# **Lecture 18:** Fitting geometric models

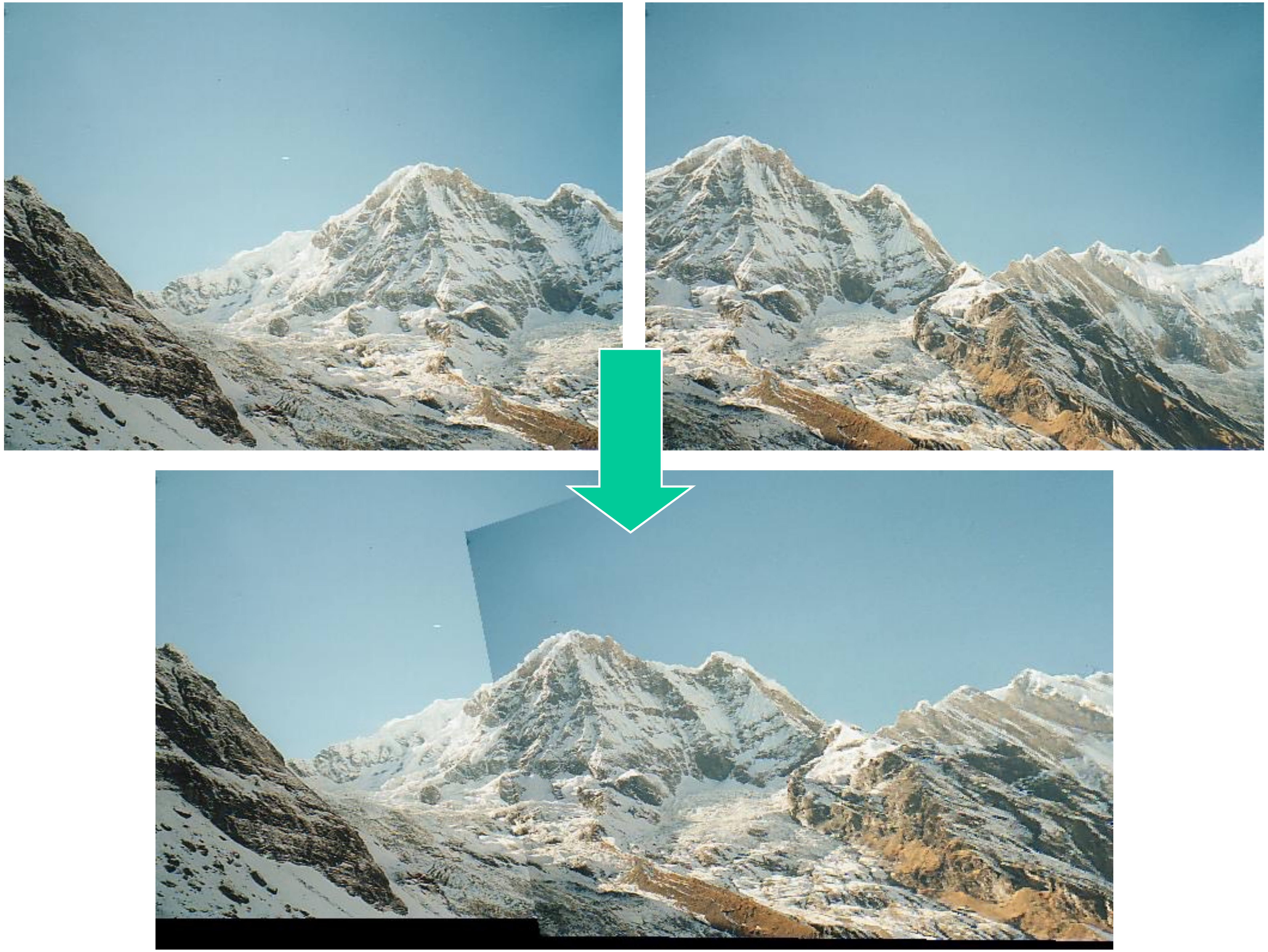
# Announcements

- Section this week: project office hours
- PS8: panorama stitching

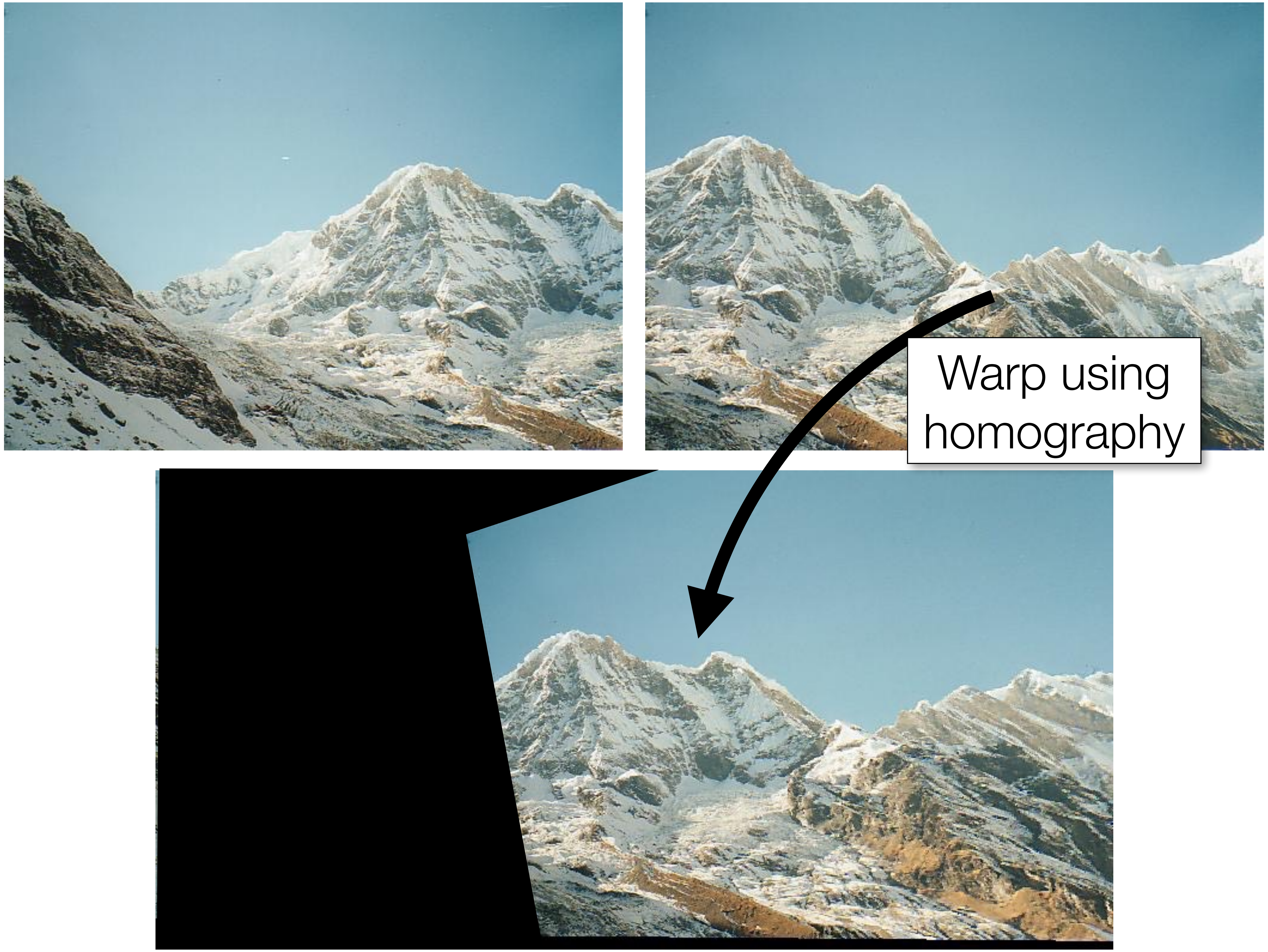
# Today

- Finding correspondences
- Fitting a homography
- RANSAC
- Triangulation

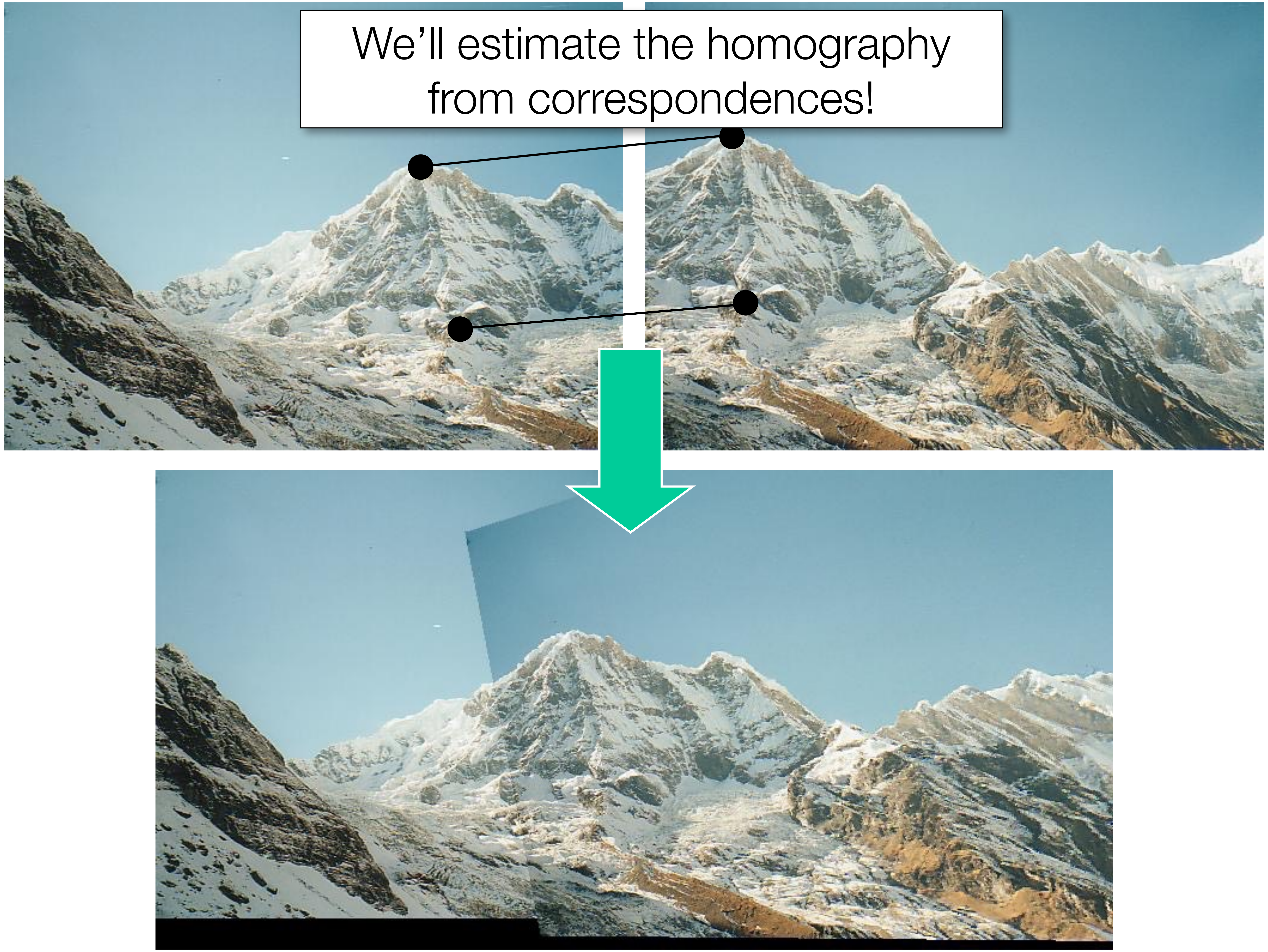
# Panorama stitching (PS9)



# Panorama stitching

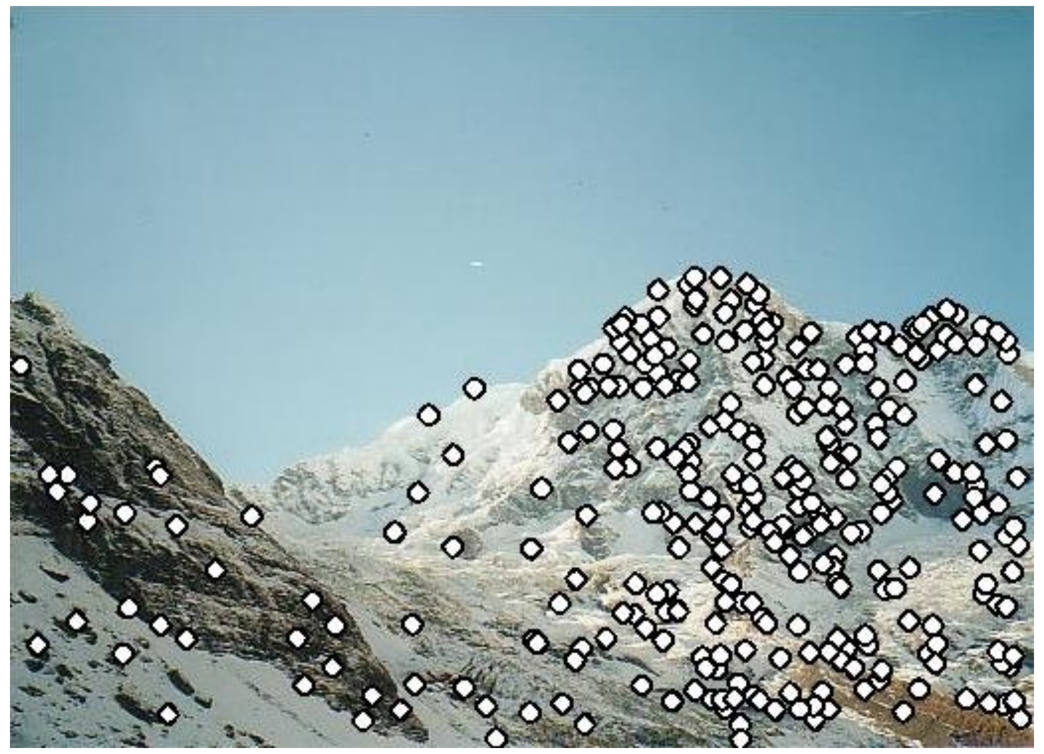


# Panorama stitching



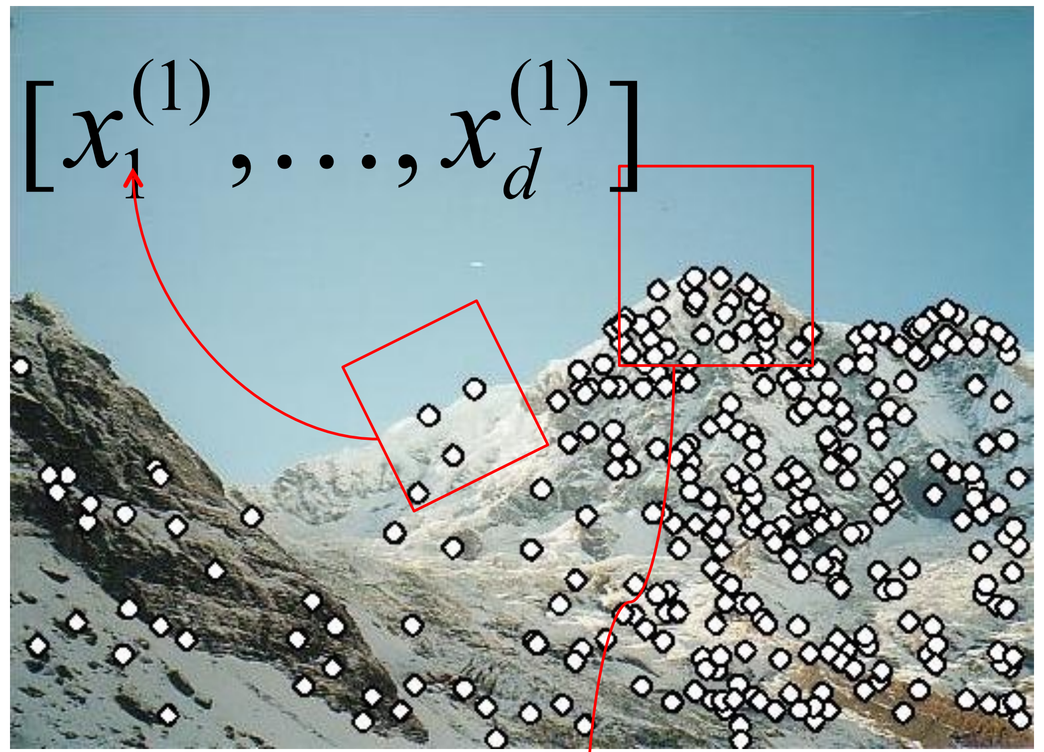
# Finding correspondences with local features

1) **Detection:** Identify the interest points (a.k.a. keypoints), the candidate points to match.



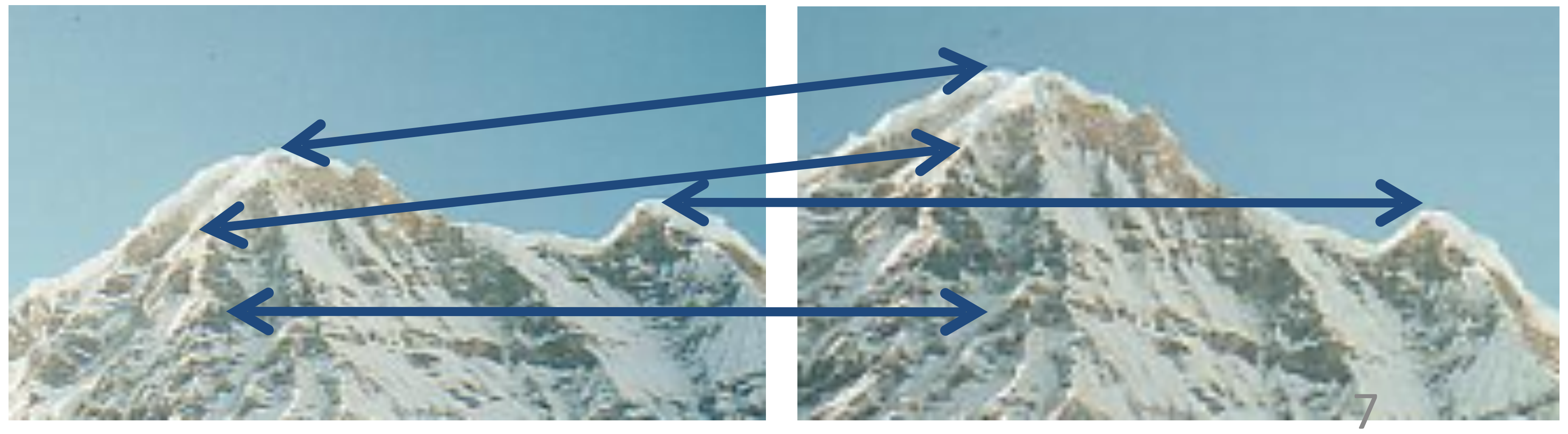
2) **Description:** Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) **Matching:** Determine correspondence between descriptors in two views



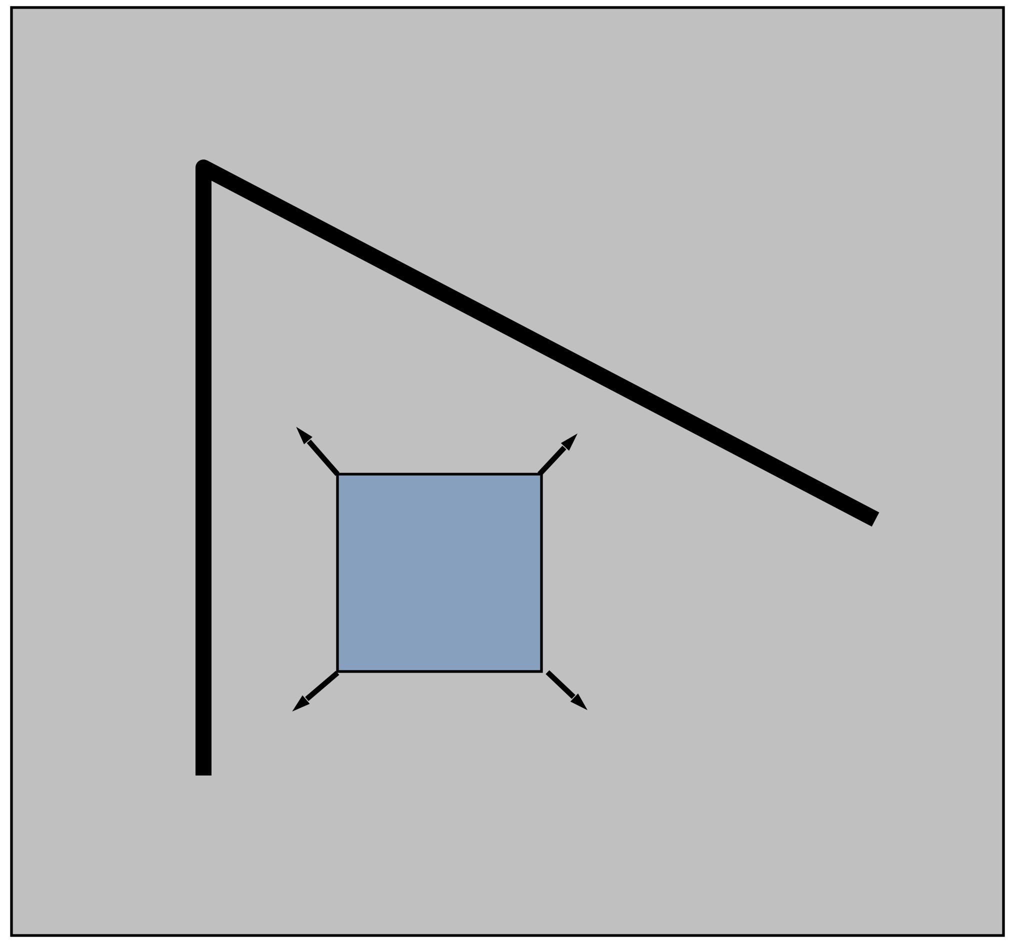


Source: K. Grauman

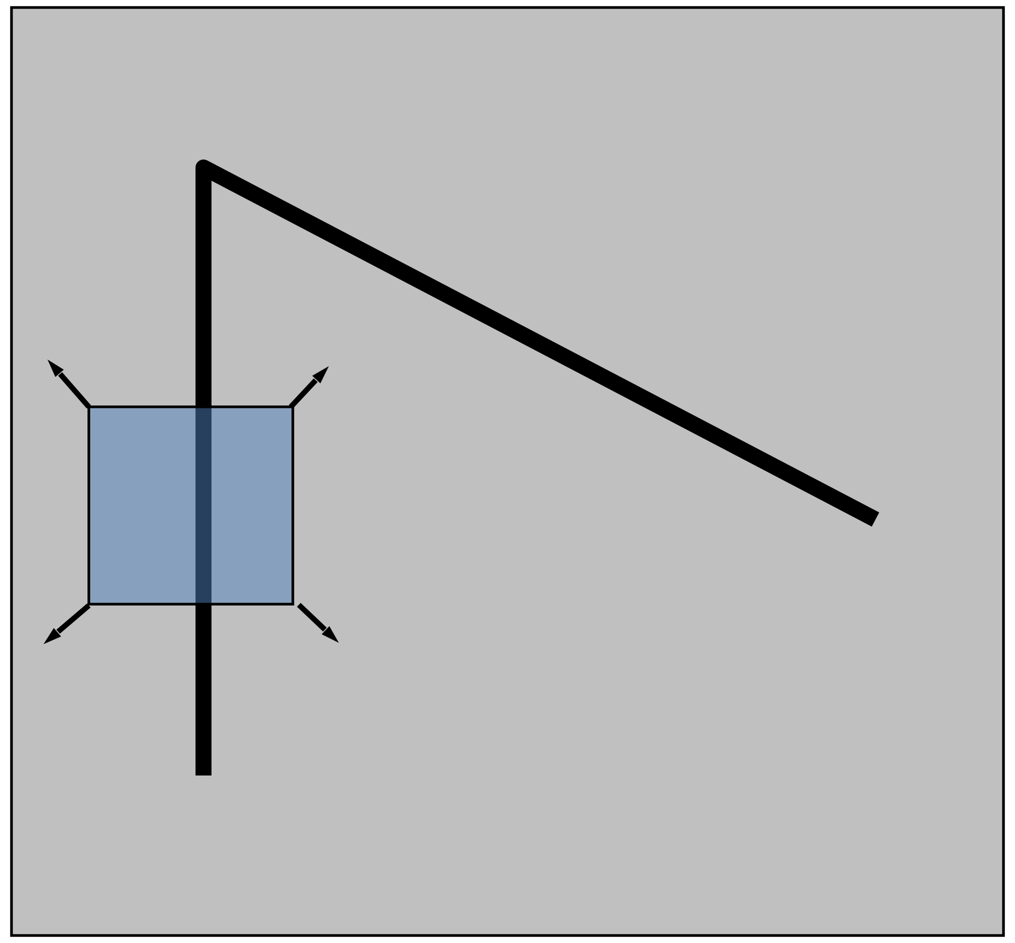
# What are good regions to match?

