x Q(x)$	**premise**
2) $\exists x Q(x)$	**simplification (1)**
3) $Q(c)$	**existential instantiation (2)**
4) $\forall x P(x)$	**simplification (1)**
5) $P(c)$	**universal instantiation (4)**
6) $P(c) \wedge Q(c)$	**conjunction (3,5)**
7) $\exists x (P(x) \wedge Q(x))$	**existential generalization (6)**

Problem 27. 10 points.

(a) Show that if $f : A \to B$ and $S, T \subseteq A$, then

$$(f(S) - f(T)) \subseteq f(S - T).$$

Solution:

Let $x \in f(S) - f(T)$. Then $x \in f(S)$ and $x \notin f(T)$. Since $x \in f(S)$, there exists some $y \in S$ such that $f(y) = x$. Furthermore, since $x \notin f(T)$, it must be the case that $y \notin T$. Therefore, $y \in S - T$ and so $x \in f(S - T)$. Thus, $(f(S) - f(T)) \subseteq f(S - T)$.

(b) Is it also true that $f(S - T) \subseteq (f(S) - f(T))$? Circle one:

Yes **No**

Prove your answer.

Solution:

There are many counterexamples that could work here. One is given below.

Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$. Let $S = \mathbb{R}$ and let $T = \mathbb{R}^-$. Then $S - T = \mathbb{R}^+ \cup \{0\}$. Note that in this case,

$$f(S - T) = \mathbb{R}^+ \cup \{0\}.$$

However, we have that

$$f(S) = \mathbb{R}^+ \cup \{0\}$$

and

$$f(T) = \mathbb{R}^+$$

and therefore $f(S) - f(T) = \{0\}$ and so $f(S - T) \not\subseteq (f(S) - f(T))$.

Problem 28. 10 points.

Let m and n be positive integers. Prove by contraposition that if $m \cdot n + 4$ is odd, then m is odd and n is odd.

Solution:

By means of contraposition, assume that it is not the case that m is odd and n is odd. This means that either m or n is even. Without loss of generality, we may assume that m is even. Therefore, we can write $m = 2k$ for some integer k. But then $m \cdot n + 4 = 2k \cdot n + 4 = 2(k \cdot n + 2)$ and therefore we see that $m \cdot n$ is even.

Problem 29. 10 points.

Let $L(x, y)$ be the statement "x loves y", and let $M(x, y)$ be the statement "x is the mother of y" where the domain discourse consists of all the people on earth. Use quantifiers to formally express the following statements.

(a) Everybody loves their mother.

$$\forall x \forall y (M(x, y) \rightarrow L(y, x))$$

(b)Nobody loves everybody.

$$\forall x \exists y (\neg L(x, y))$$

(c) All mothers love all of their children.

$$\forall x \forall y (M(x, y) \rightarrow L(x, y))$$

(d) Everyone loves exactly one person.

$$\forall x \exists y (L(x, y) \wedge \forall z ((y \neq z) \rightarrow \neg L(x, z)))$$

(e) There is someone who loves no one except himself or herself.

$$\exists x \forall y ((x \neq y) \rightarrow \neg L(x, y))$$

Problem 30. 10 points.

Use the rules of logic to prove that $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology.

Solution:

$$
\begin{aligned}
(\neg p \wedge (p \vee q)) \rightarrow q &\equiv \neg(\neg p \wedge (p \vee q)) \vee q && \textbf{definition of implication} \\
&\equiv \neg(\neg p) \vee \neg(p \vee q) \vee q && \textbf{DeMorgan's} \\
&\equiv p \vee \neg(p \vee q) \vee q && \textbf{double negation} \\
&\equiv p \vee (\neg p \wedge \neg q) \vee q && \textbf{DeMorgan's} \\
&\equiv ((p \vee \neg p) \wedge (p \vee \neg q)) \vee q && \textbf{distribution} \\
&\equiv (T \wedge (p \vee \neg q)) \vee q && \textbf{negation} \\
&\equiv (p \vee \neg q) \vee q && \textbf{identity} \\
&\equiv p \vee (\neg q \vee q) && \textbf{associative} \\
&\equiv p \vee T && \textbf{negation} \\
&\equiv T && \textbf{domination}
\end{aligned}
$$