Quine-McCluskey

- Note: The vast majority of this is taken from Prof. Steven Nowick of Columbia. http://www.cs.columbia.edu/~cs6861/handouts/quine-mccluskey-handout.pdf
- A briefer overview, including showing a relationship to K-maps, can be found at http://tinyurl.com/QM-dick. It is written by our own Dr. Robert Dick.

Quine-McCluskey is a tabular method for finding the minimal sum-of-products. It is a method that is useful for two primary reasons:

1. It gives us a much more easy-to-program method than K-maps
2. It provides more formal detail about how to select prime implicants.

Steps are:

1. Generate Prime Implicants
2. Construct Prime Implicant Table
3. Reduce Prime Implicant Table (iterate until done…)
    a. Remove Essential Prime Implicants
    b. Row Dominance
    c. Column Dominance
4. Solve Prime Implicant Table

AB/CD	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Step 1: Generate Prime Implicants

$F(A,B,C,D) = \Sigma m(0,2,5,6,7,8,10,12,13,14,15)$

List minterms by their "Hamming weight"—that is the # of 1s in them. Consider:

List Minterms

Column I	
0	0000
2	0010
8	1000
5	0101
6	0110
10	1010
12	1100
7	0111
13	1101
14	1110
15	1111

Column I			*Column II*	
0	0000	√	(0,2)	00-0
2	0010	√	(0,8)	-000
8	1000	√	(2,6)	0-10
5	0101	√	(2,10)	-010
6	0110	√	(8,10)	10-0
10	1010	√	(8,12)	1-00
12	1100	√	(5,7)	01-1
7	0111	√	(5,13)	-101
13	1101	√	(6,7)	011-
14	1110	√	(6,14)	-110
15	1111	√	(10,14)	1-10
			(12,13)	110-
			(12,14)	11-0
			(7,15)	-111
			(13,15)	11-1
			(14,15)	111-

Now do a pair-wise check to see which terms can be combined with the grouping in front of them. Put a check next to each minterm that can be combined with another minterm.

Continue into column III, doing the same thing.

Column I			*Column II*			*Column III*	
0	0000	√	(0,2)	00-0	√	(0,2,8,10)	-0-0
2	0010	√	(0,8)	-000	√	(0,8,2,10)	-0-0
8	1000	√	(2,6)	0-10	√	(2,6,10,14)	–10
5	0101	√	(2,10)	-010	√	(2,10,6,14)	–10
6	0110	√	(8,10)	10-0	√	(8,10,12,14)	1–0
10	1010	√	(8,12)	1-00	√	(8,12,10,14)	1–0
12	1100	√	(5,7)	01-1	√	(5,7,13,15)	-1-1
7	0111	√	(5,13)	-101	√	(5,13,7,15)	-1-1
13	1101	√	(6,7)	011-	√	(6,7,14,15)	-11-
14	1110	√	(6,14)	-110	√	(6,14,7,15)	-11-
15	1111	√	(10,14)	1-10	√	(12,13,14,15)	11–
			(12,13)	110-	√	(12,14,13,15)	11–
			(12,14)	11-0	√		
			(7,15)	-111	√		
			(13,15)	11-1	√		
			(14,15)	111-	√		

Column III contains a number of duplicate entries, *e.g.* (0,2,8,10) and (0,8,2,10). Duplicate entries appear because a product in Column III can be formed in several ways. For example, (0,2,8,10) is formed by combining products (0,2) and (8,10) from Column II, and (0,8,2,10) (the same product) is formed by combining products (0,8) and (2,10). **Cross out the duplicate entries.**
That leaves: (0,2,8,10), (2,6,10,14), (5,7,13,15), (6,7,14,15), (8,10,12,14) and (12,13,14,15) or

$B'D'$, CD', BD, BC, AD' and AB.

Step 2: Construct Prime Implicant Table.

	$B'D'$ (0,2,8,10)	CD' (2,6,10,14)	BD (5,7,13,15)	BC (6,7,14,15)	AD' (8,10,12,14)	AB (12,13,14,15)
0	X					
2	X	X				
5			X			
6		X		X		
7			X	X		
8	X				X	
10	X	X			X	
12					X	X
13			X			X
14		X		X	X	X
15			X	X		X

Step 3: Reduce Prime Implicant Table.

Iteration #1.