# What are good regions to match?

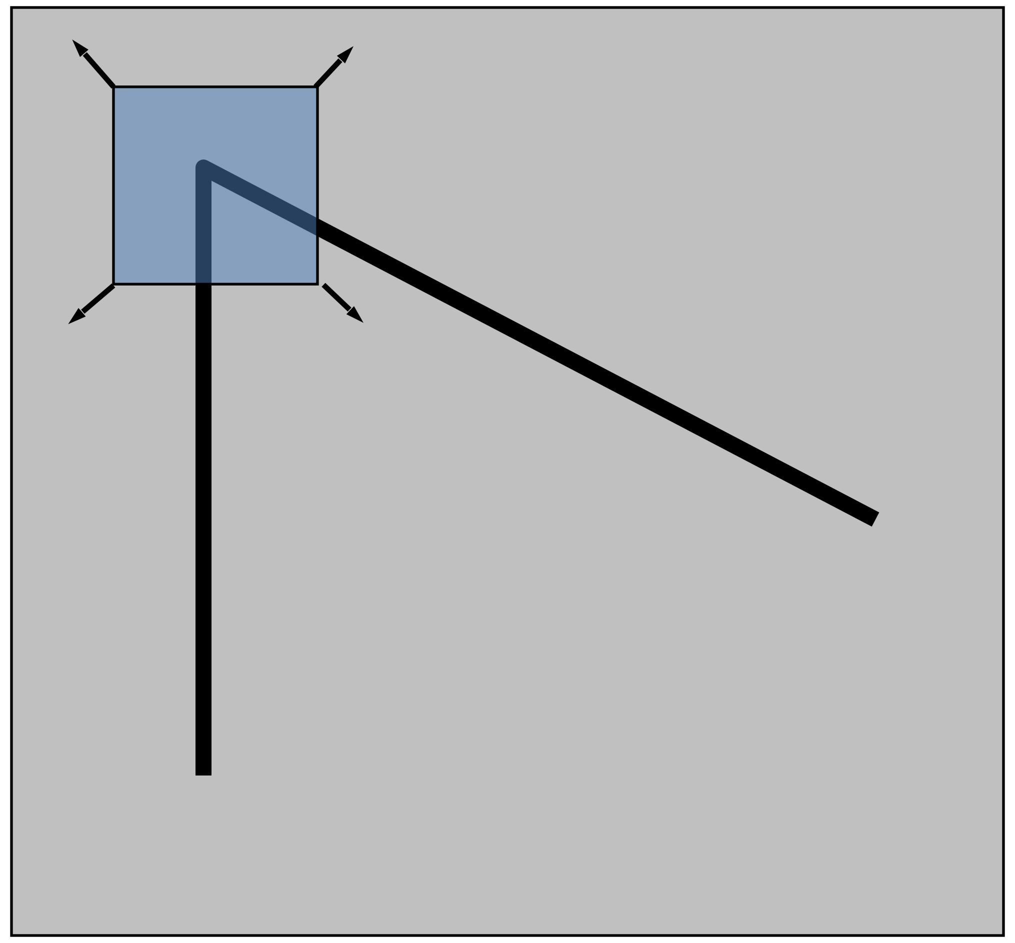
- How does the window change when you shift it?
- Shifting the window in any direction causes a big change



"flat" region:
no change in all directions



"edge":
no change along the edge direction



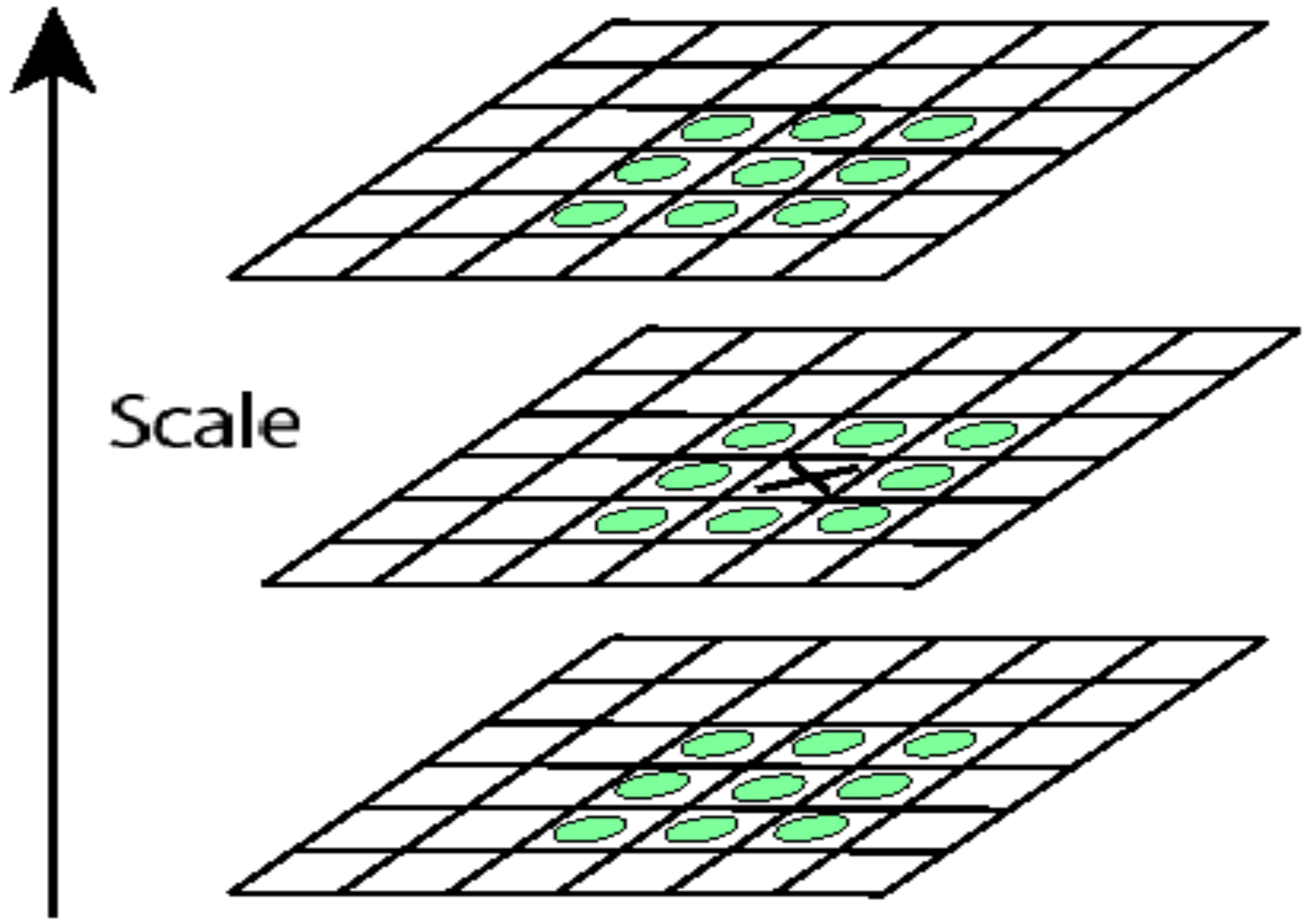
"corner":
significant change in all directions



Source: S. Seitz, D. Frolova, D. Simakov, N. Snavely

# Finding good key points to match

$$\circ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} =$$



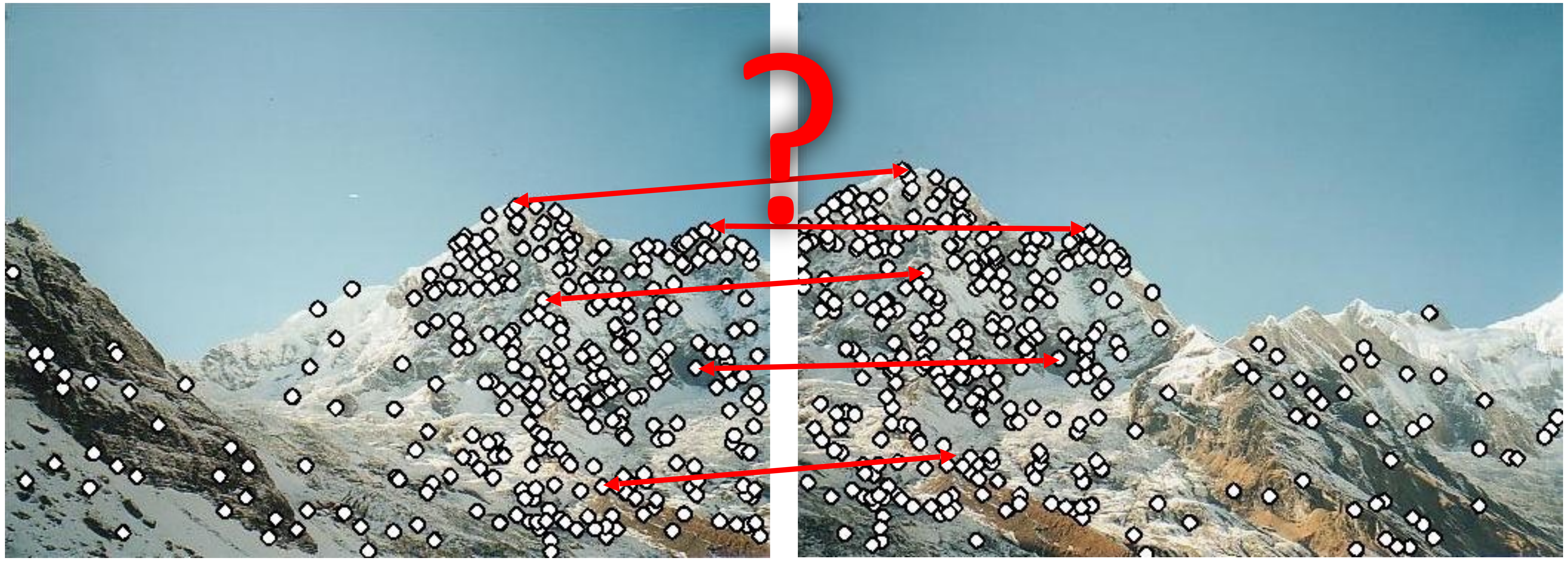

Compute difference-of-Gaussians filter (approx. to Laplacian).

Find local optima in space and scale using Laplacian pyramid.

# Feature descriptors

We know how to detect good points

Next question: **How to match them?**



Come up with a *descriptor* (feature vector) for each point, find similar descriptors between the two images



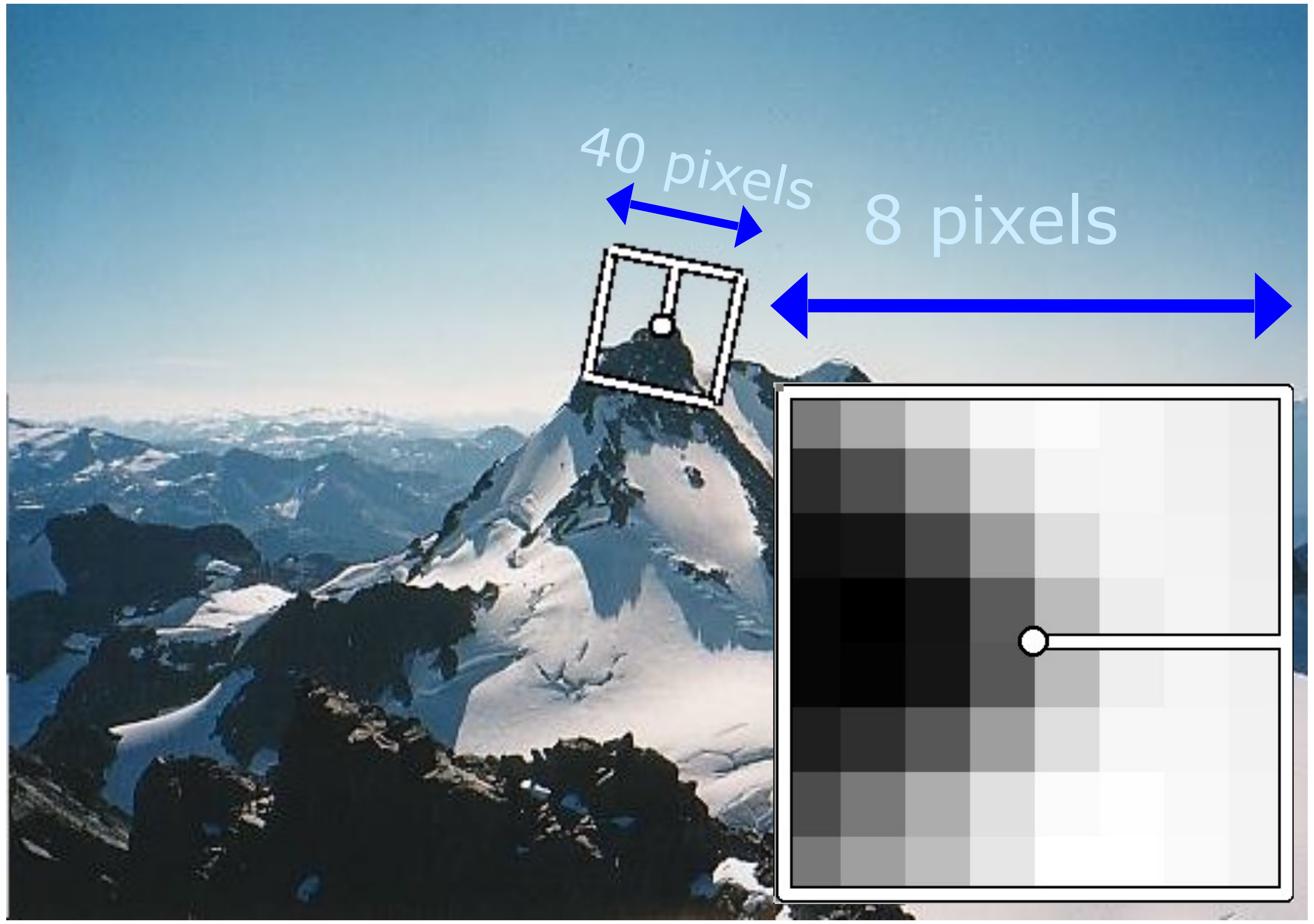
Source: N. Snavely

# Simple idea: normalized image patch

We want invariance to rotation, lighting, and tiny spatial shifts.

Take 40x40 window around feature

- Find dominant orientation
- Rotate to horizontal
- Downsample to 8x8
- Intensity normalize the window by subtracting the mean, dividing by the standard deviation in the window






Source: N. Snavely, M. Brown

# Scale Invariant Feature Transform (SIFT)

- Compute histograms of oriented gradients
- Take 16x16 square window around detected feature
- Compute edge orientation for each pixel
- Looks like a small, hand-crafted CNN

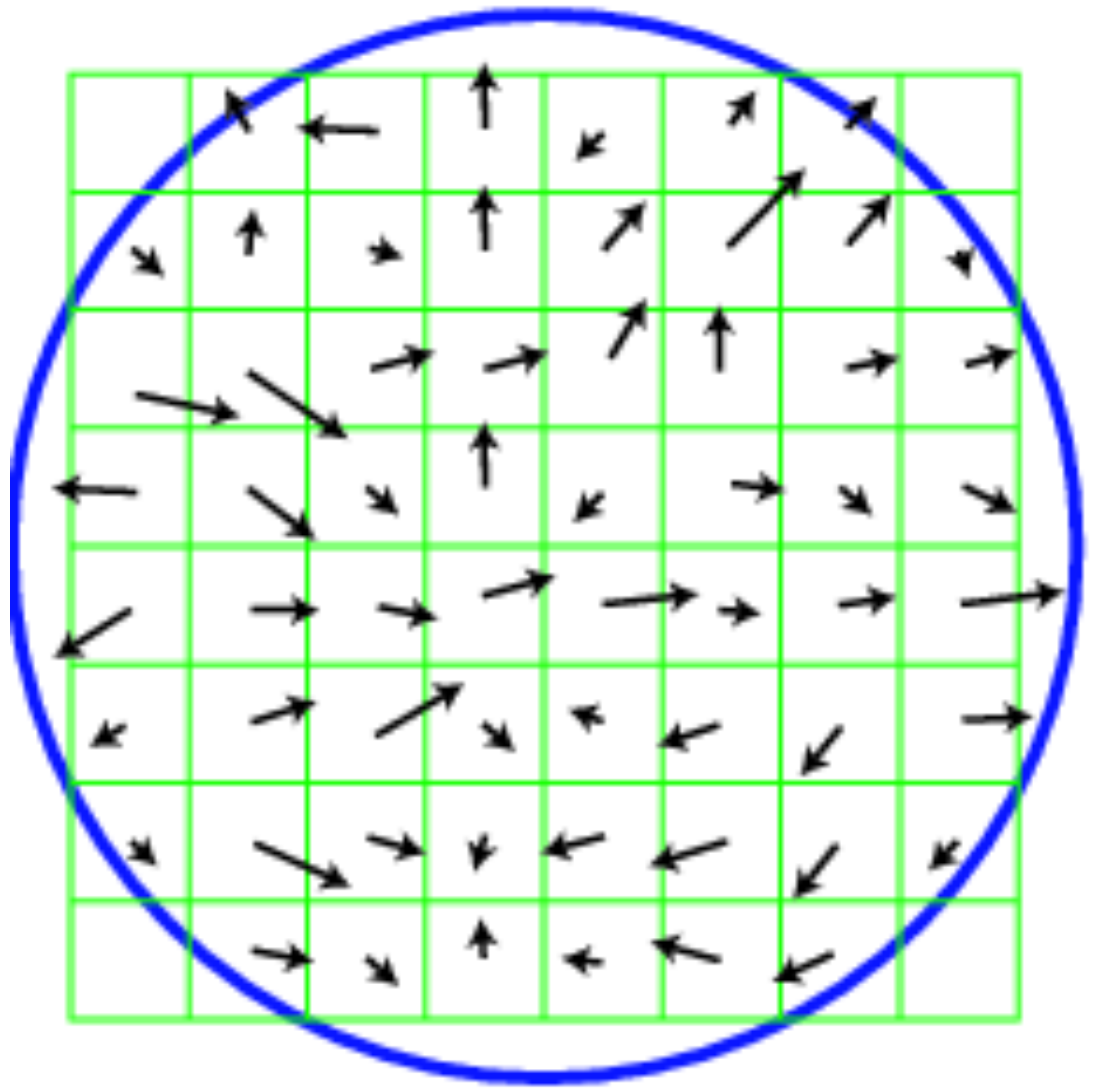
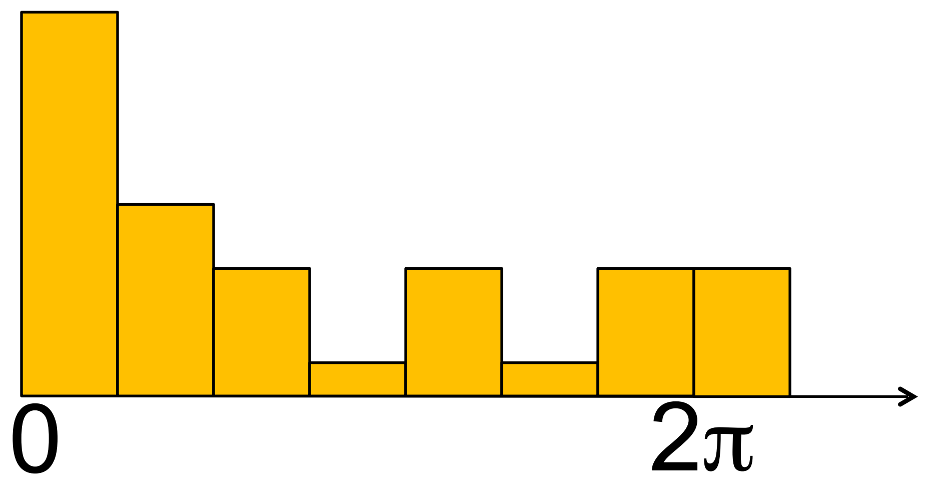
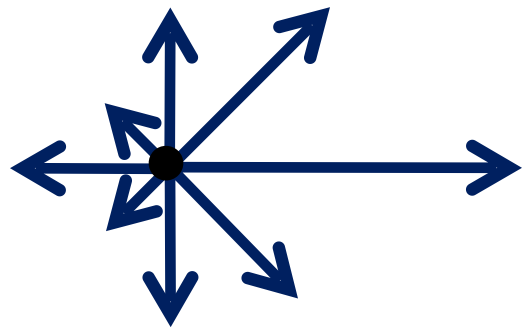


Image gradients




angle histogram



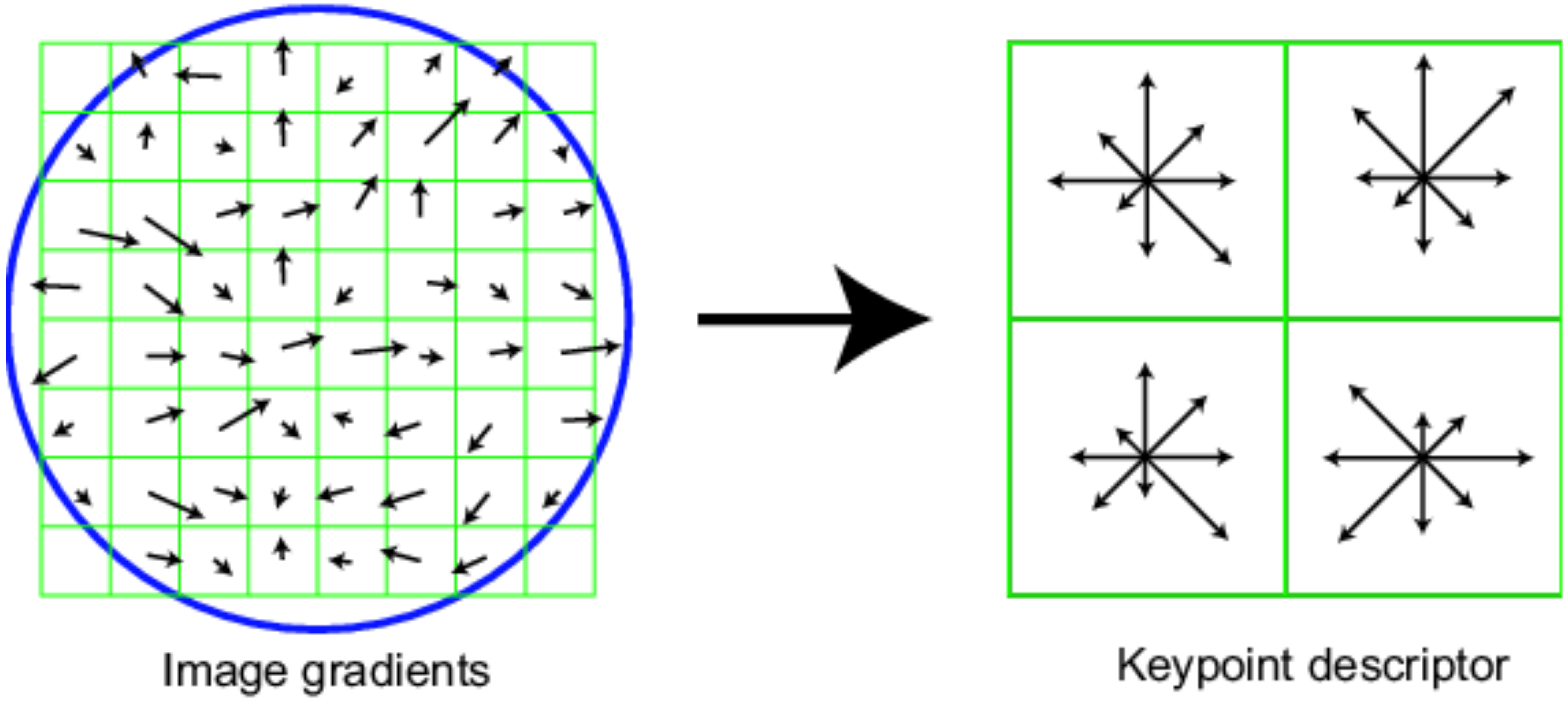


Source: N. Snavely, D. Lowe

# Scale Invariant Feature Transform

Create the descriptor:

- Rotation invariance: rotate by “dominant” orientation
- Spatial invariance: spatial pool to 2x2
- Compute an orientation histogram for each cell
- (4 x 4) cells x 8 orientations = 128 dimensional descriptor






