QUICKSORT

Worst Case Analysis

Recurrence Relation:
\[ T(0) = T(1) = 0 \quad (\text{base case}) \]
\[ T(N) = N + T(N-1) \]

Solving the RR:

\[
\begin{align*}
T(N) &= N + T(N-1) \\
T(N-1) &= (N-1) + T(N-2) \\
T(N-2) &= (N-2) + T(N-3) \\
&\quad \vdots \\
T(3) &= 3 + T(2) \\
T(2) &= 2 + T(1) \\
T(1) &= 0
\end{align*}
\]

Hence,

\[
T(N) = N + (N-1) + (N-2) \ldots + 3 + 2 
\approx \frac{N^2}{2}
\]

which is \( O(N^2) \)
QUICKSORT

Best Case Analysis

Recurrence Relation:

\[ T(0) = T(1) = 0 \quad \text{(base case)} \]
\[ T(N) = 2T(N/2) + N \]

Solving the RR:

\[
\frac{T(N)}{N} = \frac{N}{N} + \frac{2T(N/2)}{N} \quad \text{Note: Divide both side of recurrence relation by } N
\]

\[
\frac{T(N)}{N} = 1 + \frac{T(N/2)}{N/2}
\]

\[
\frac{T(N/2)}{N/2} = 1 + \frac{T(N/4)}{N/4}
\]

\[
\frac{T(N/4)}{N/4} = 1 + \frac{T(N/8)}{N/8}
\]

...

\[
\frac{T\left(\frac{N}{N/2}\right)}{N} = 1 + \frac{T\left(\frac{N}{N/2}\right)}{N} = 1 + \frac{T(1)}{1}
\]

same as

\[
\frac{T(2)}{2} = 1 + \frac{T(1)}{1} \quad \text{Note: } T(1) = 0
\]

Hence,

\[
\frac{T(N)}{N} = 1 + 1 + 1 + ... 1 \quad \text{Note: log}(N) \text{ terms}
\]

\[
\frac{T(N)}{N} = \log N
\]

\[
T(N) = N \log N \quad \text{which is } O(N \log N)
\]