(i) Remove Primary Essential Prime Implicants

	$B'D'(*)$ (0,2,8,10)	CD' (2,6,10,14)	$BD(*)$ (5,7,13,15)	BC (6,7,14,15)	AD' (8,10,12,14)	AB (12,13,14,15)
(∘)0	X					
2	X	X				
(∘)5			X			
6		X		X		
7			X	X		
8	X				X	
10	X	X			X	
12					X	X
13			X			X
14		X		X	X	X
15			X	X		X

* indicates an essential prime implicant

∘ indicates a distinguished row, i.e. a row covered by only 1 prime implicant

Now cross out those rows and columns which are no longer needed.

Here's what we have left:

	CD' (2,6,10,14)	BC (6,7,14,15)	AD' (8,10,12,14)	AB (12,13,14,15)
6	X	X		
12			X	X
14	X	X	X	X

Row Dominance

We note that row 14 dominates both row 6 and row 12. That is row 14 has an X in every column where row 6 has an"X" (and, in fact, row 14 has "X"'s in other columns as well). Similarly, row 14 has in "X" in every column where row 12 has an "X". Rows 6 and 12 are said to be *dominated by* row 14.

A *dominating* row can always be eliminated. To see this, note that every product which covers row 6 also covers row 14. That is, if some product covers row 6, row 14 is *guaranteed* to be covered. Similarly, any product which covers row 12 will also cover row 14. Therefore, row 14 can be crossed out.

(iii) Column Dominance

	CD' (2,6,10,14)	BC (6,7,14,15)	AD' (8,10,12,14)	AB (12,13,14,15)
6	X	X		
12			X	X

Column CD' *dominates* column BC. That is, column CD' has an "X" in every row where column BC has an "X". In fact, in this example, column BC also dominates column CD', so each is *dominated by* the other. (Such columns are said to *co-dominate* each other.) Similarly, columns AD' and AB dominate each other, and each is dominated by the other.

A *dominated* column can always be eliminated. To see this, note that every row covered by the dominated column is also covered by the dominating column. For example, $C'D$ covers every row which BC covers. Therefore, the dominating column can always replace the dominated column, so the dominated column is crossed out. In this example, CD' and BC dominate each other, so either column can be crossed out (but not both). Similarly, AD' and AB dominate each other, so either column can be crossed out.

Iteration #2

(i) Remove Secondary Essential Prime Implicants

	$CD'(**)$ (2,6,10,14)	$AD'(**)$ (8,10,12,14)
$(\circ)$6	X	
$(\circ)$12		X

** indicates a secondary essential prime implicant

$\circ$ indicates a distinguished row

In iteration #2 and beyond, *secondary essential prime implicants* are identified. These are implicants which will appear in *any* solution, *given* the choice of column-dominance used in the previous steps (if 2 columns co-dominated each other in a previous step, the choice of which was deleted can affect what is an "essential"at this step). As before, a row which is covered by only 1 prime implicant is called a *distinguished row*. The prime implicant which covers it is a *(secondary) essential prime implicant*.

Secondary essential prime implicants are identified and removed. The corresponding columns are crossed out. Also, each row where the column contains an *X* is completely crossed out, since these minterms are now covered. These essential implicants will be added to the final solution. In this example, both *CD'* and *AD'* are secondary essentials.

Step 4: Solve Prime Implicant Table.

No other rows remain to be covered, so no further steps are required. Therefore, the minimum-cost solution consists of the primary and secondary essential prime implicants $B'D'$, BD, CD' and AD':

$$F = B'D' + BD + CD' + AD'$$

Let's work our own problem:

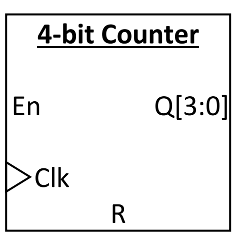
$\Sigma_{(A,B,C,D)}$=0,1,4,5,6,11,14. This example is more interesting when it comes to generating prime implicants, but much less interesting when reducing the prime implicant table.

Step 1: Find prime implicants

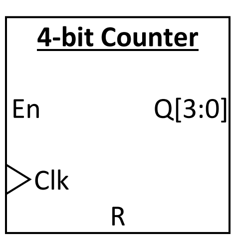
Step 2/3: Construct Prime Implicant table and reduce

Clock dividers

Say I have a 1MHz input clock and I want a *glitch-free* 100KHz output clock. How can I use a counter and gates to achieve that goal? First worry about getting the clock divided, then worry about the glitch-free part!



How do I change that if I want a 20% duty cycle? (Duty cycle means the percent of time that the signal is high). Still needs to be glitch free!



And finally, how do I handle the generic problem (divide by X, high for Y cycles)?