Source: N. Snavely, D. Lowe

# SIFT invariances

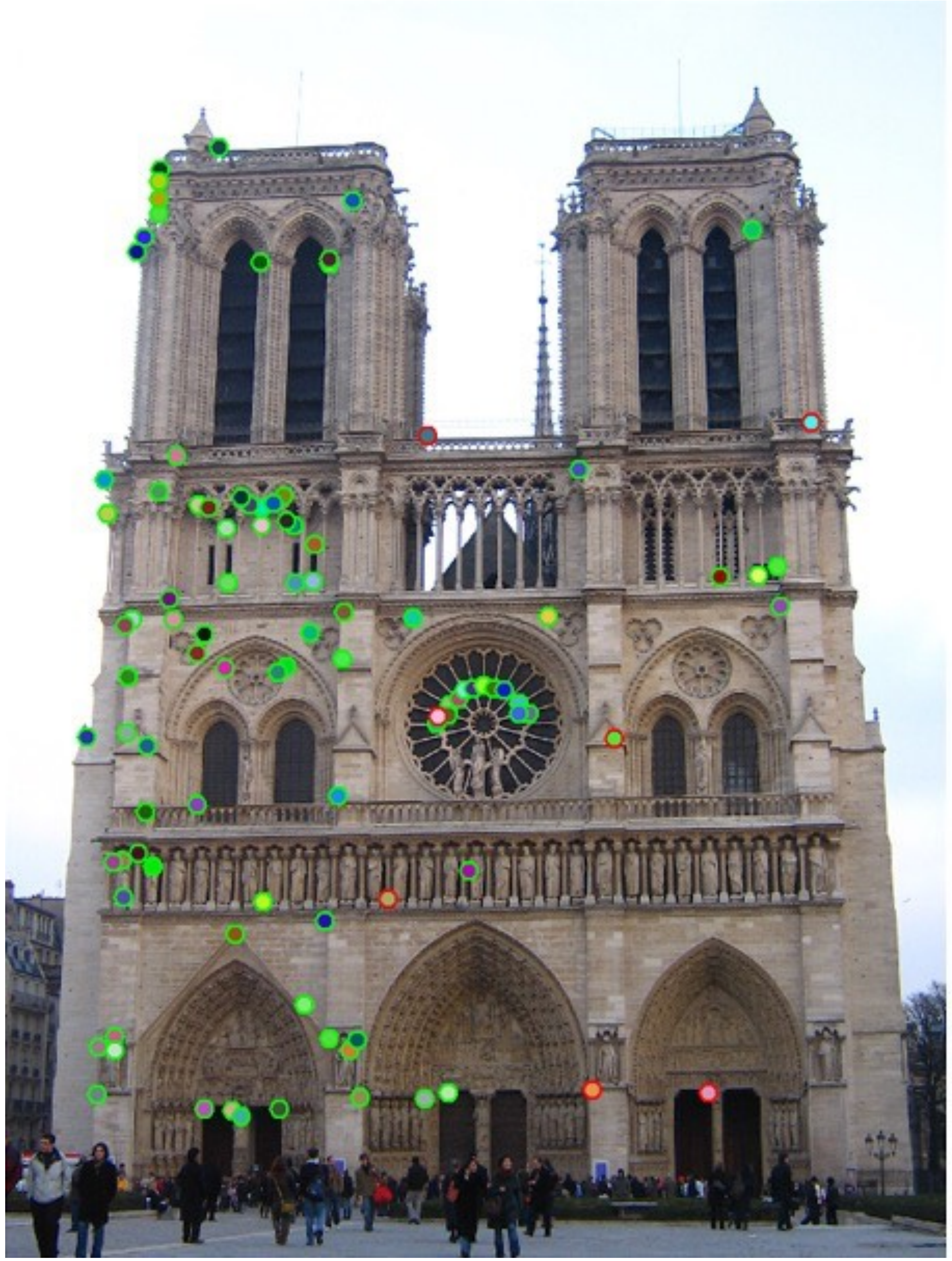


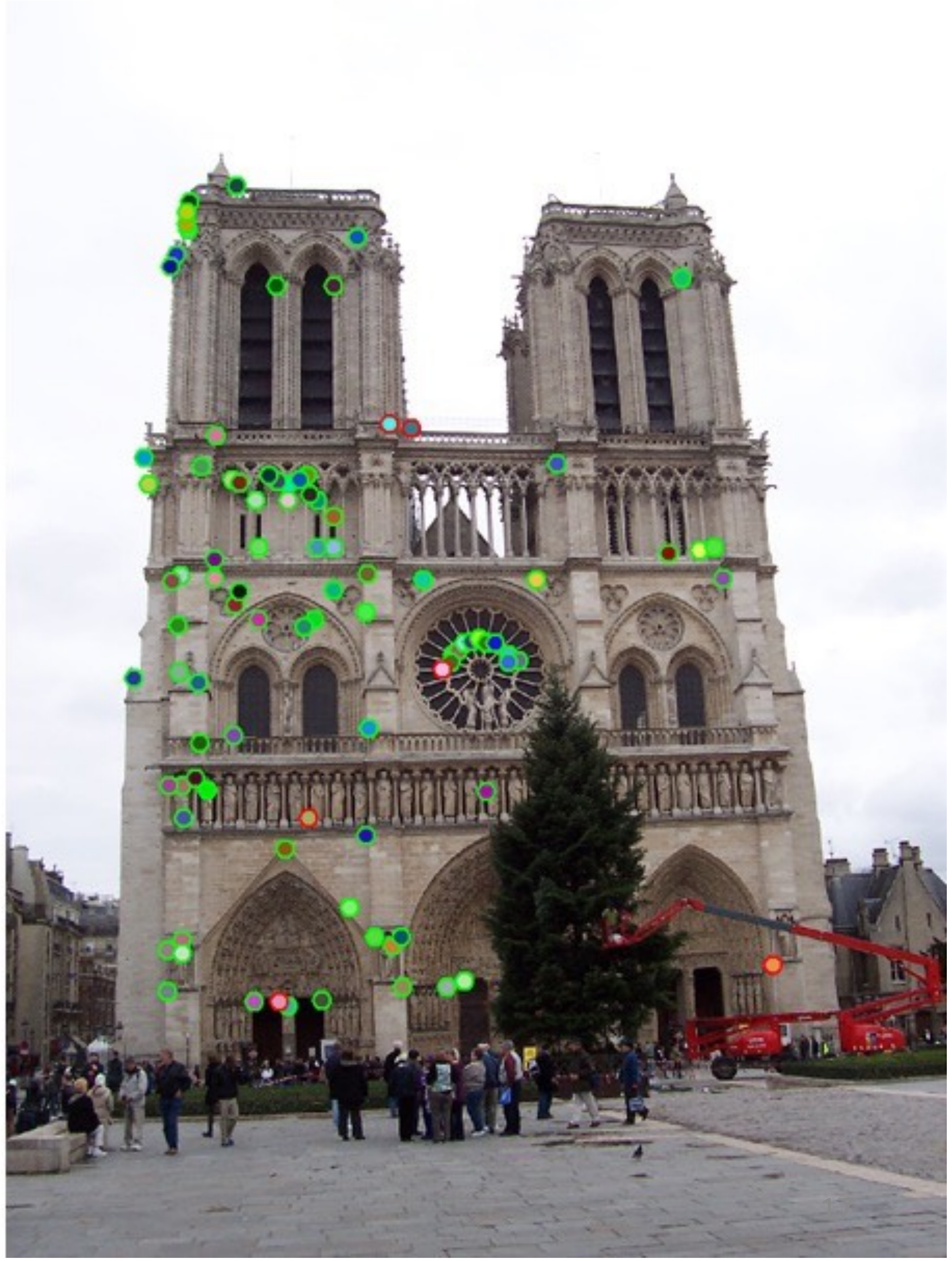

Source: N. Snavely

# Today

- Finding correspondences
  - Computing local features
  - **Matching**
- Fitting a homography
- RANSAC

# How can we tell if two features match?





Source: N. Snavely

# Feature matching

Given a feature in $I_1$, how do we find the best match in $I_2$?

1. Define distance function that compares two descriptors
2. Test all the features in $I_2$, find the closest one.

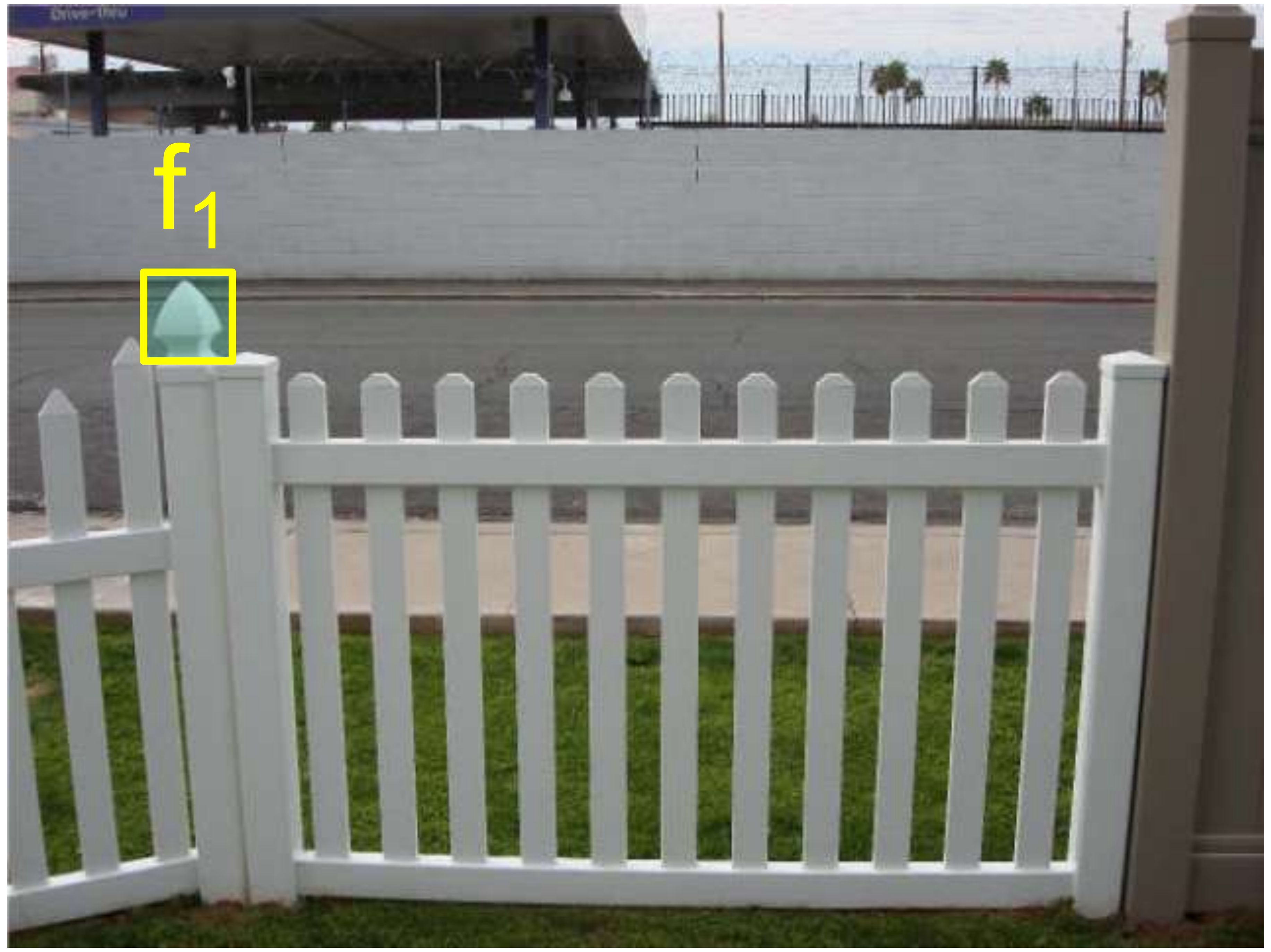


Source: N. Snavely

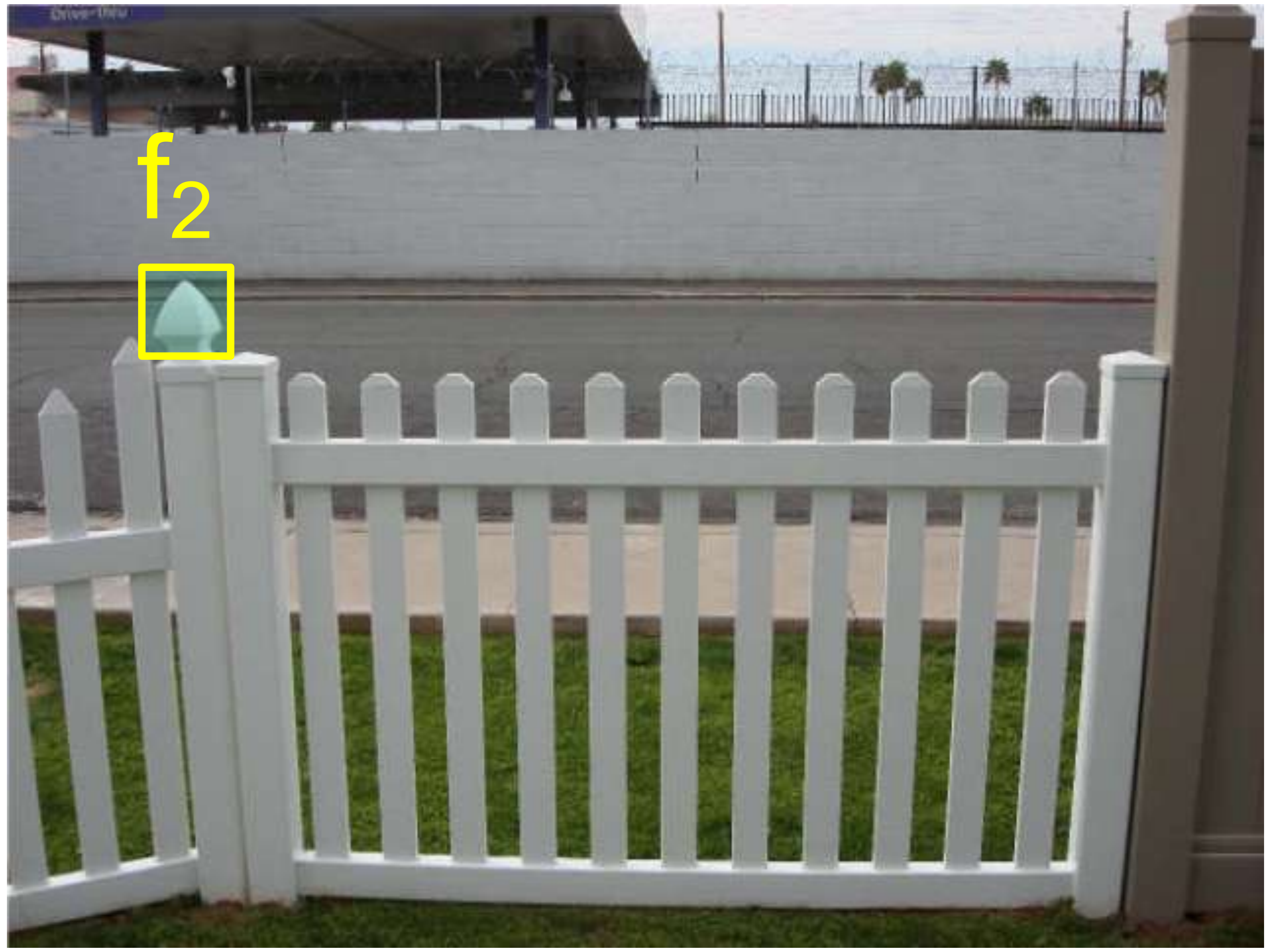
# Finding matches

How do we know if two features match?

– Simple approach: are they the nearest neighbor in $L_2$ distance, $||f_1 - f_2||$?




$I_1$




$I_2$

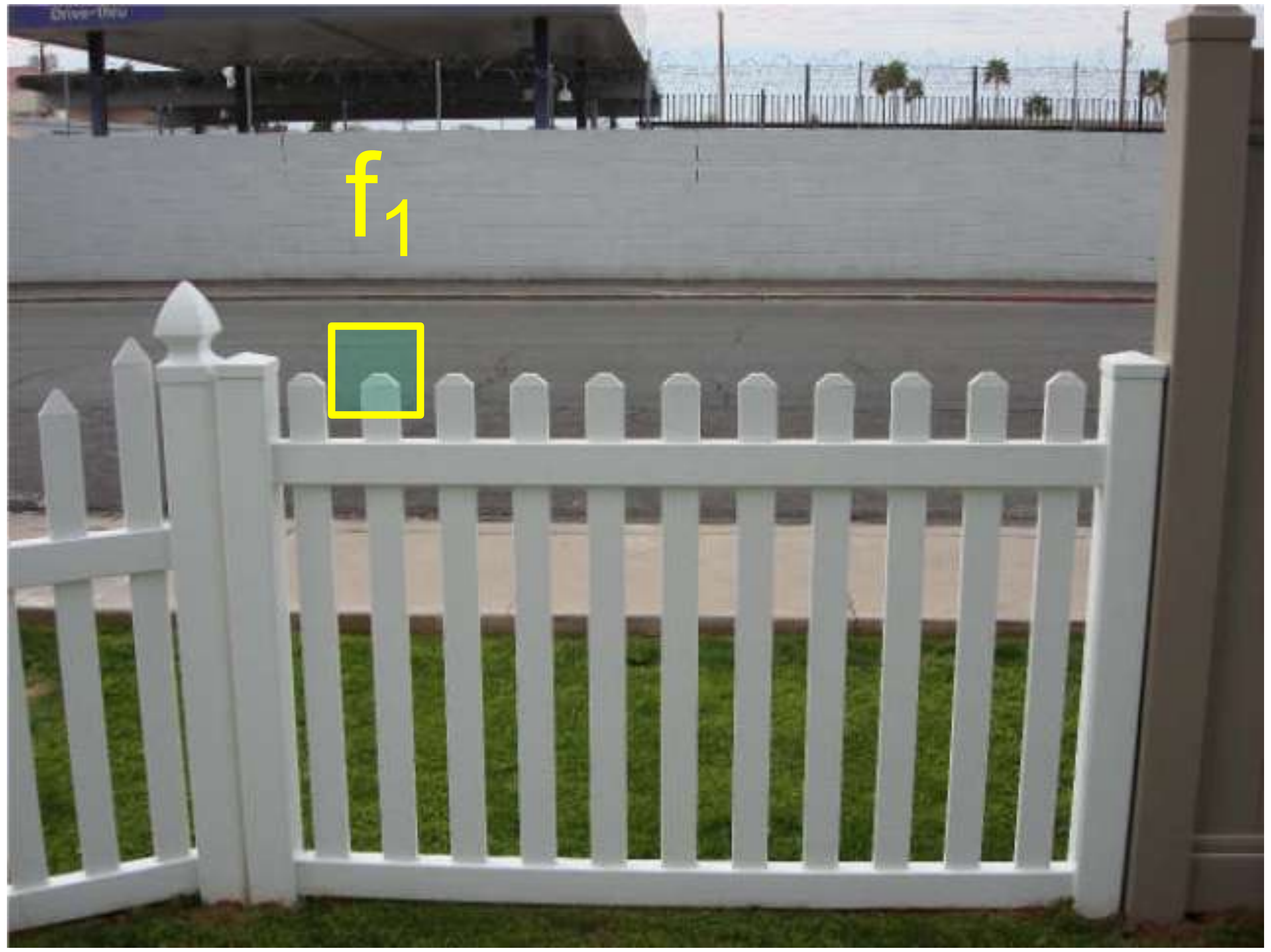


Source: N. Snavely

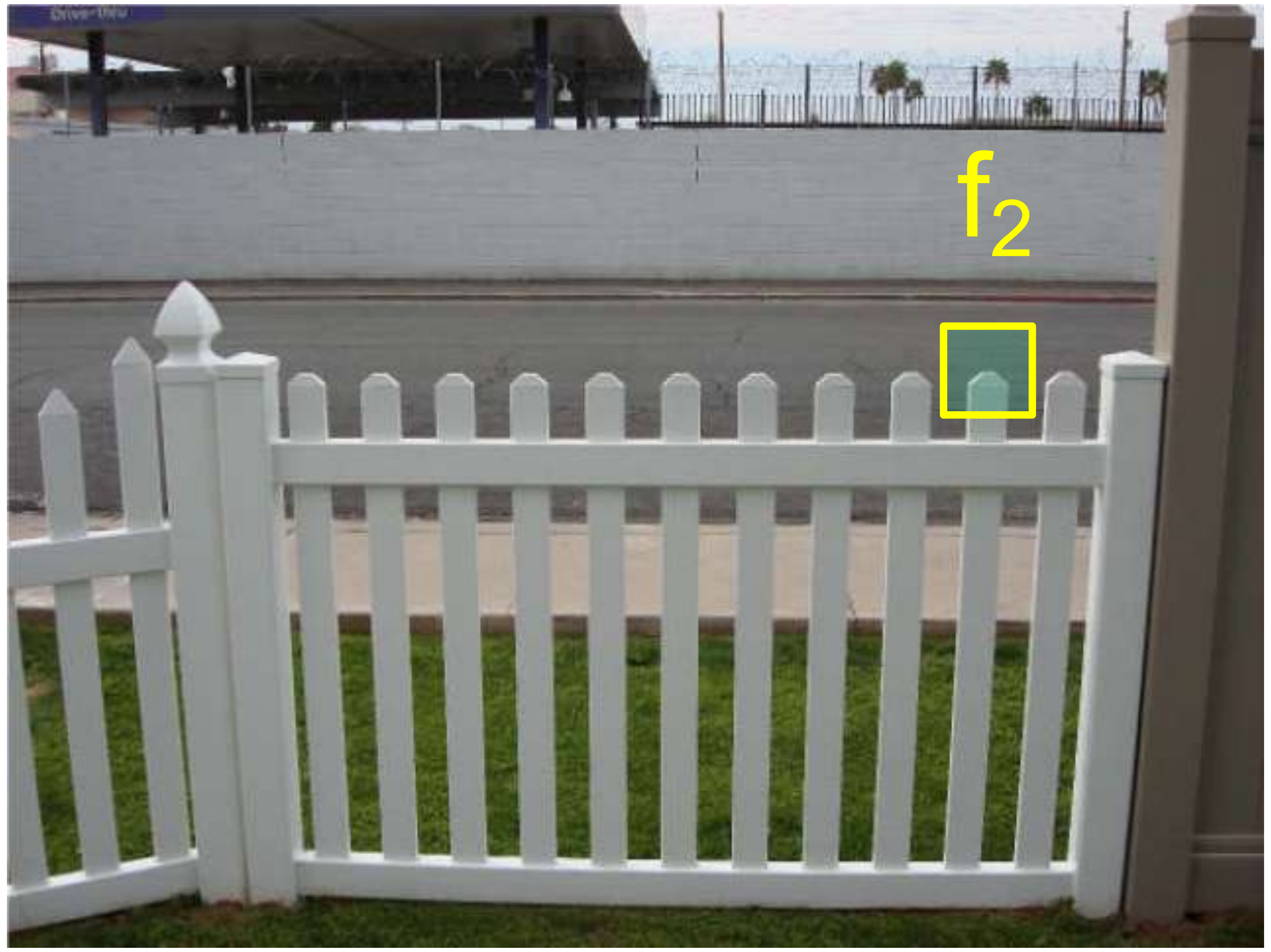
# Finding matches

How do we know if two features match?

- Simple approach: are they the nearest neighbor in $L_2$ distance, $||f_1 - f_2||$?
- Can give good scores to ambiguous (incorrect) matches




$I_1$




$I_2$

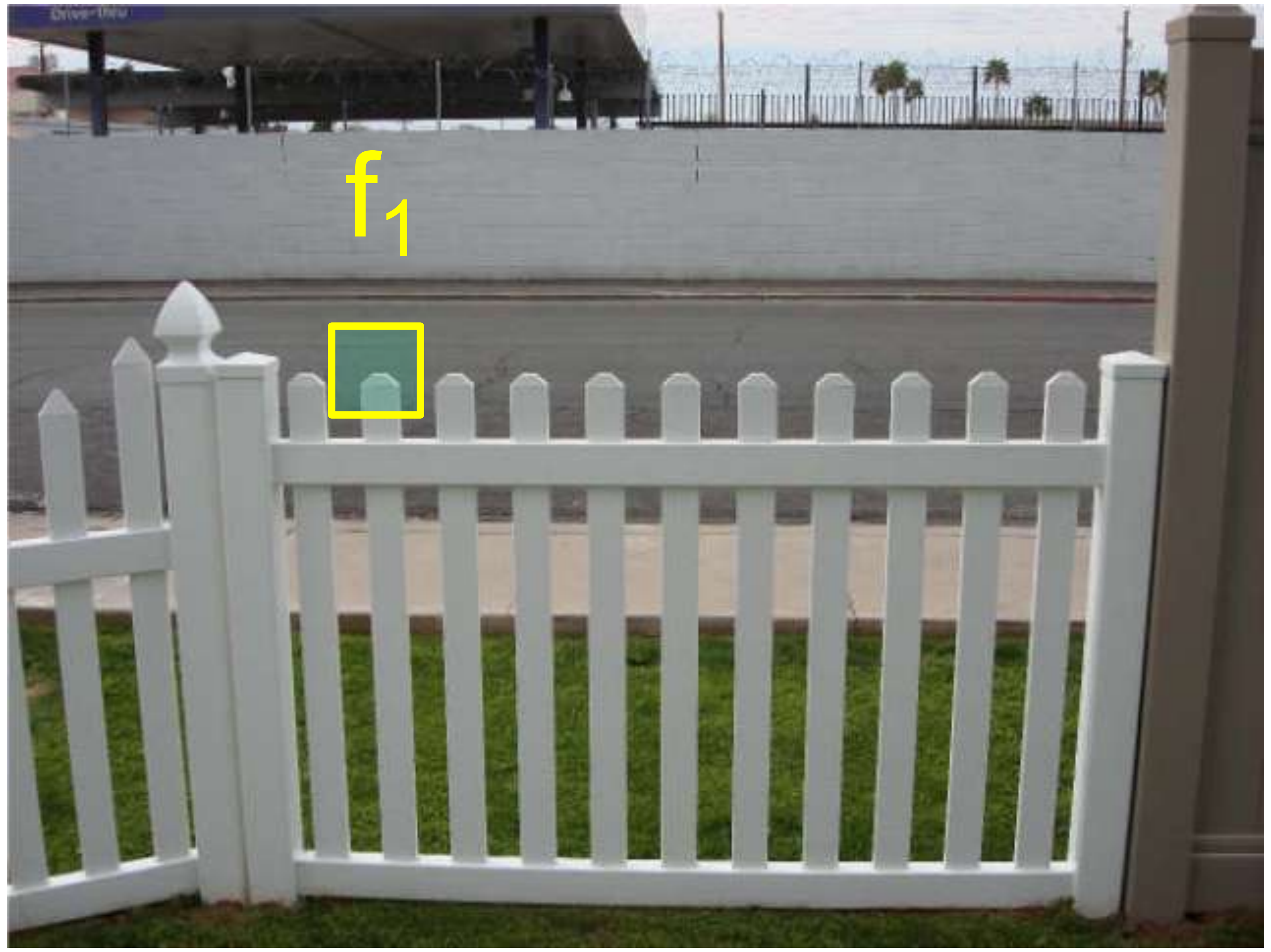


Source: N. Snavely

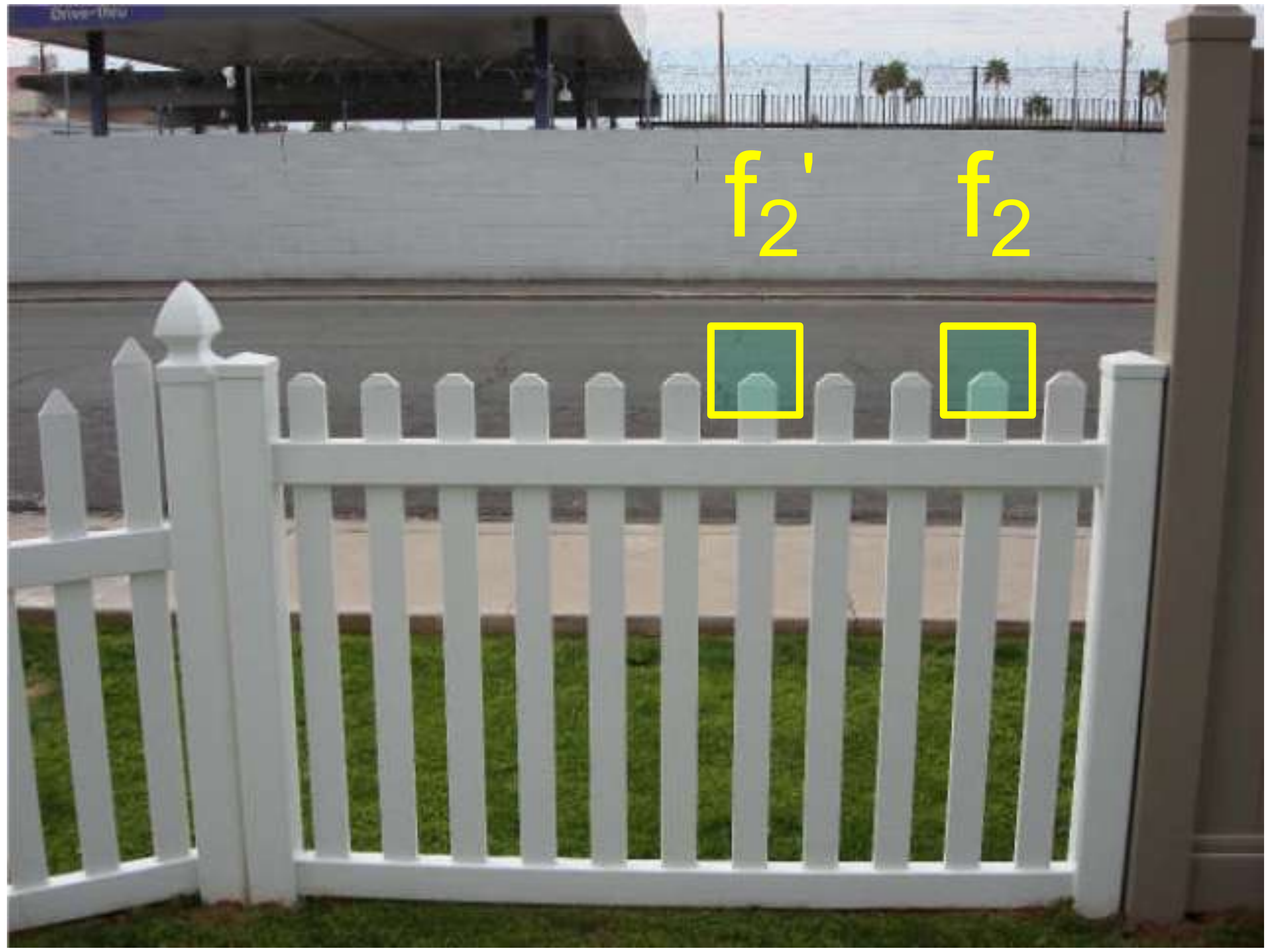
# Finding matches

Throw away matches that fail tests:

- **Ratio test**: this *by far* the best match? Compare best and 2nd-best matches.
  - Ratio distance = $||f_1 - f_2|| / || f_1 - f_2' ||$
  - $f_2$ is best SSD match to $f_1$ in $I_2$
  - $f_2'$ is 2nd best SSD match to $f_1$ in $I_2$
- **Forward-backward consistency**: $f_1$ should also be nearest neighbor of $f_2$




$I_1$

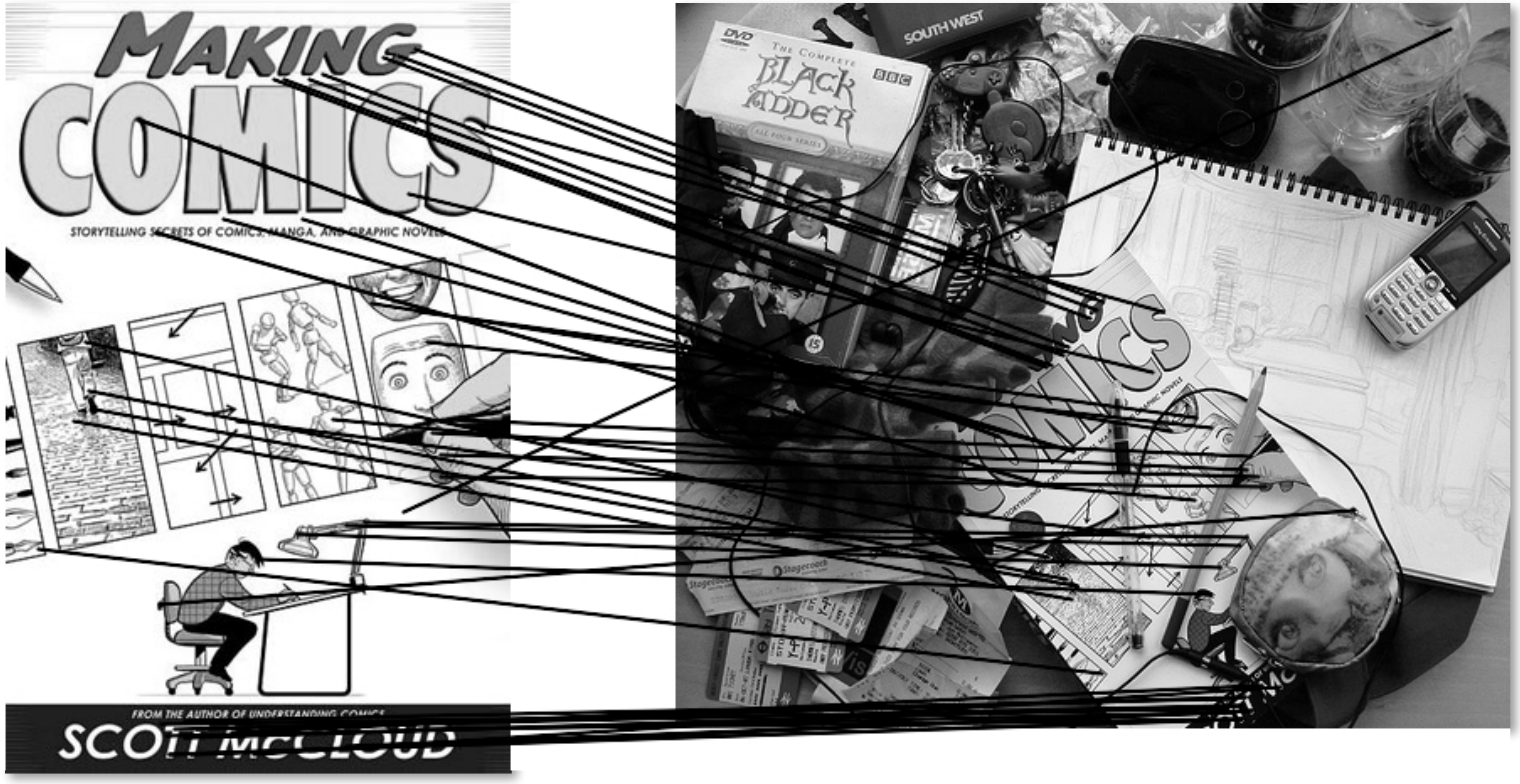



$I_2$



Source: N. Snavely

# Feature matching example

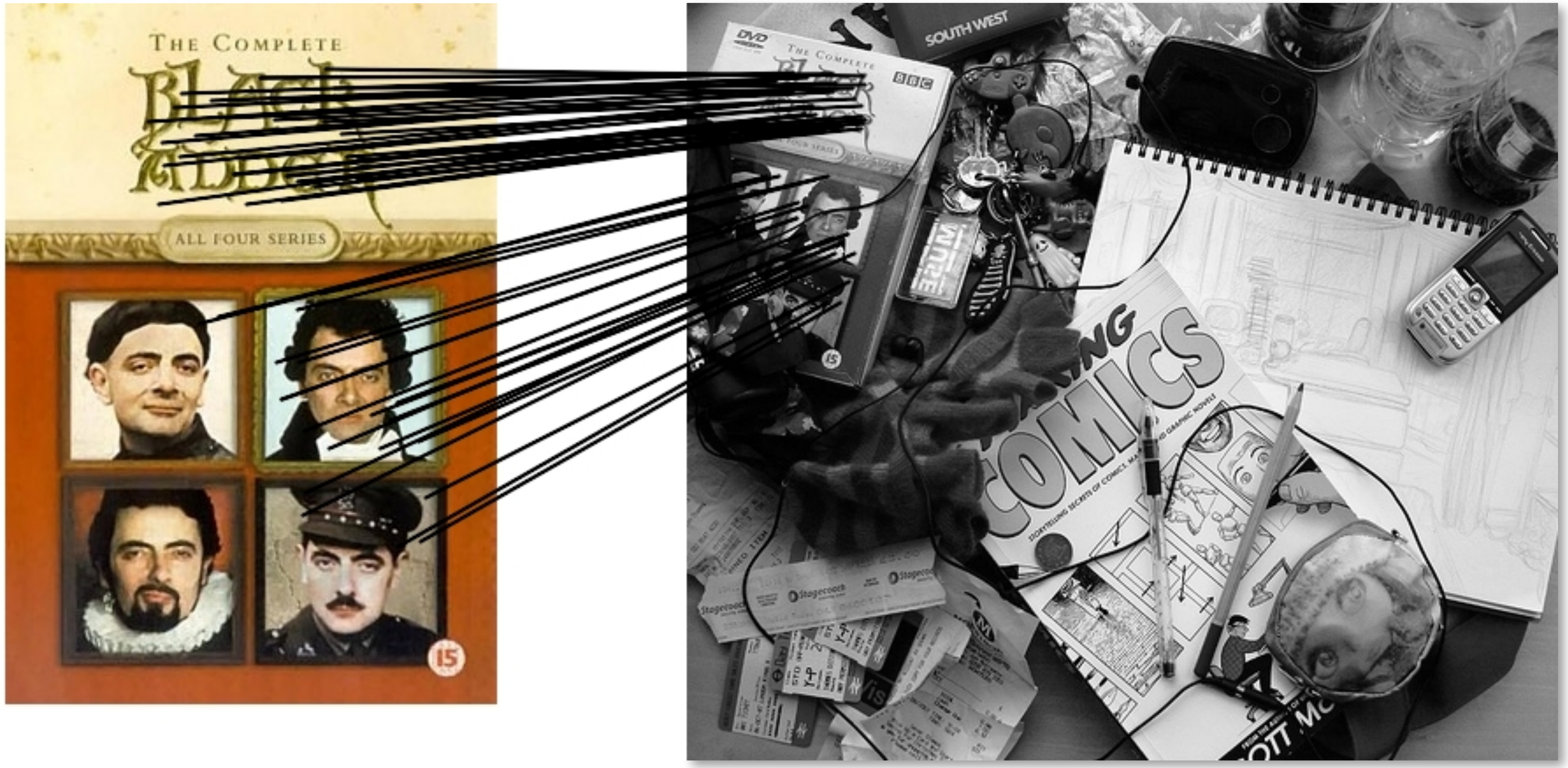



**51 feature matches after ratio test**



Source: N. Snavely

# Feature matching example




**58 feature matches after ratio test**

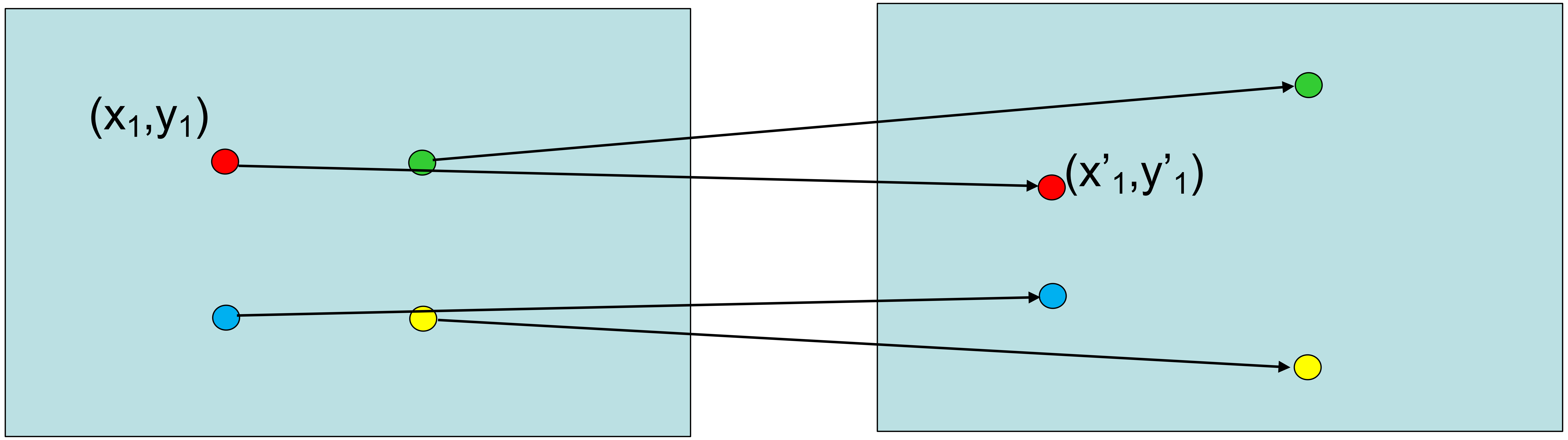


Source: N. Snavely

# Today

- Finding correspondences
  - Computing local features
  - Matching
- **Fitting a homography**
- RANSAC

# From matches to a homography




$$\begin{bmatrix} x_1' \\ y_1' \\ w_1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$



Source: Torralba, Isola, Freeman

# From matches to a homography

Point in 1st image

Matched point in 2nd

minimize $J(H) = \sum_i ||f_H(p_i) - p_i'||^2$

where $f_H(p_i) = Hp_i/(H_3^T p_i)$ applies homography

Remember: homogenous coordinates.

$H_3$ is the third row of $H$

# Option #1: Direct linear transform

$$\begin{bmatrix} x_1' \\ y_1' \\ w_1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Leaving homogeneous coordinates:

$$x_1' = \frac{ax_1 + by_1 + c}{gx_1 + hy_1 + i}$$

$$y_1' = \frac{dx_1 + ey_1 + f}{gx_1 + hy_1 + i}$$

Re-arranging the terms:

$$gx_1x'_1 + hy_1x'_1 + ix_1' = ax_1 + by_1 + c$$

$$gx_1y'_1 + hy_1y'_1 + ix_1' = dx_1 + ey_1 + f$$



Source: Torralba, Freeman, Isola

# Option #1: Direct linear transform

$$g x_1 x'_1 + h y_1 x'_1 + i x_1' = a x_1 + b y_1 + c$$

$$g x_1 y'_1 + h y_1 y'_1 + i x_1' = d x_1 + e y_1 + f$$

More rearranging:

$$g x_1 x'_1 + h y_1 x'_1 + i x'_1 - a x_1 - b y_1 - c = 0$$
$$g x_1 y'_1 + h y_1 y'_1 + i y'_1 - d x_1 - e y_1 - f = 0$$

In matrix form:

$$\begin{bmatrix} -x_1 & -y_1 & -1 & 0 & 0 & 0 & x_1 x'_1 & y_1 x'_1 & x'_1 \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & x_1 y'_1 & y_1 y'_1 & y'_1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

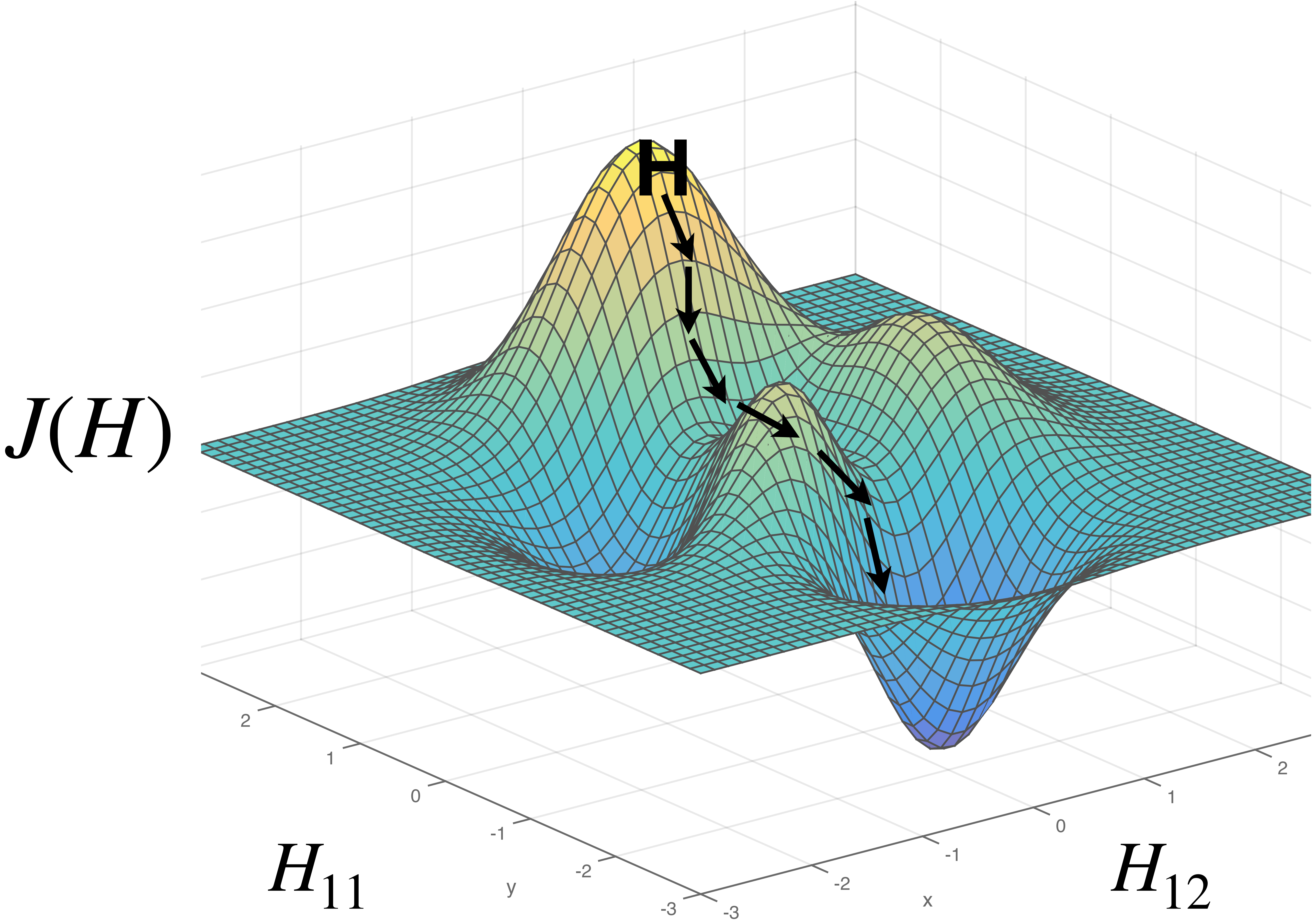
Can solve using Singular Value Decomposition (SVD).

Fast to solve (but not using “right” loss function). Uses an algebraic trick.
Often used in practice for initial solutions!



Source: Torralba, Freeman, Isola

# Option #2: Optimization



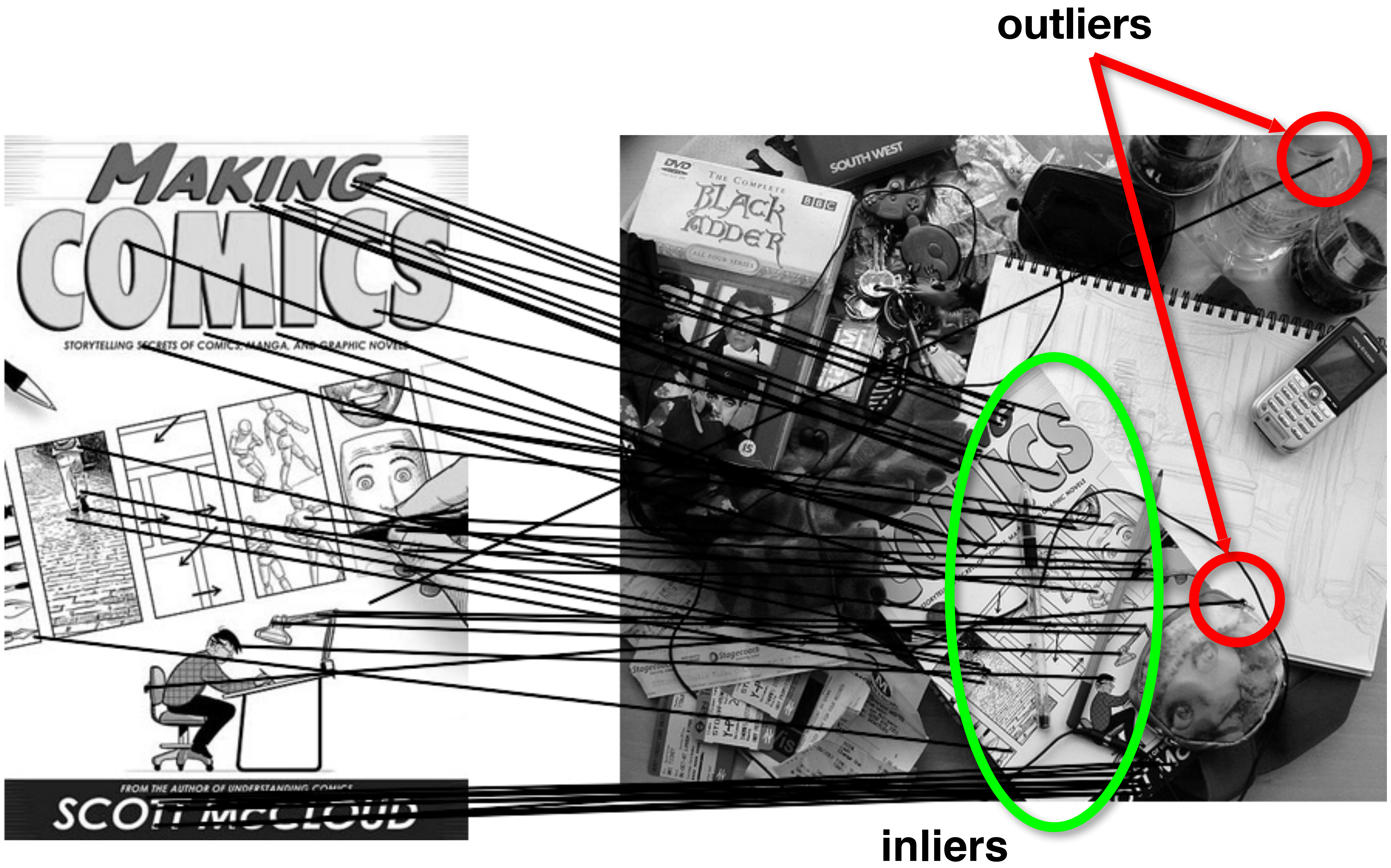

minimize $\quad J(H) = \sum_i ||f_H(p_i) - p_i'||^2$

# Optimization

$$\text{minimize} \quad J(H) = \sum_i ||f_H(p_i) - p_i'||^2$$

- Can use gradient descent, just like when learning neural nets
- These problems are **smaller scale** than deep learning problems but have **more local optima**:
  - Use 2nd derivatives to improve optimization
  - Can use finite differences or autodiff
- Can use special-purpose **nonlinear least squares** methods.
  - Exploits structure in the problem for a sum-of-squares loss.

# Problem: outliers






Source: N. Snavely

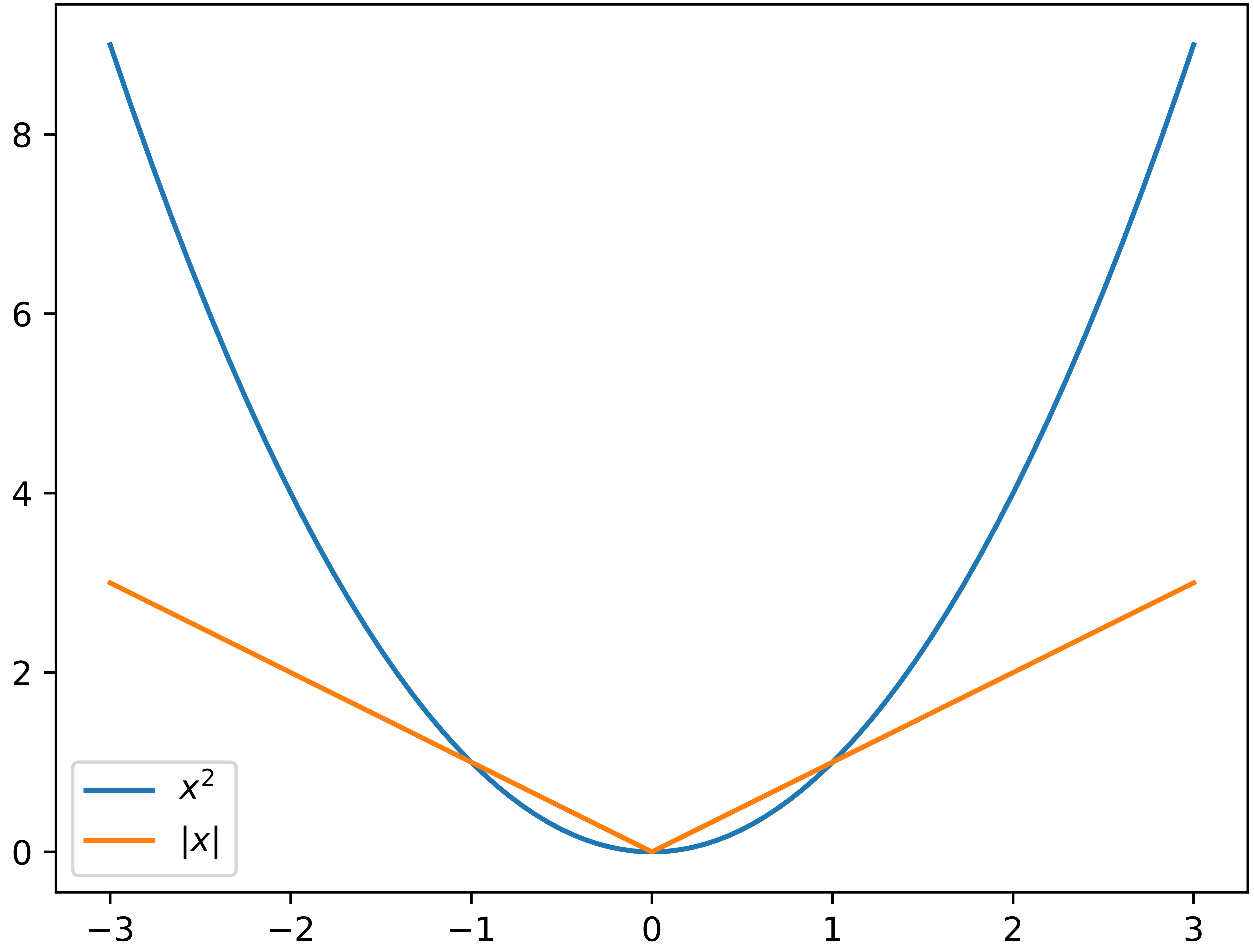
# One idea: robust loss functions

minimize $J(H) = \sum_{i=1}^{N} \sum_{j=1}^{2} \rho(f_H(p_{ij}) - p'_{ij})$

where $\rho(x)$ is a **robust** loss.

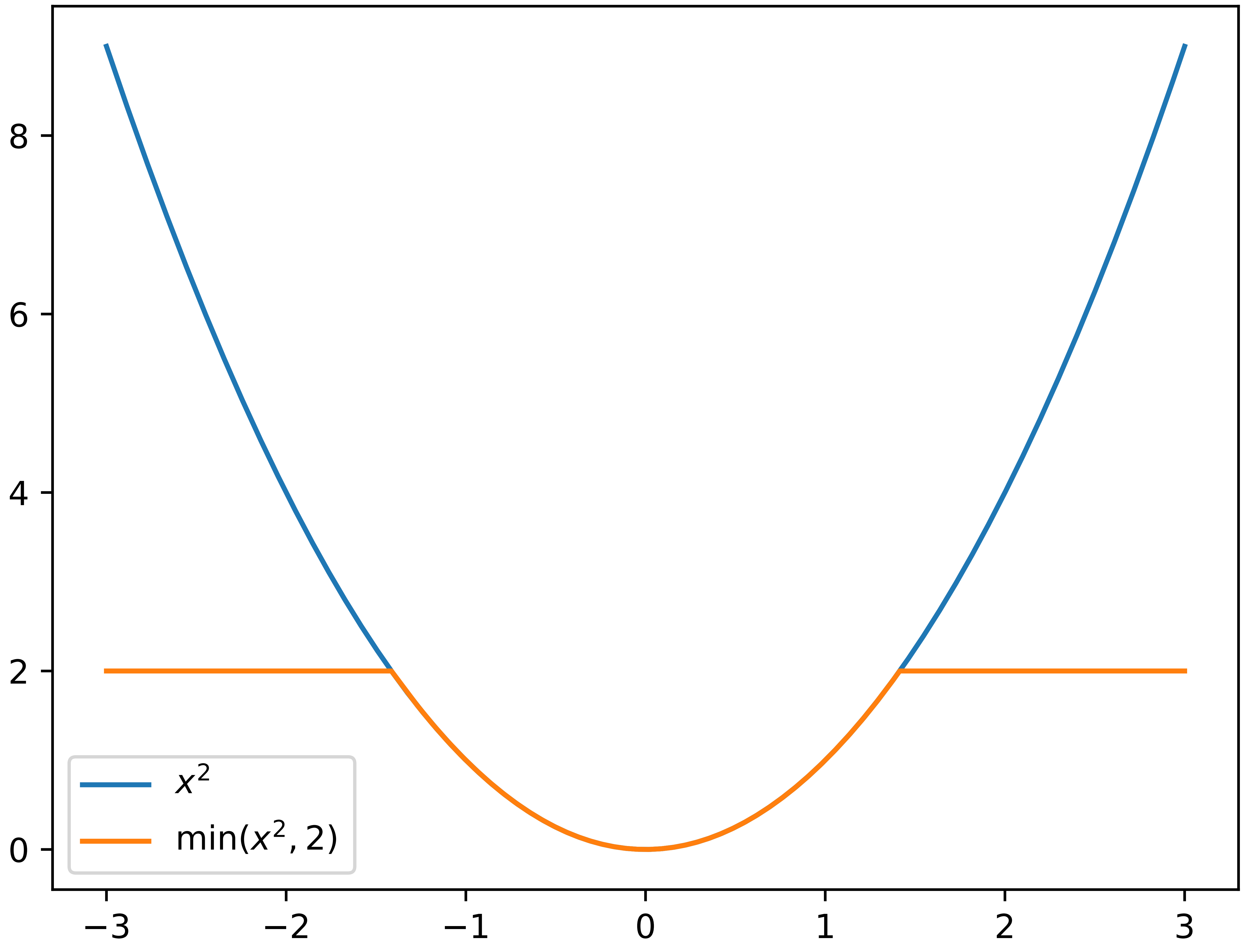
Special case: $\rho(x) = x^2$ is L2 loss (same as before)

# Robust loss functions



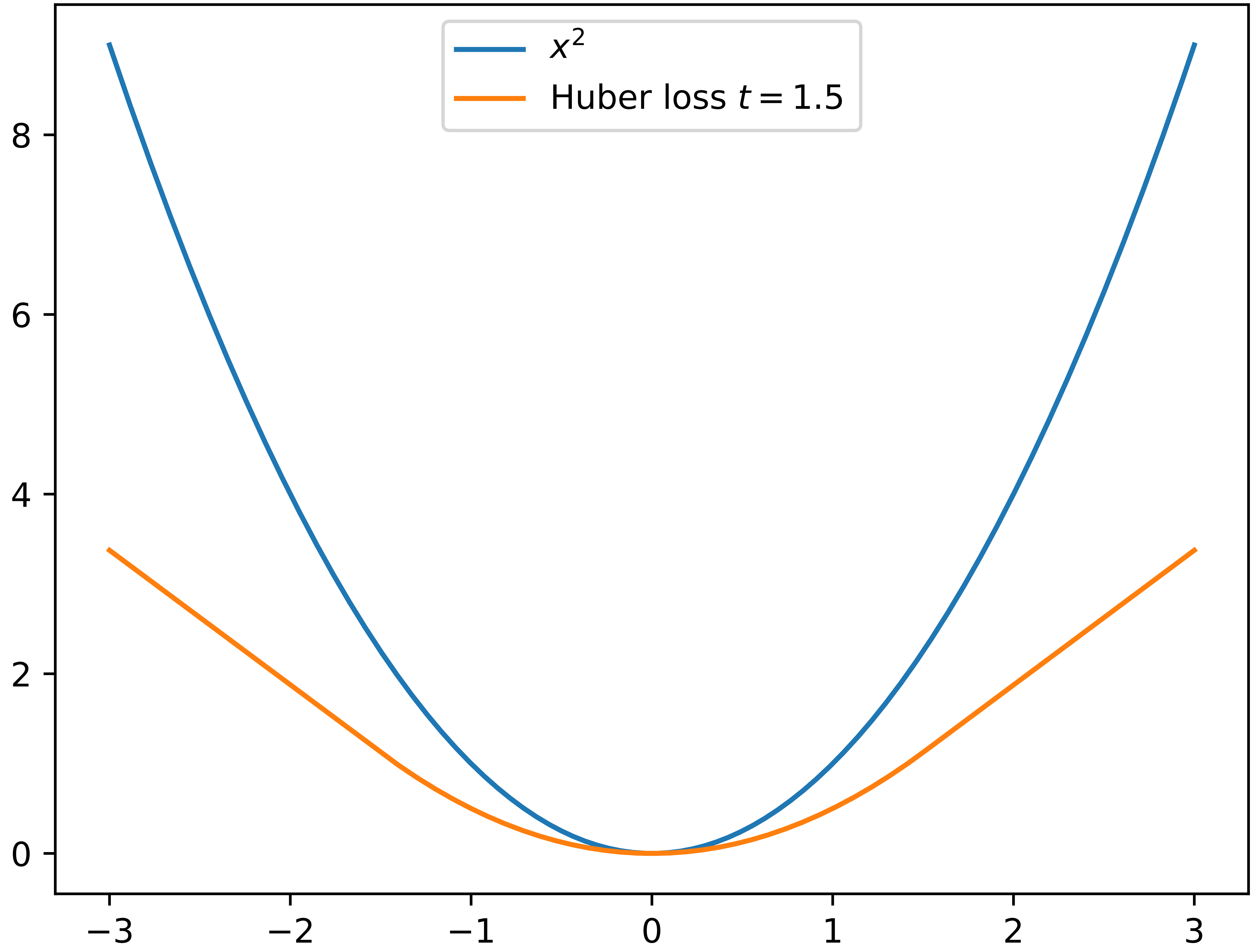

L1 loss: $\rho(x) = |x|$

# Robust loss functions




Truncated quadratic: $\rho(x) = \min(x^2, \tau)$

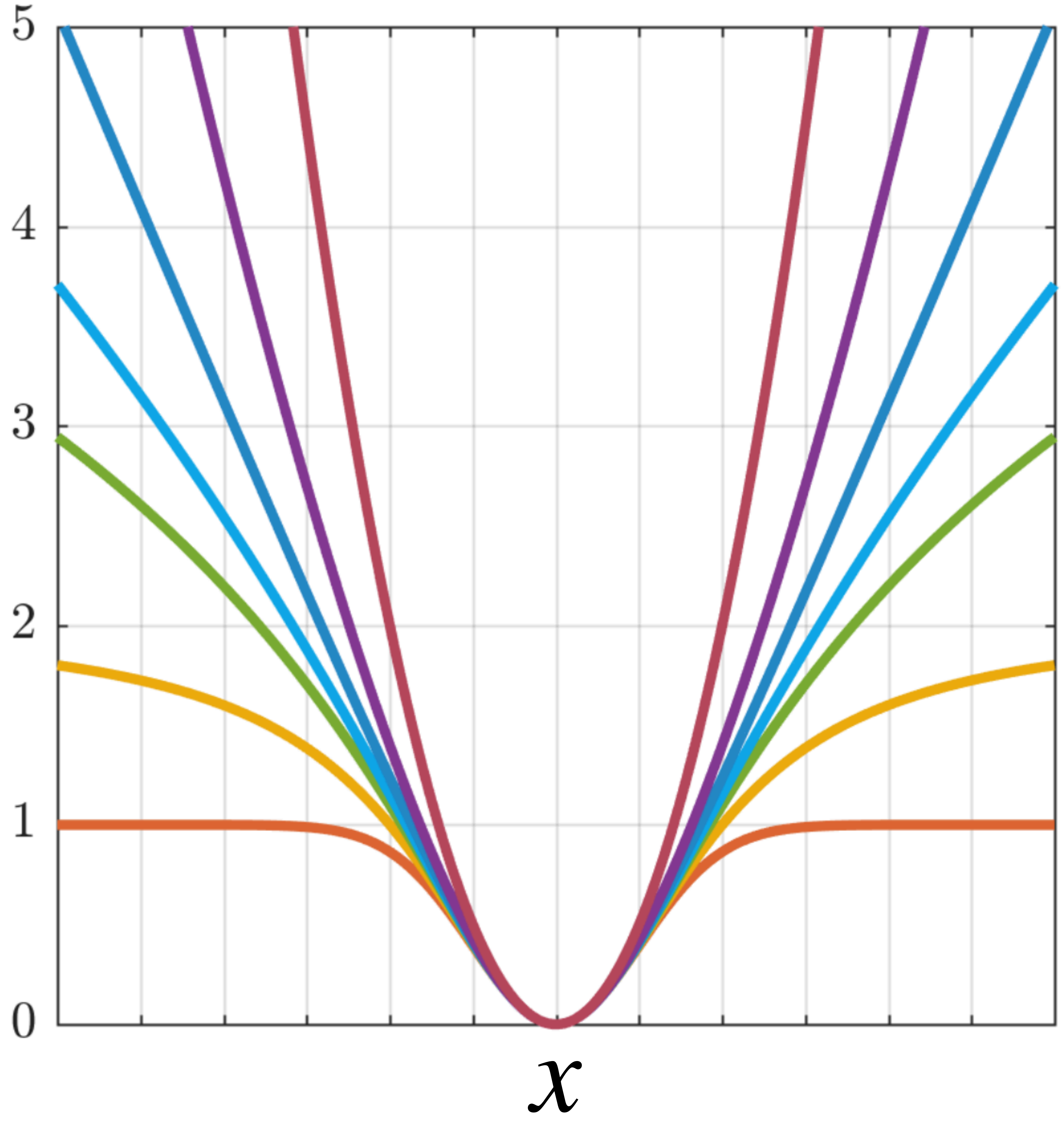
# Robust loss functions




## Huber loss:

$$\rho(x) = \begin{cases} \frac{1}{2}x^2 & \textbf{if } |x| \leq \tau, \\ \tau(|x| - \frac{1}{2}\tau), & \textbf{else} \end{cases}$$

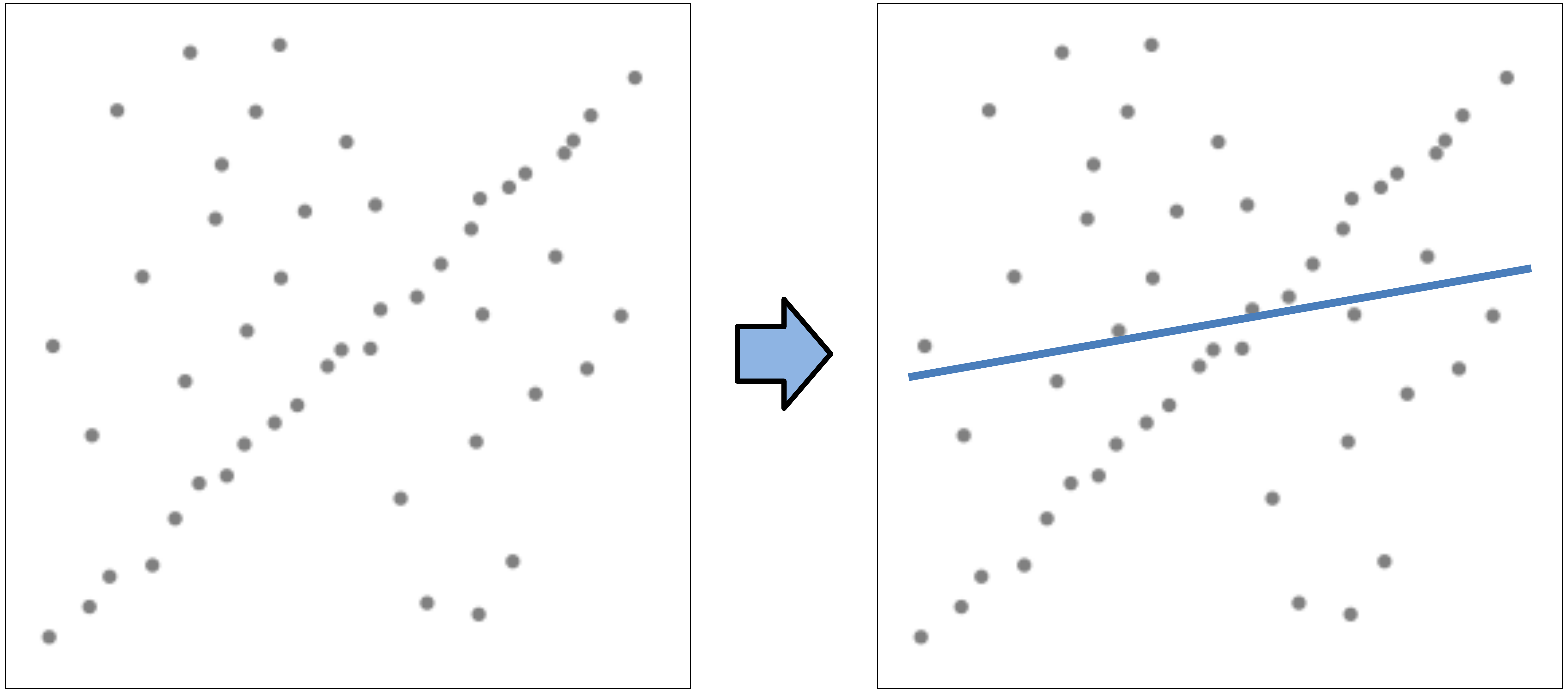
# Robust loss functions




Source: [Barron 2019, "A General and Adaptive Robust Loss Function"]

# Handling outliers

- Can be hard to fit a robust loss, e.g., due to local minima
- Another idea: trial and error!
- Let's consider the problem of linear regression



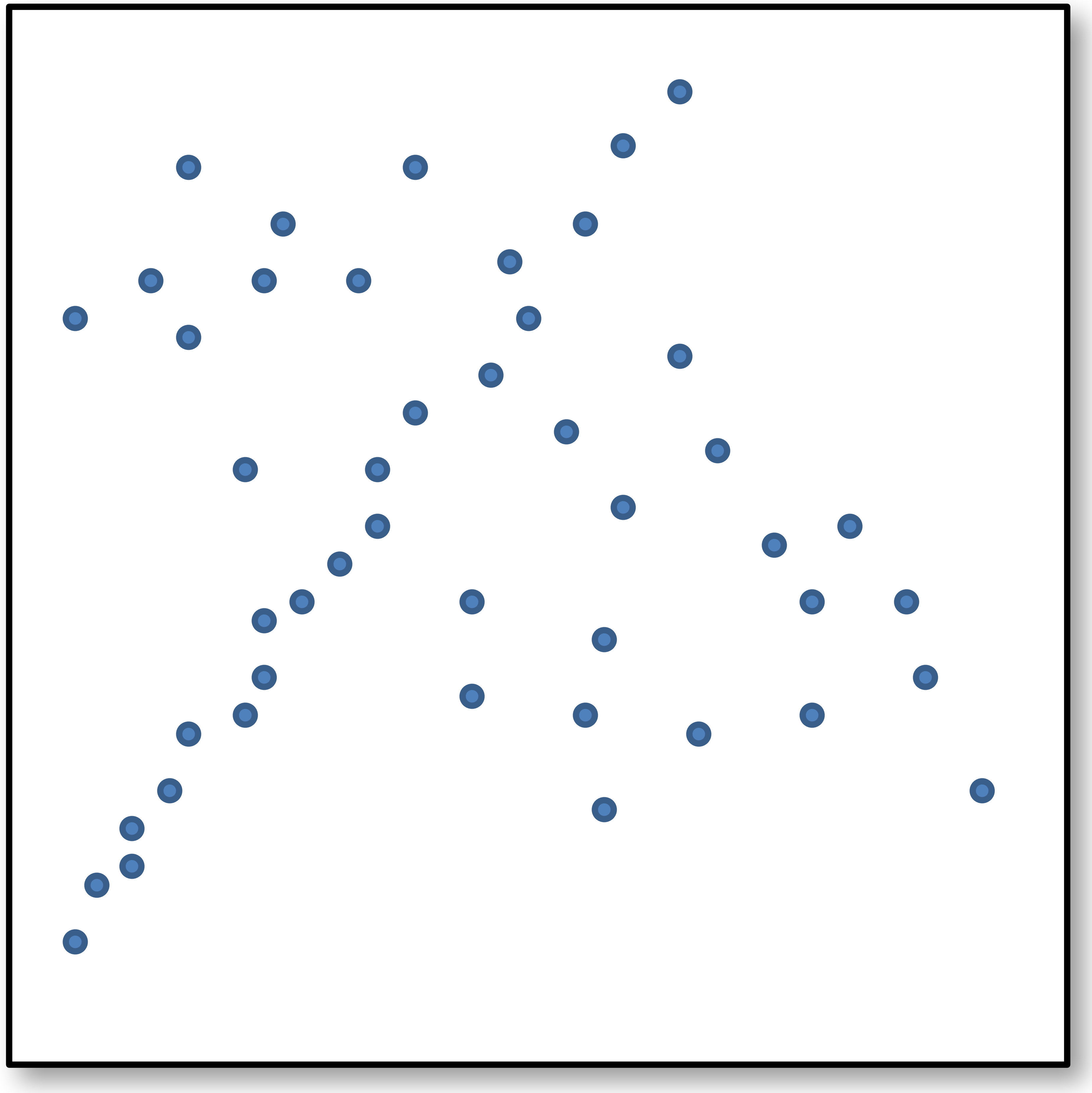
Problem: Fit a line to these data points

Least squares fit



Source: N. Snavely

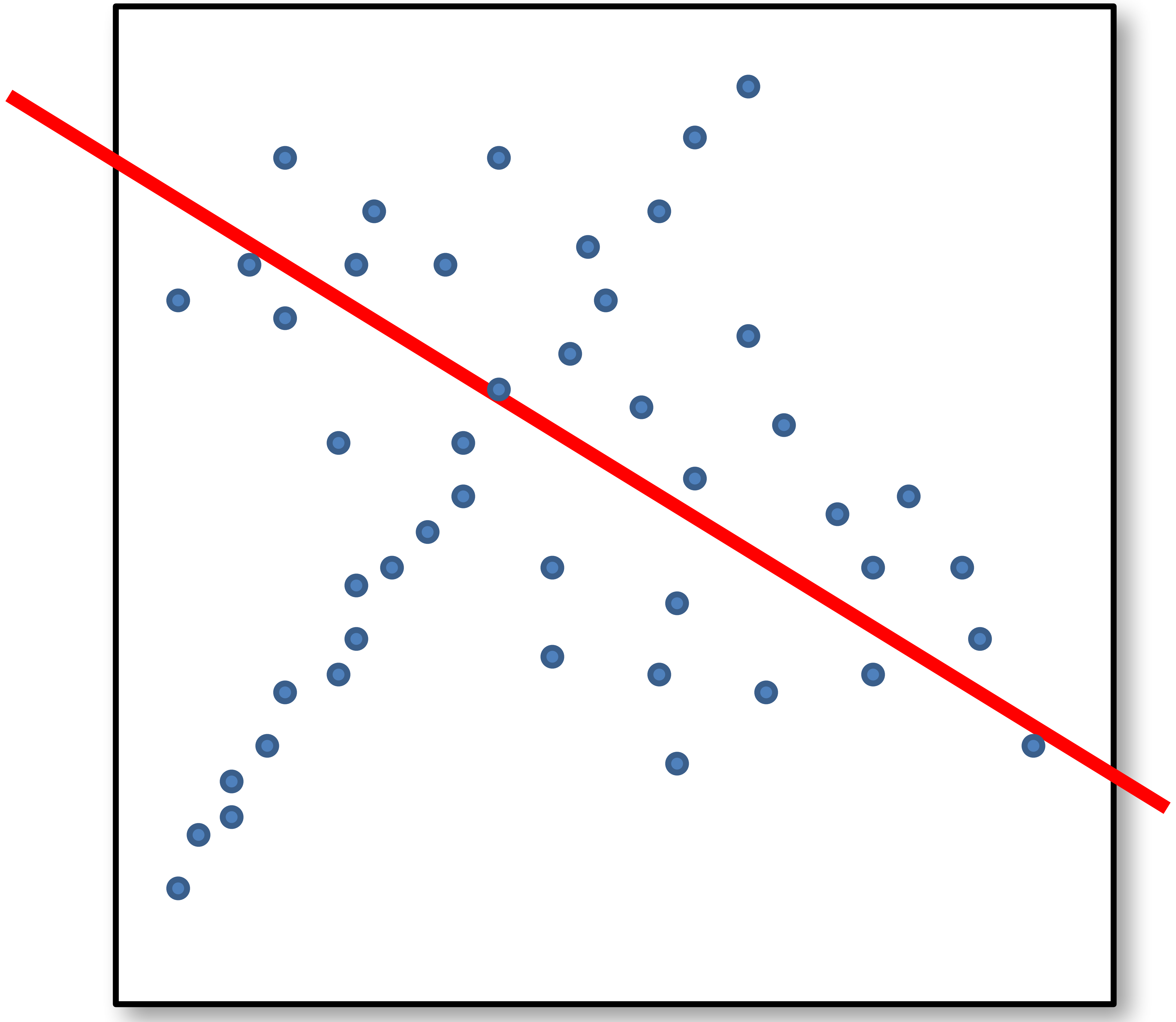
# Counting inliers





Source: N. Snavely

# Counting inliers

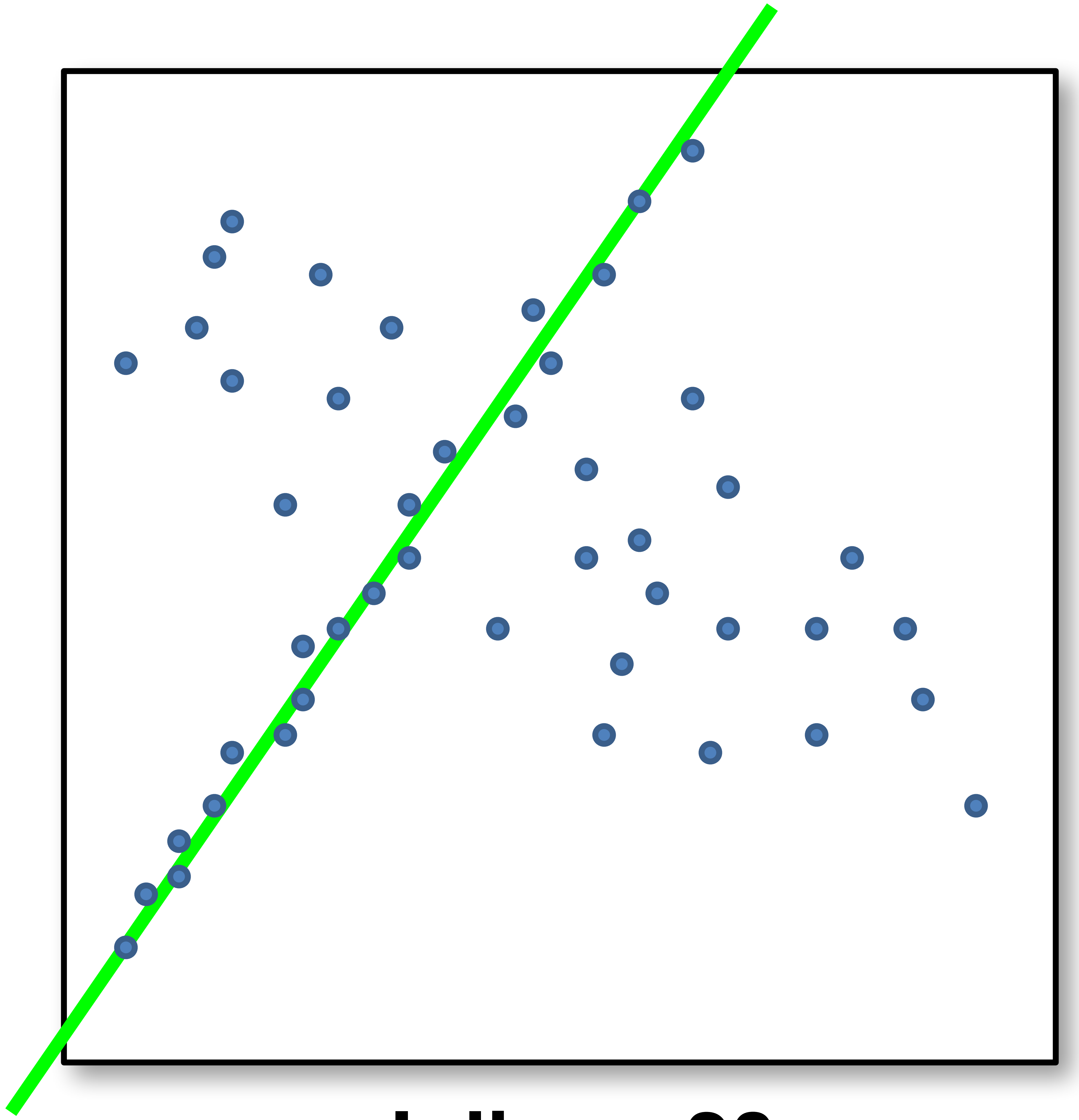


**Inliers: 3**



Source: N. Snavely

# Counting inliers



**Inliers: 20**



Source: N. Snavely

# RANSAC

- Idea:
  - All the inliers will agree with each other on the solution; the (hopefully small) number of outliers will (hopefully) disagree with each other
    - RANSAC only has guarantees if there are < 50% outliers
  - “All good matches are alike; every bad match is bad in its own way.”

– Tolstoy via Alyosha Efros

Source: N. Snavely

# RANSAC: random sample consensus

RANSAC loop (for N iterations):

- Select four feature pairs (at random)
- Compute homography H
- Count **inliers** where $||p_i' - f_H(p_i)|| < \varepsilon$

Afterwards:

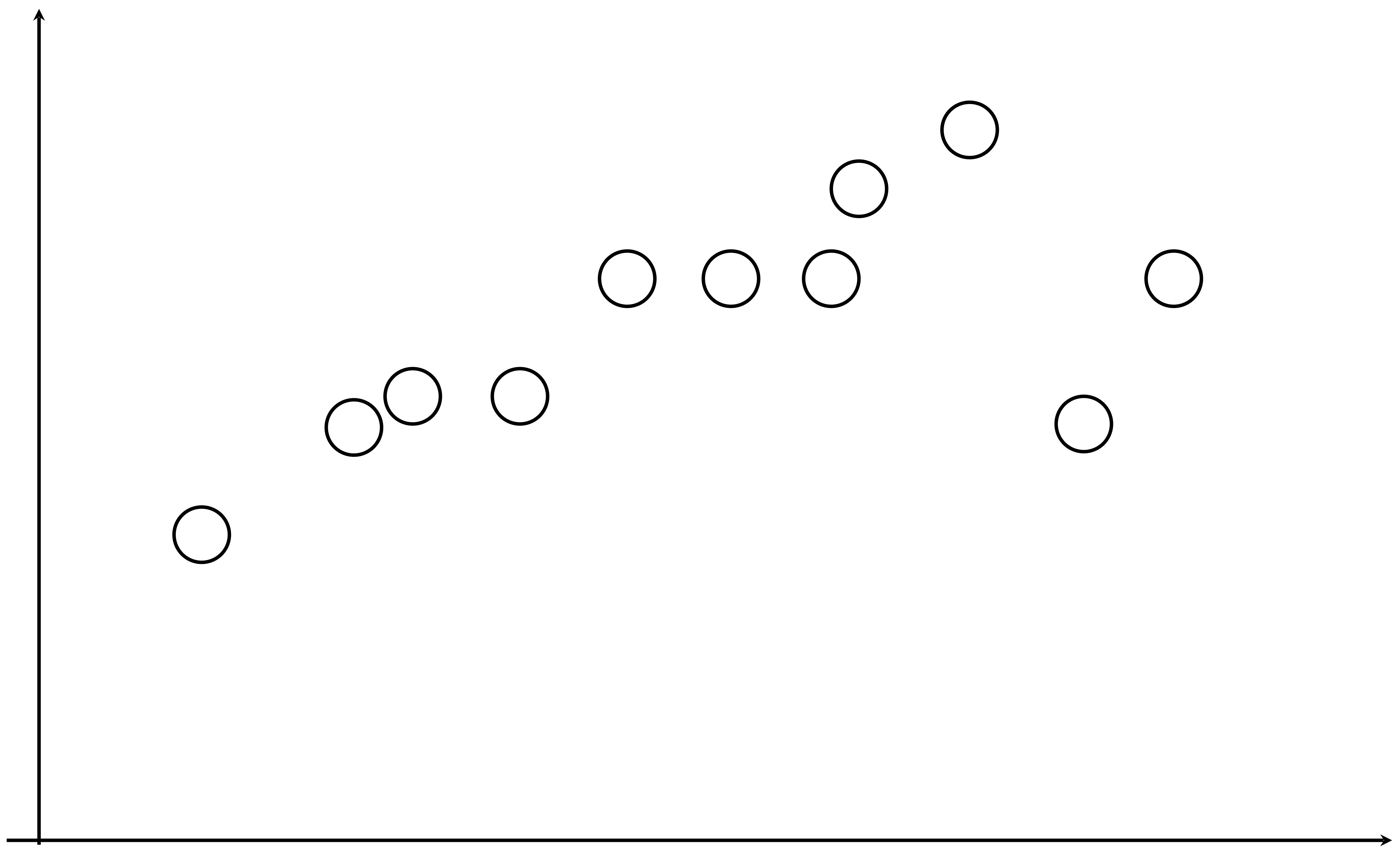
- Choose $\boldsymbol{H}$ with largest set of inliers
- Recompute $\boldsymbol{H}$ using only those inliers (often using high-quality nonlinear least squares)



Source: Torralba, Freeman, Isola

# Simple example: fit a line

- Rather than homography H (8 numbers) fit y=ax+b (2 numbers a, b) to 2D pairs

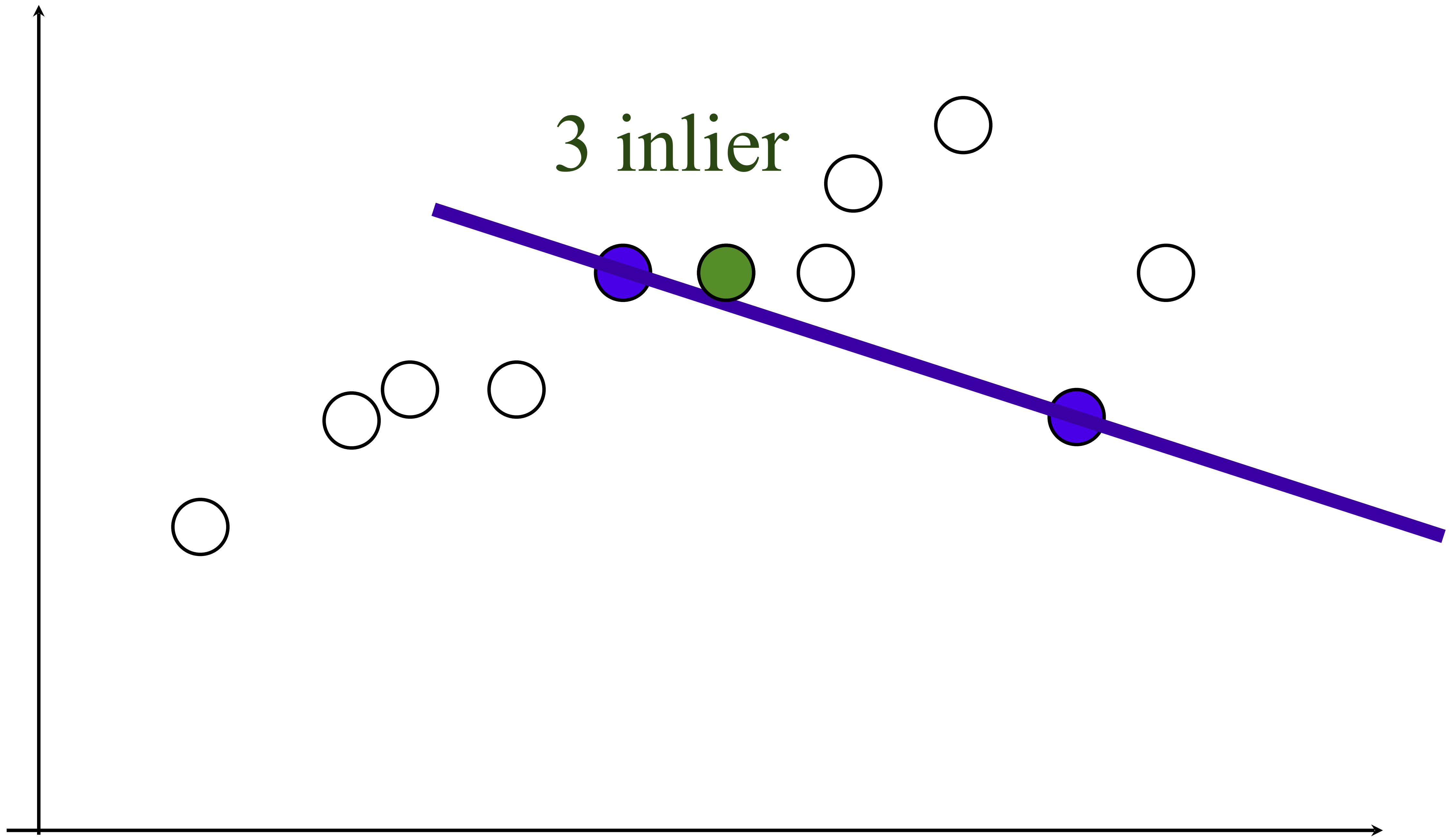




Source: Torralba, Freeman, Isola

# Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers

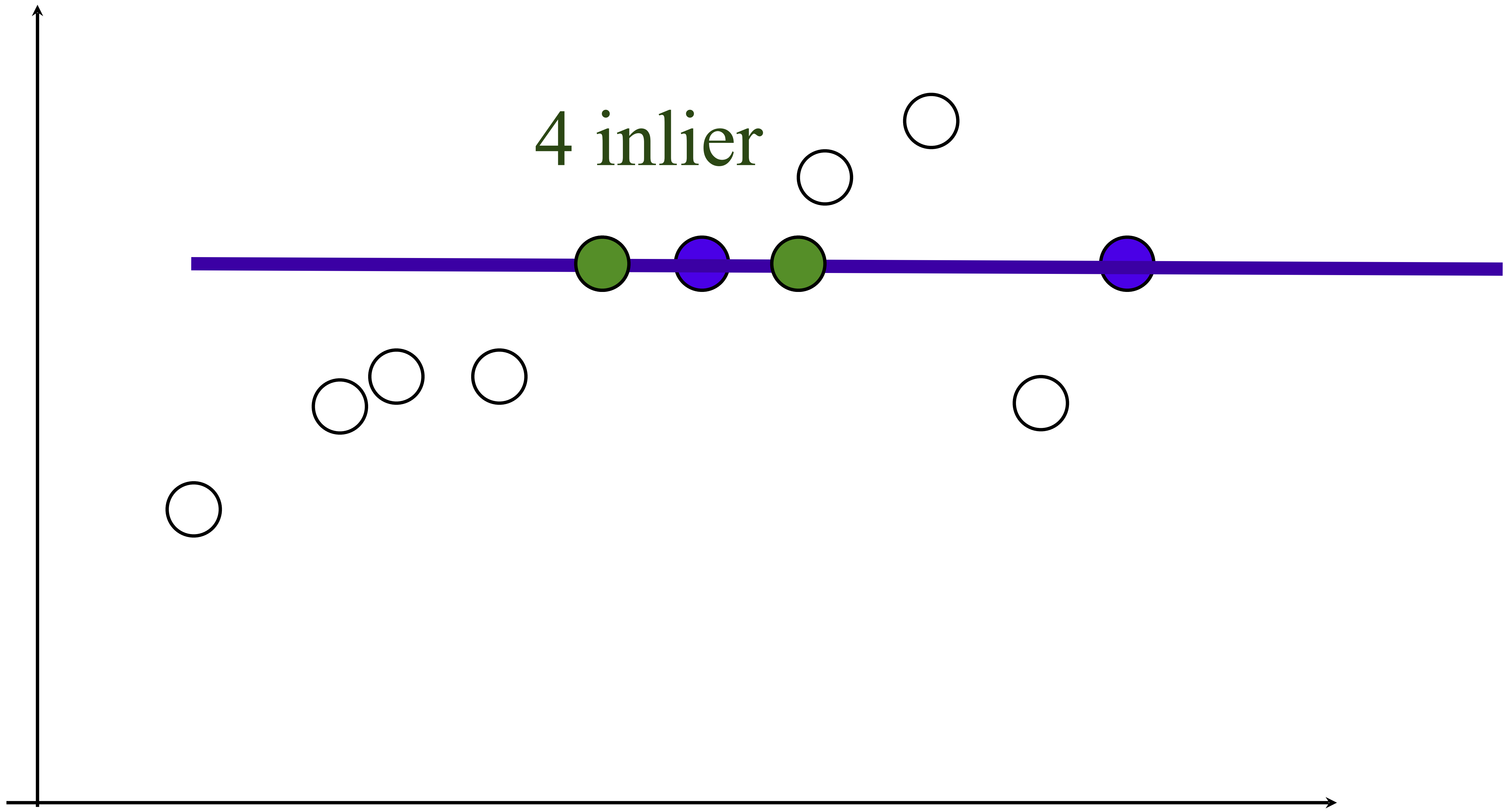





Source: Torralba, Freeman, Isola

# Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers

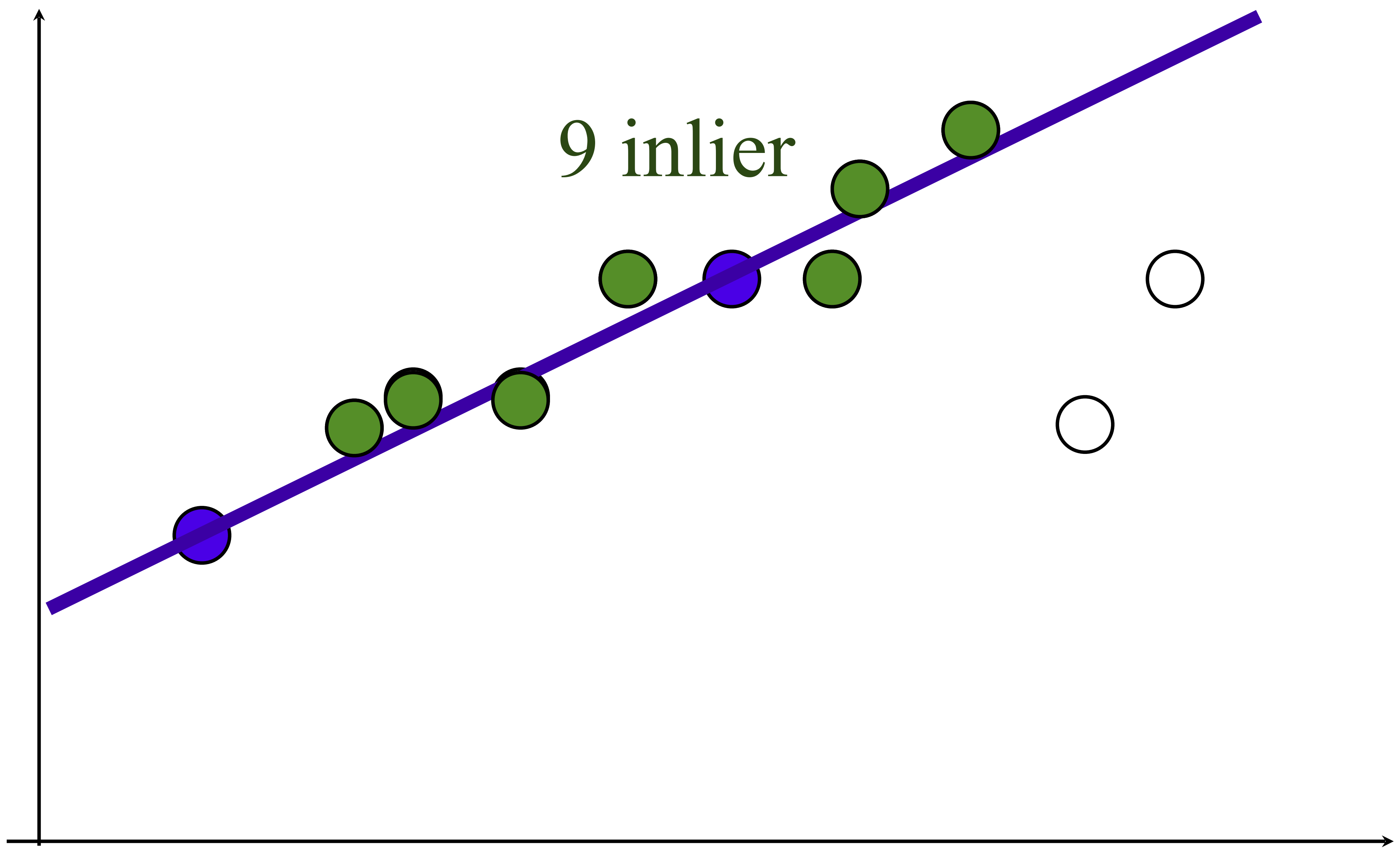





Source: Torralba, Freeman, Isola

# Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers

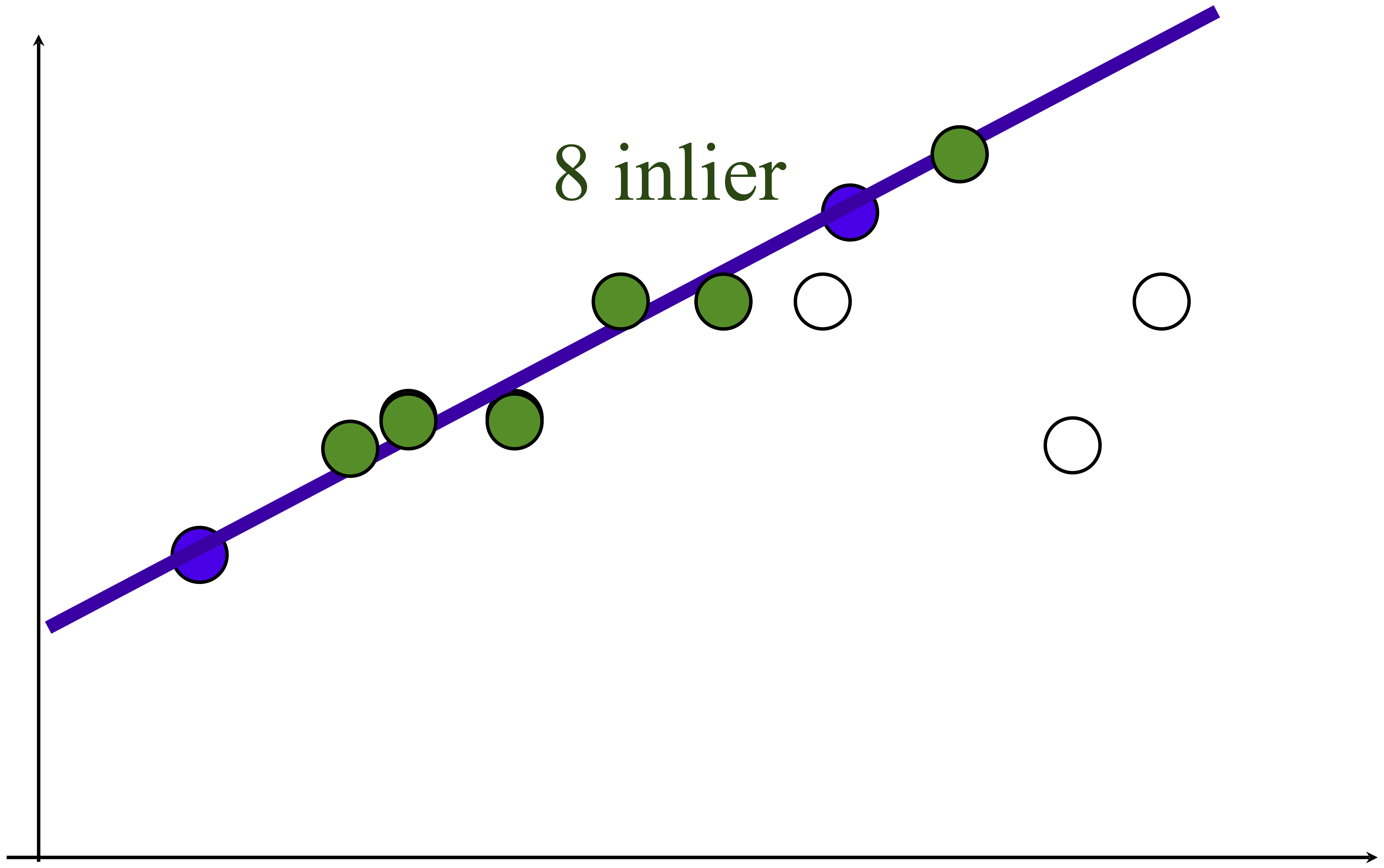





Source: Torralba, Freeman, Isola

# Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers

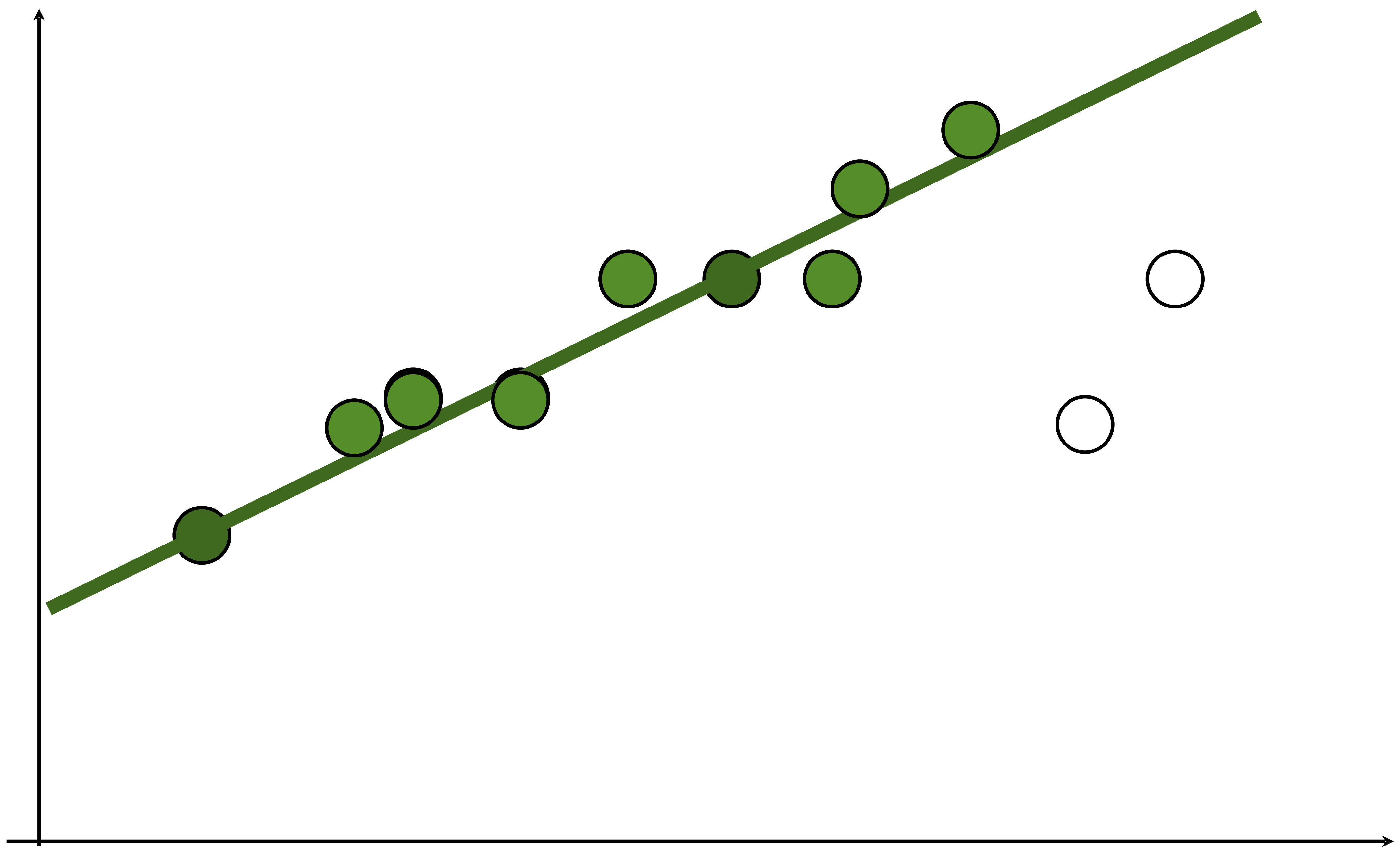





Source: Torralba, Freeman, Isola

# Simple example: fit a line

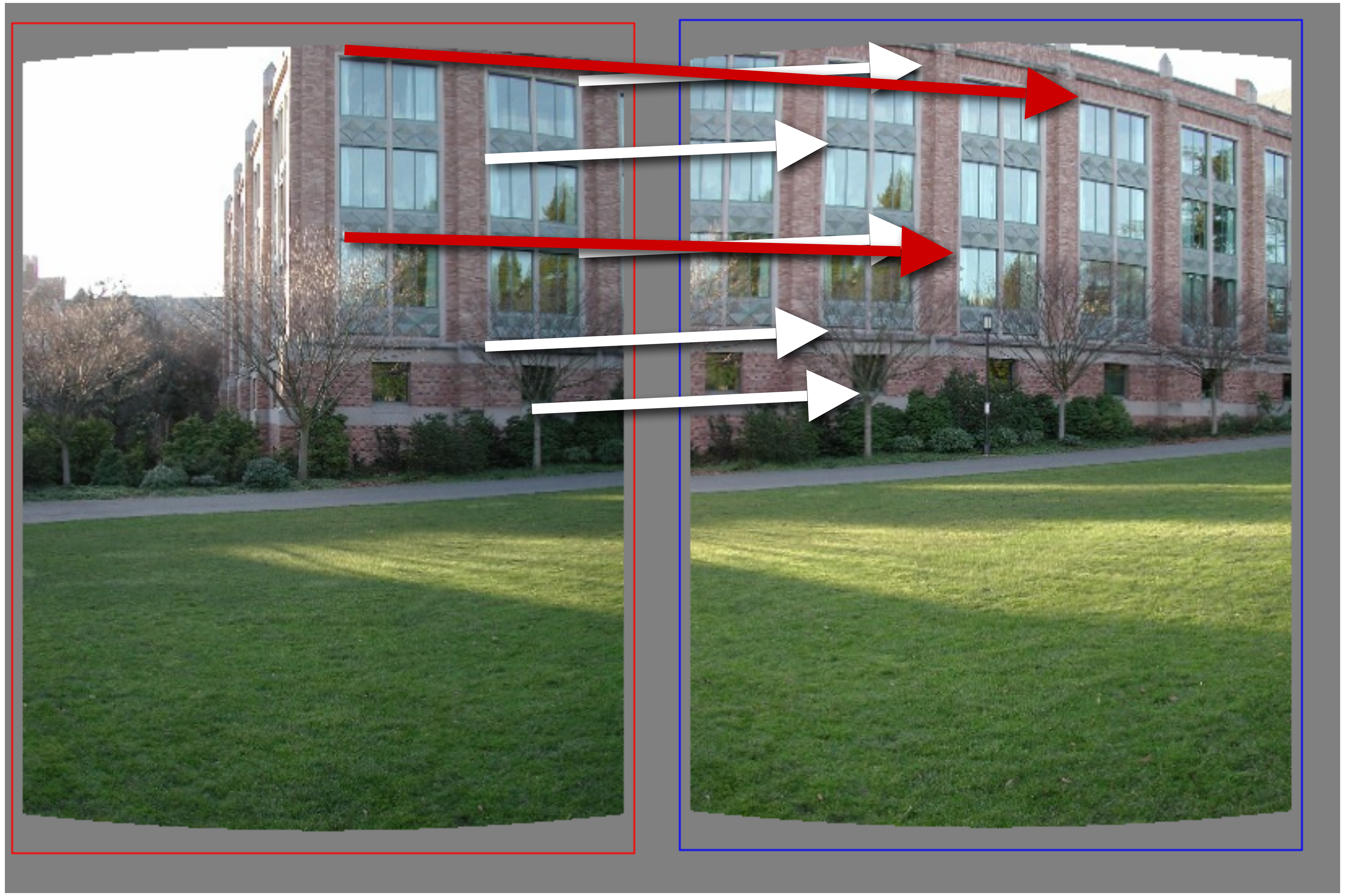
- Use biggest set of inliers
- Do least-square fit





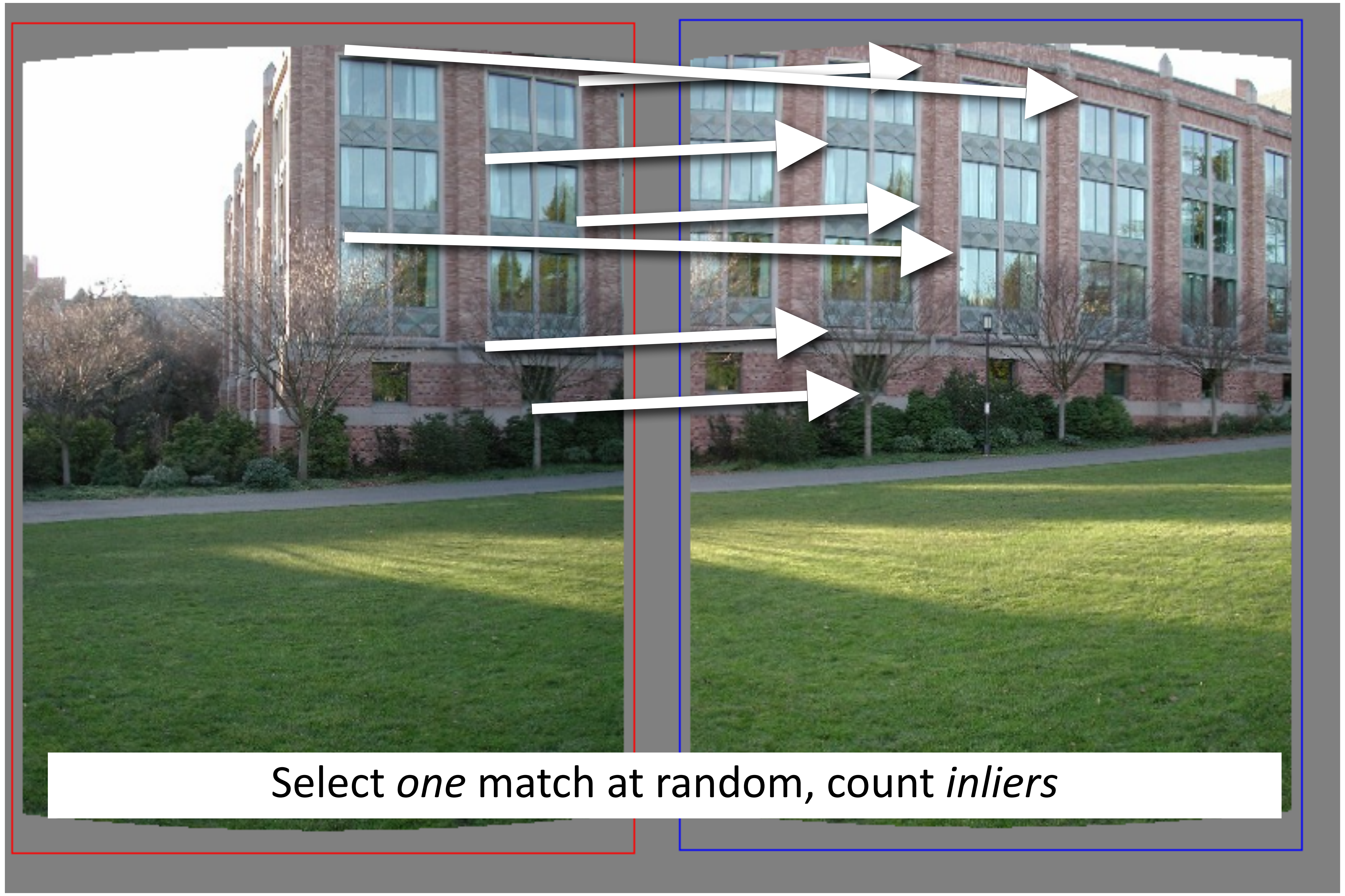
Source: Torralba, Freeman, Isola

# Example: fitting a translation



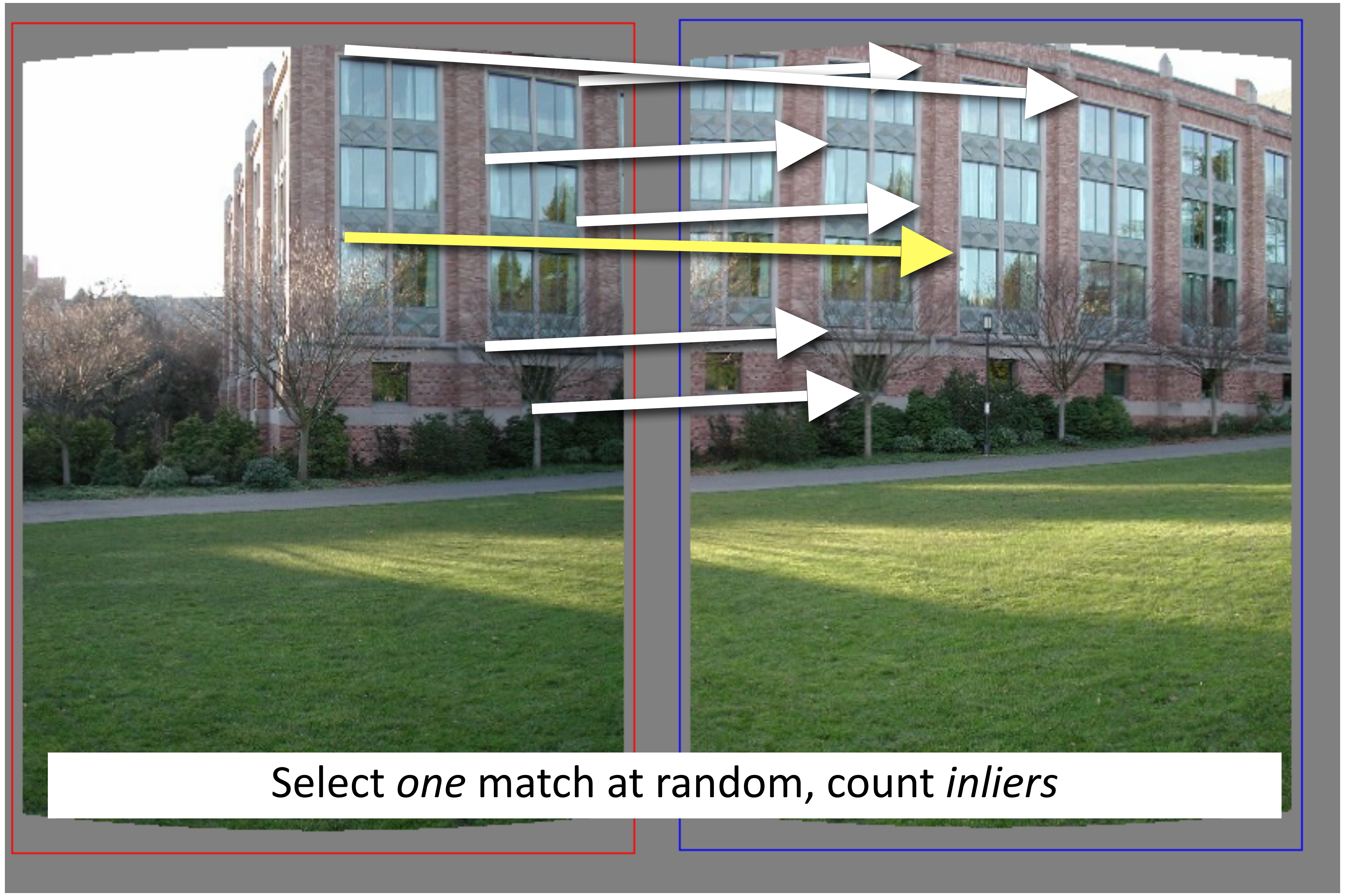
Source: N. Snavely

# RAndom SAmple Consensus



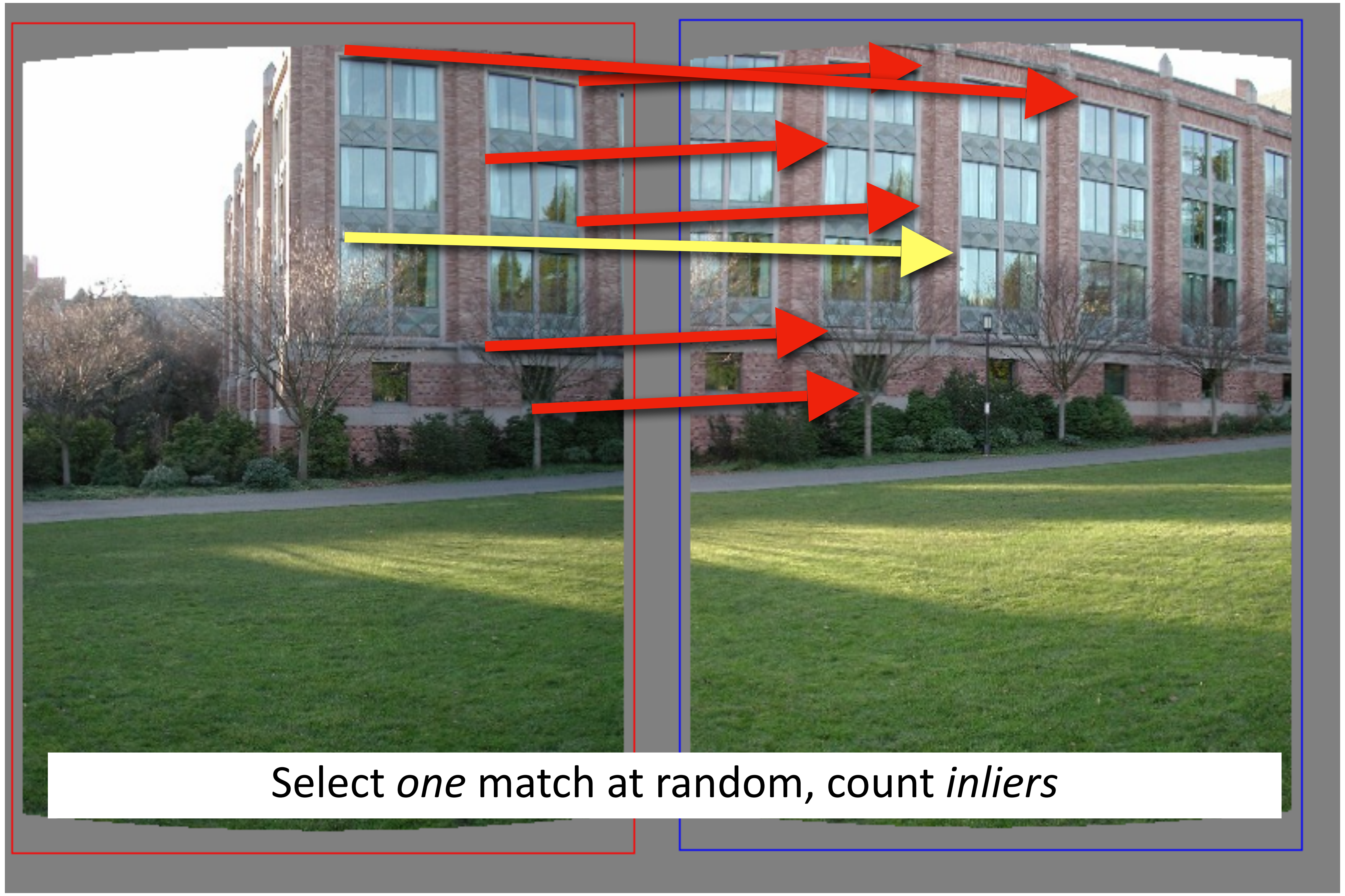

Source: N. Snavely

# RAndom SAmple Consensus



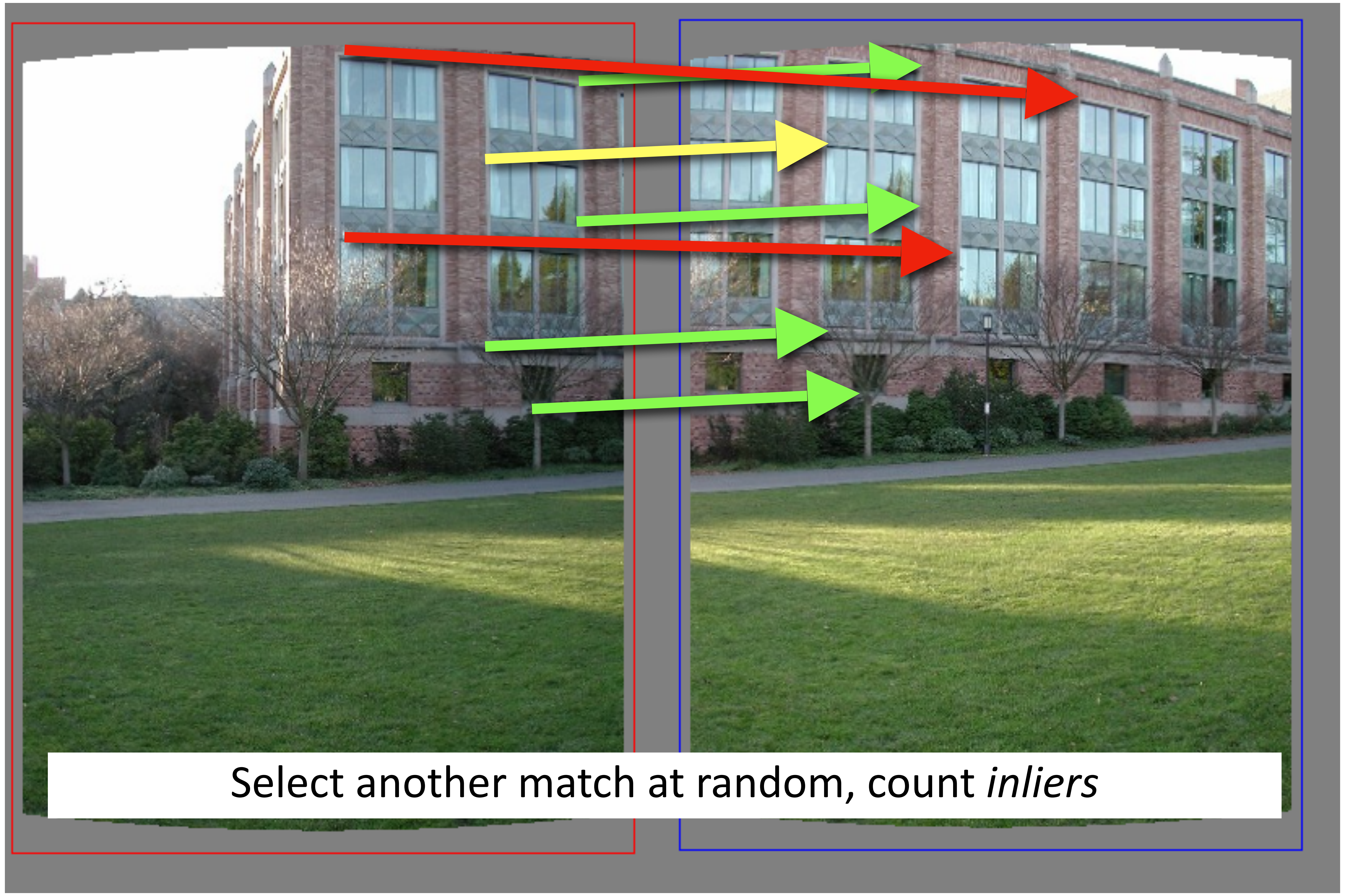

Source: N. Snavely

# RAndom SAmple Consensus



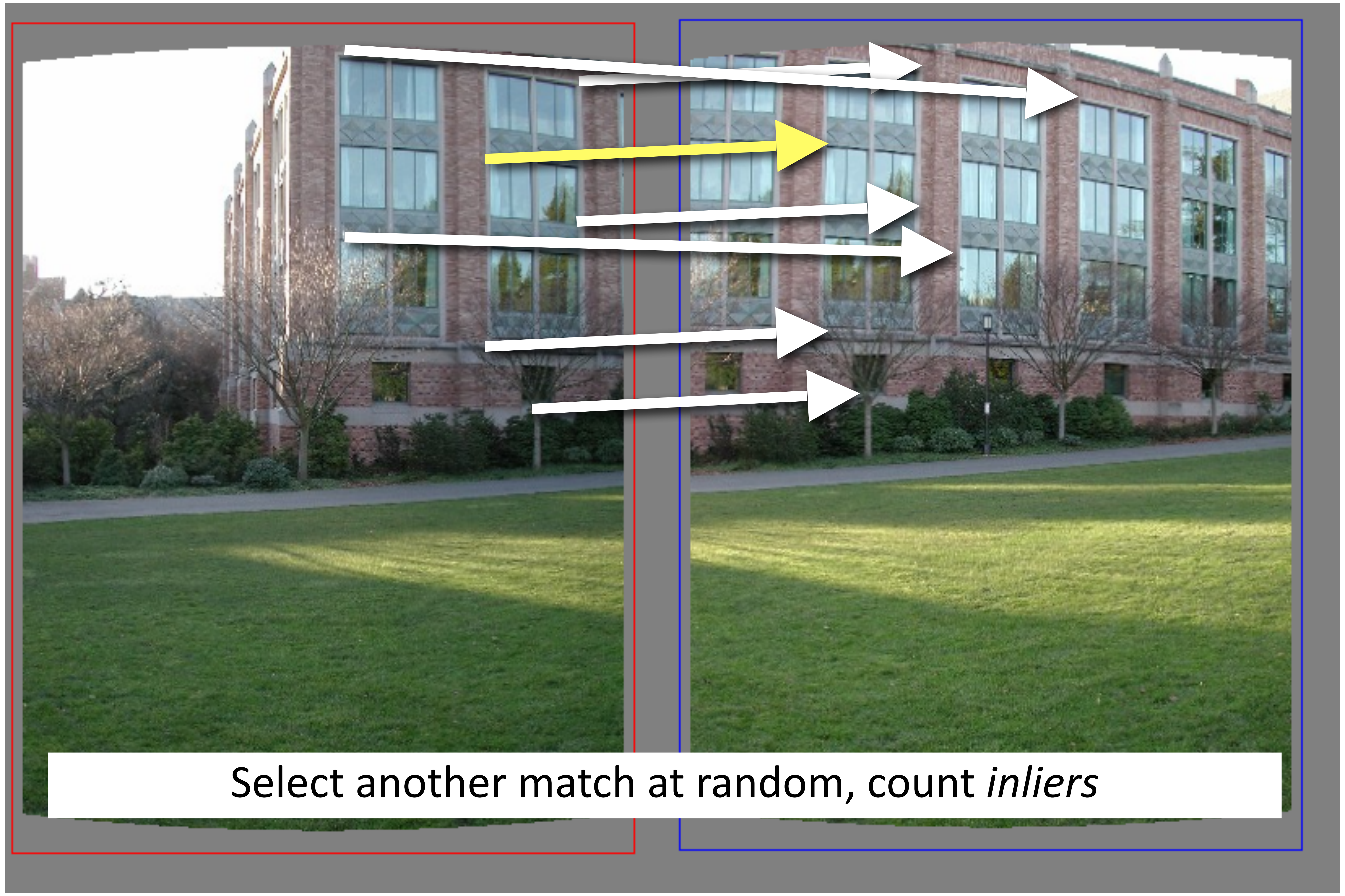

Source: N. Snavely

# RAndom SAmple Consensus



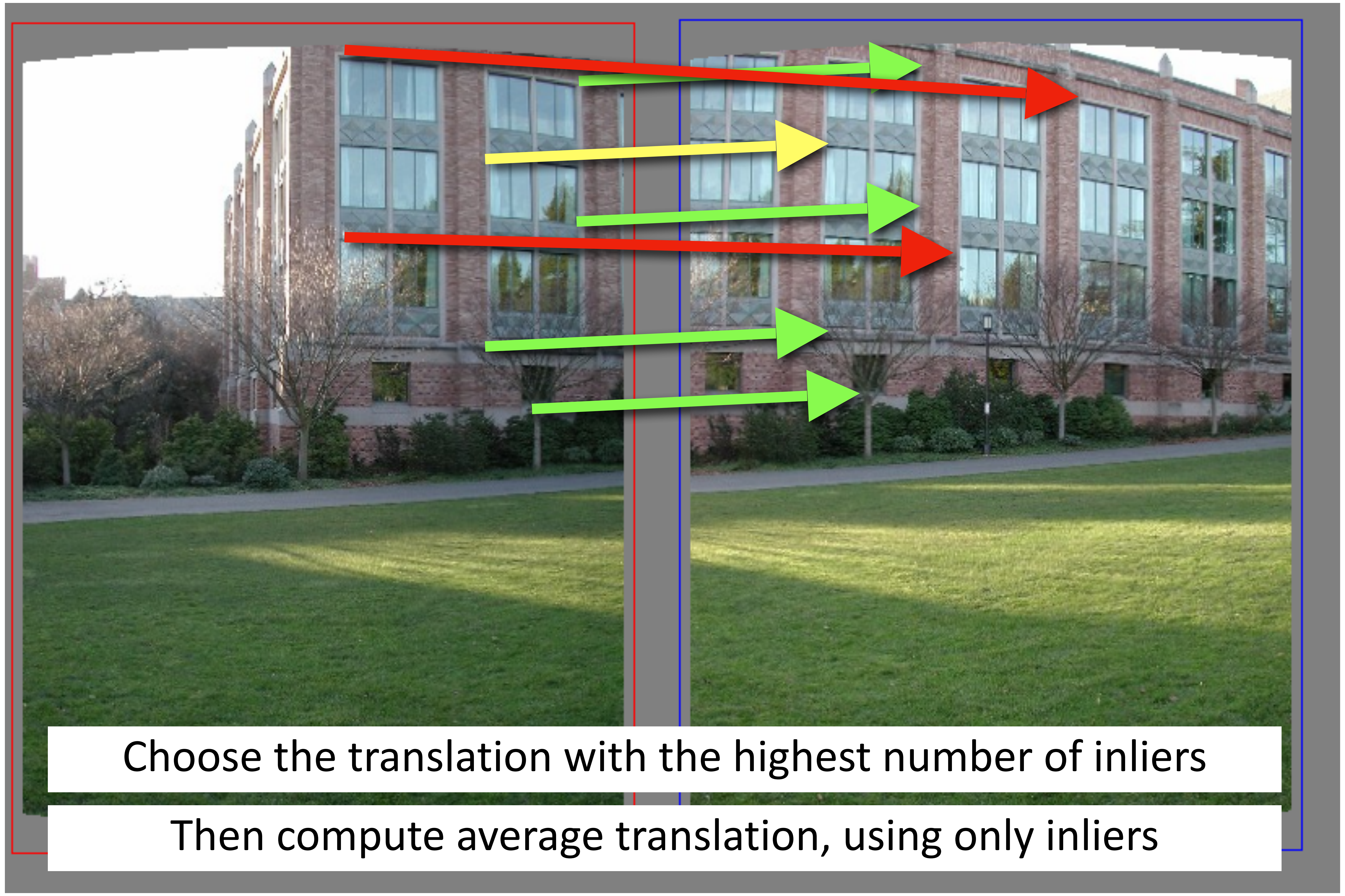

Source: N. Snavely

# <u>RA</u>ndom <u>SA</u>mple <u>C</u>onsensus



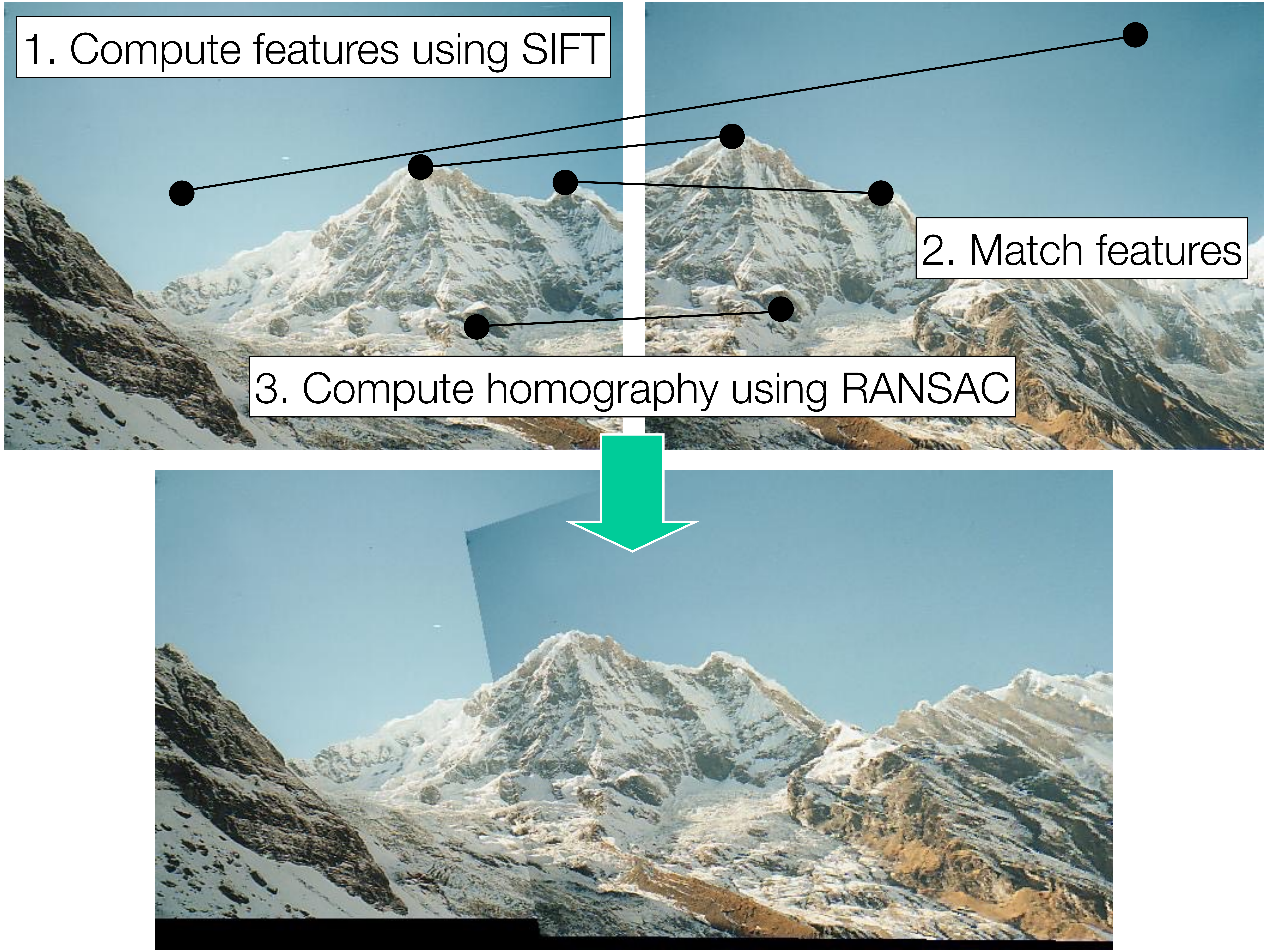

Source: N. Snavely

# RAndom SAmple Consensus




Source: N. Snavely

# Warping with a homography (PS8)






Source: N. Snavely

# A similar geometric problem: triangulation

Given projection $\mathbf{p}_i$ of unknown 3D point $\mathbf{X}$ in two or more images (with known cameras $\mathbf{P}_i$), find $\mathbf{X}$

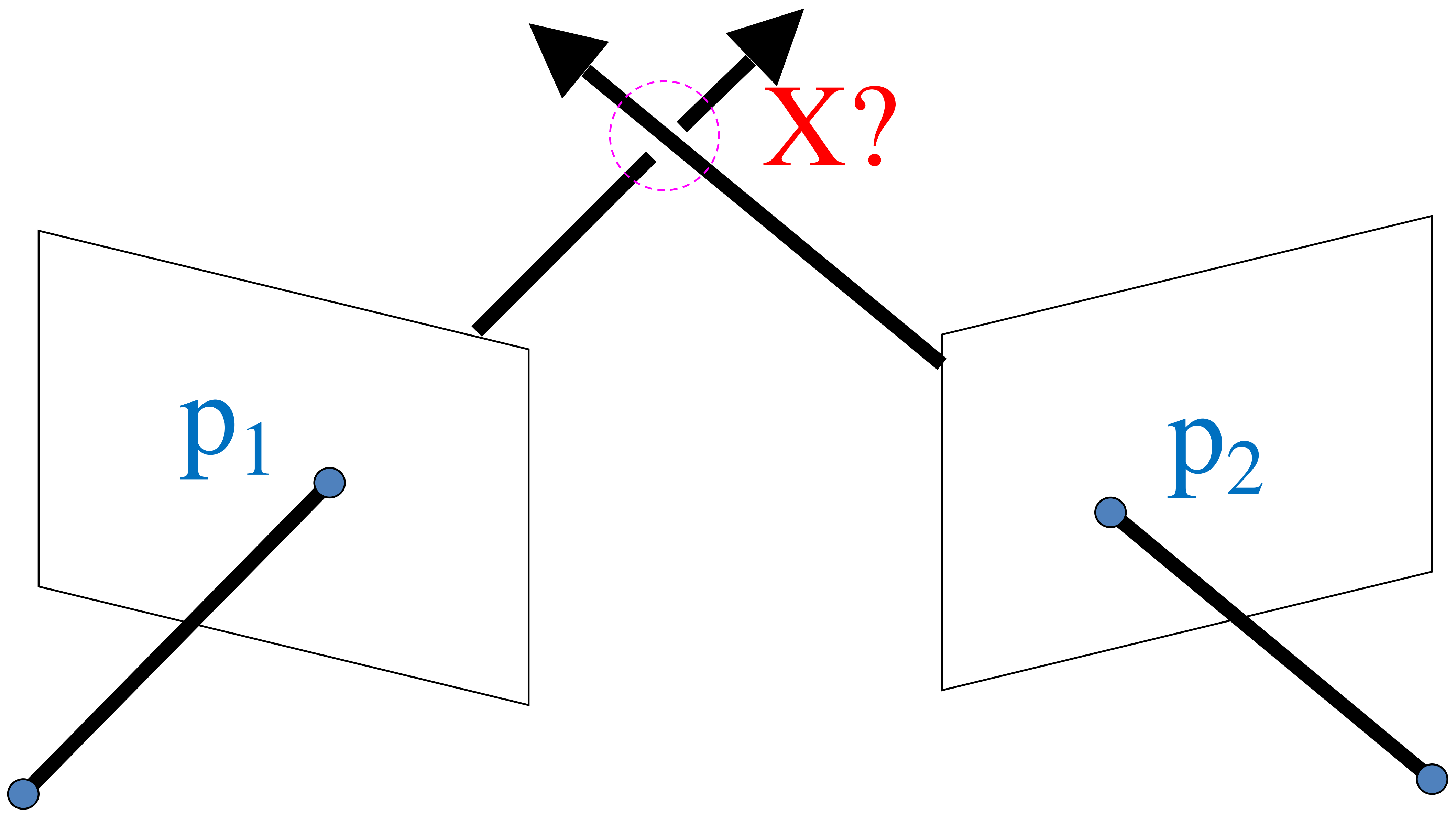


Source: D. Fouhey

# Triangulation

Given projection $\mathbf{p_i}$ of unknown 3D point $\mathbf{X}$ in two or more images (with known cameras $\mathbf{P_i}$), find $\mathbf{X}$

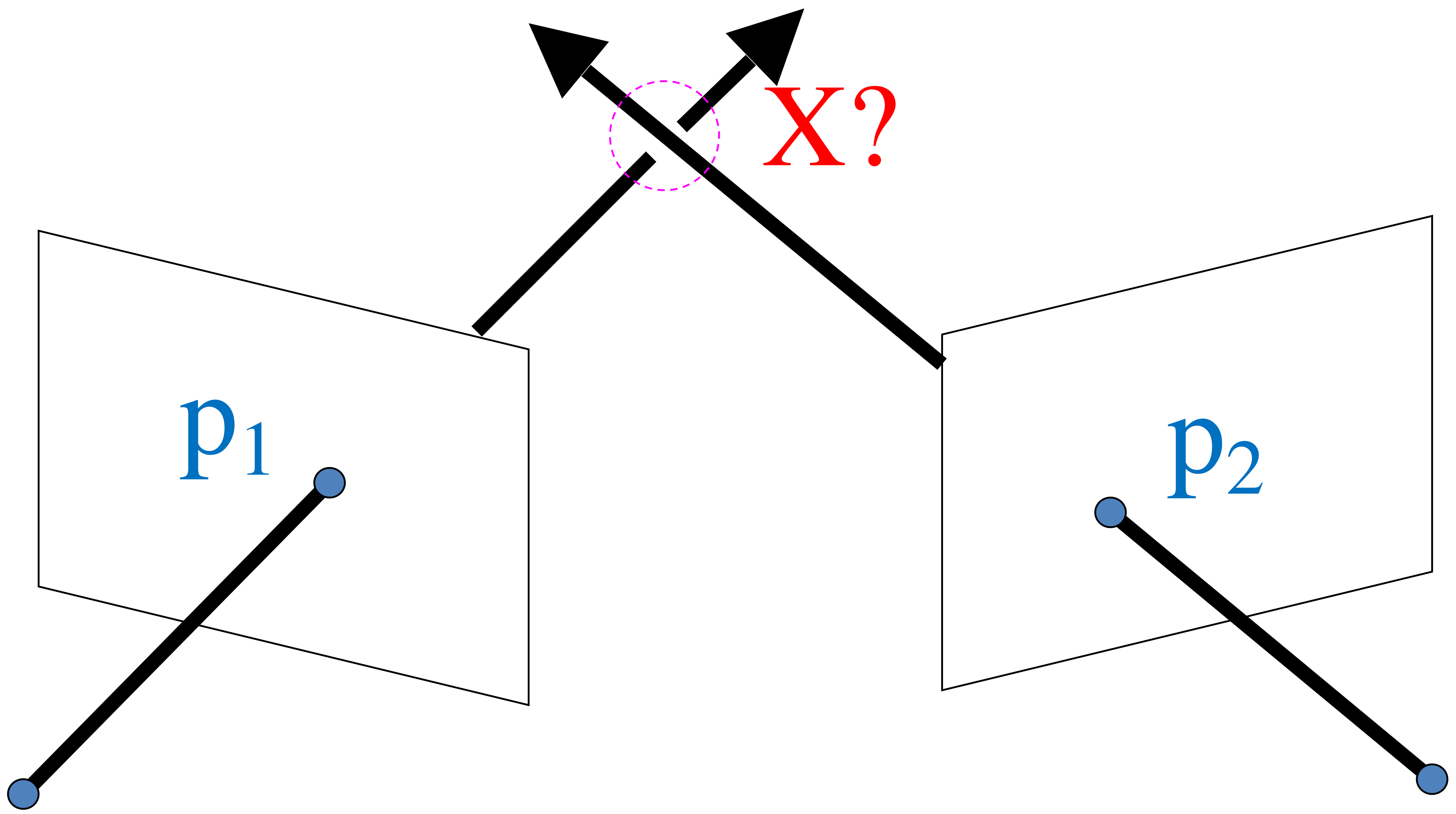
**Why is the calibration here important?**




Source: D. Fouhey

# Triangulation

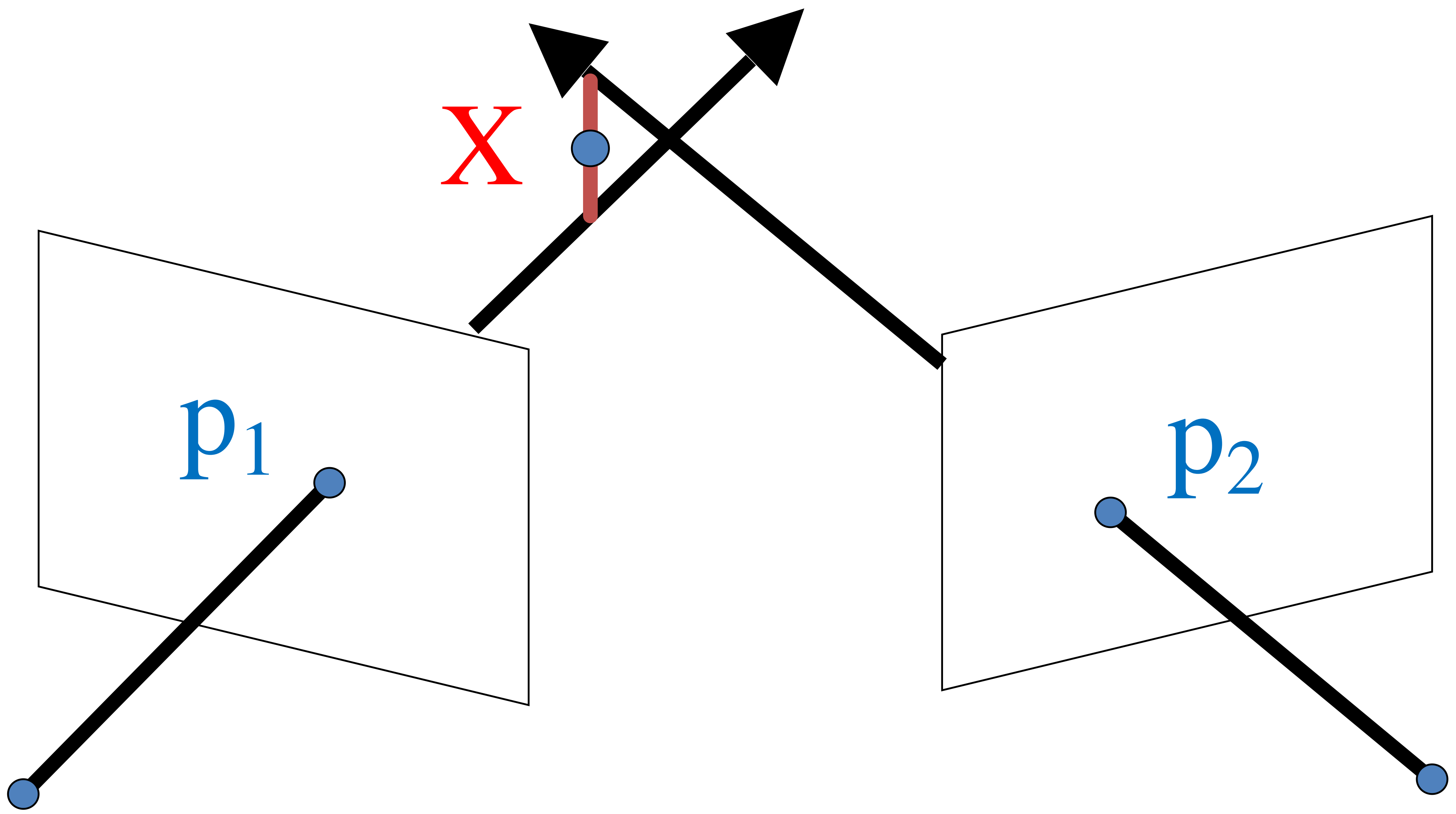
Rays in principle should intersect, but in practice usually don't exactly due to noise, numerical errors.




Source: D. Fouhey

# Triangulation – Geometry

Find shortest segment between viewing rays, set **X** to be the midpoint of the segment.

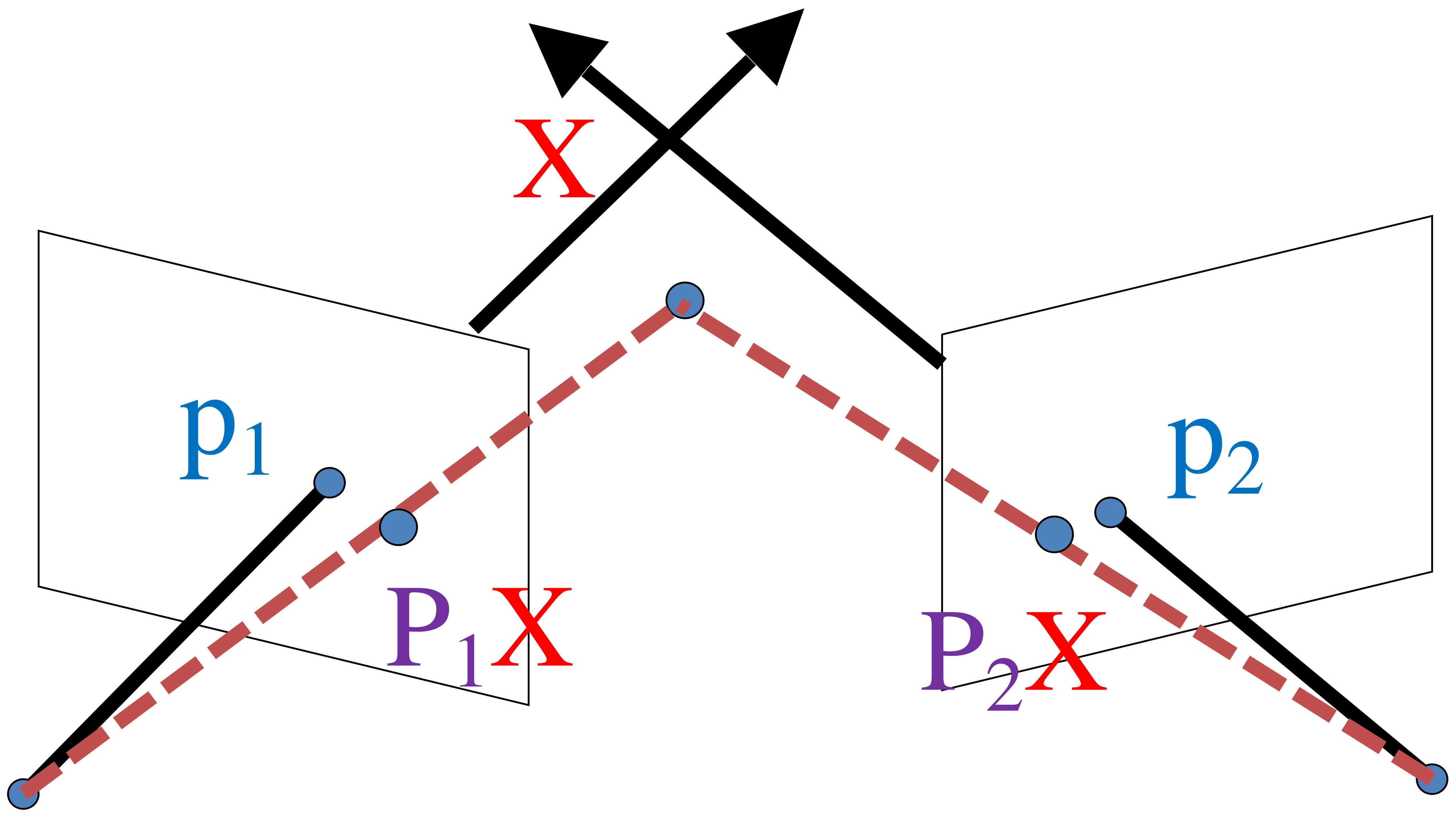



Source: D. Fouhey

# Triangulation – Non-linear Optim.

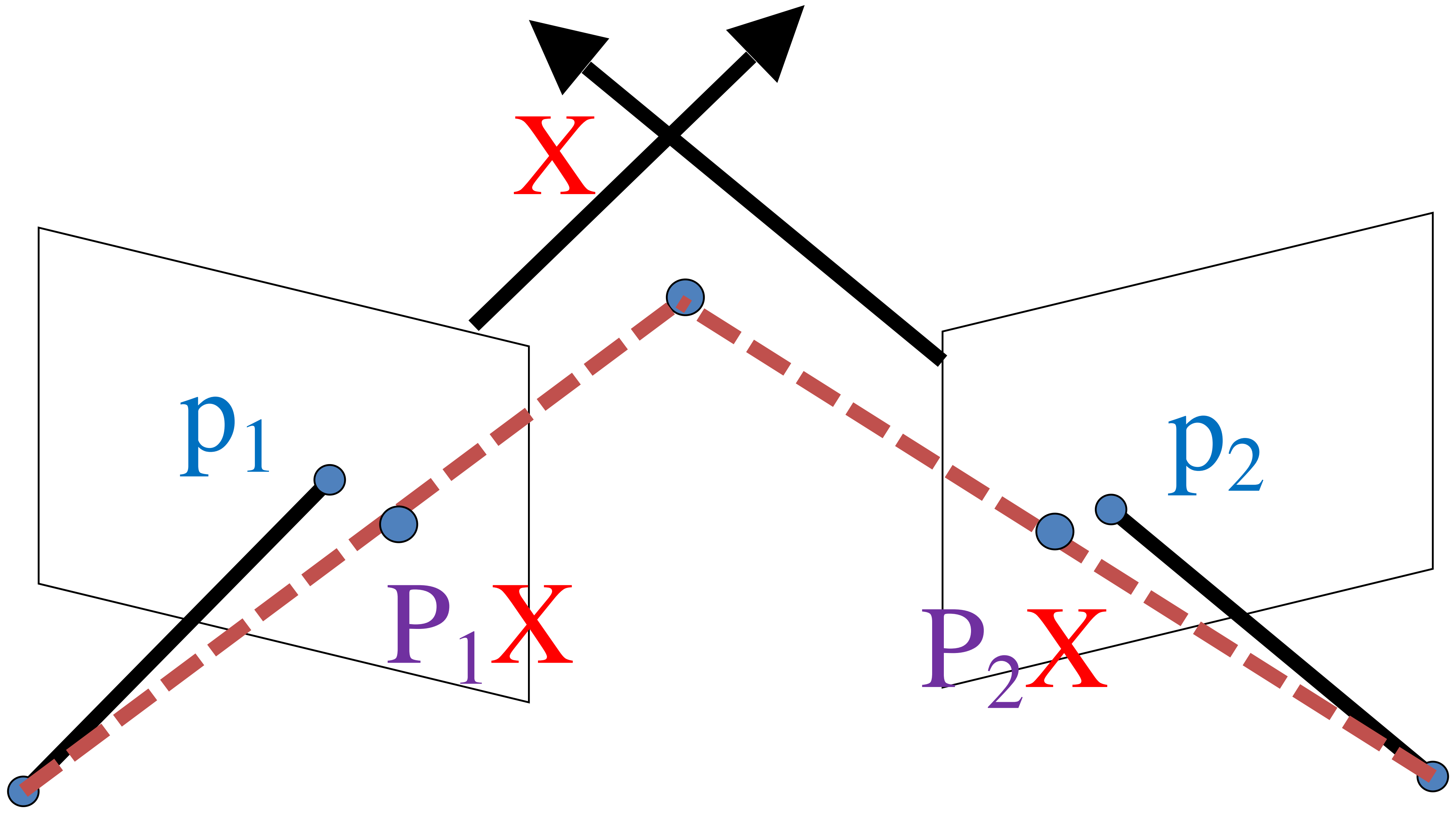
Find X minimizing $d\left(\boldsymbol{p}_1, P_1\boldsymbol{X}\right)^2 + d\left(\boldsymbol{p}_2, P_2\boldsymbol{X}\right)^2$

where $d$ is distance in image space




Source: D. Fouhey

# Triangulation – Linear Optimization




Source: D. Fouhey

# First: A better way to handle homogeneous coordinates in linear optimization

Recall: projection in homogeneous coordinates.

$$\boldsymbol{p_i} \equiv P\boldsymbol{X_i}$$

Remember: this implies **$PX_i$ & $p_i$** are proportional/scaled copies of each other

$$\boldsymbol{p_i} = \lambda P\boldsymbol{X_i},\ \lambda \neq 0$$

This implies their cross product is **0**, since $a \times b = \|a\| \|b\| \sin(\theta)$.

$$\boldsymbol{p_i} \times P\boldsymbol{X_i} = \boldsymbol{0}$$

Handles the "divide by 0" issue we saw before.

Source: D. Fouhey

# Triangulation – Linear Optimization

$$p_1 \equiv P_1 \mathbf{X} \qquad p_2 \equiv P_2 \mathbf{X}$$

$$\rightarrow \quad p_1 \times P_1 \mathbf{X} = \mathbf{0} \qquad p_2 \times P_2 \mathbf{X} = \mathbf{0}$$

$$\rightarrow \quad [p_{1x}] P_1 \mathbf{X} = \mathbf{0} \qquad [p_{2x}] P_2 \mathbf{X} = \mathbf{0}$$

Cross Prod.
as matrix

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [\boldsymbol{a}_x] \boldsymbol{b}$$

$$[p_{1x}] P_1 \mathbf{X} = \mathbf{0} \qquad [p_{2x}] P_2 \mathbf{X} = \mathbf{0}$$

$$\rightarrow \quad ([p_{1x}] P_1) \mathbf{X} = \mathbf{0} \qquad ([p_{2x}] P_2) \mathbf{X} = \mathbf{0}$$

$\rightarrow$ Two eqns per camera for 3 unknown in X

Source: D. Fouhey

# Next time: Estimating 3D structure

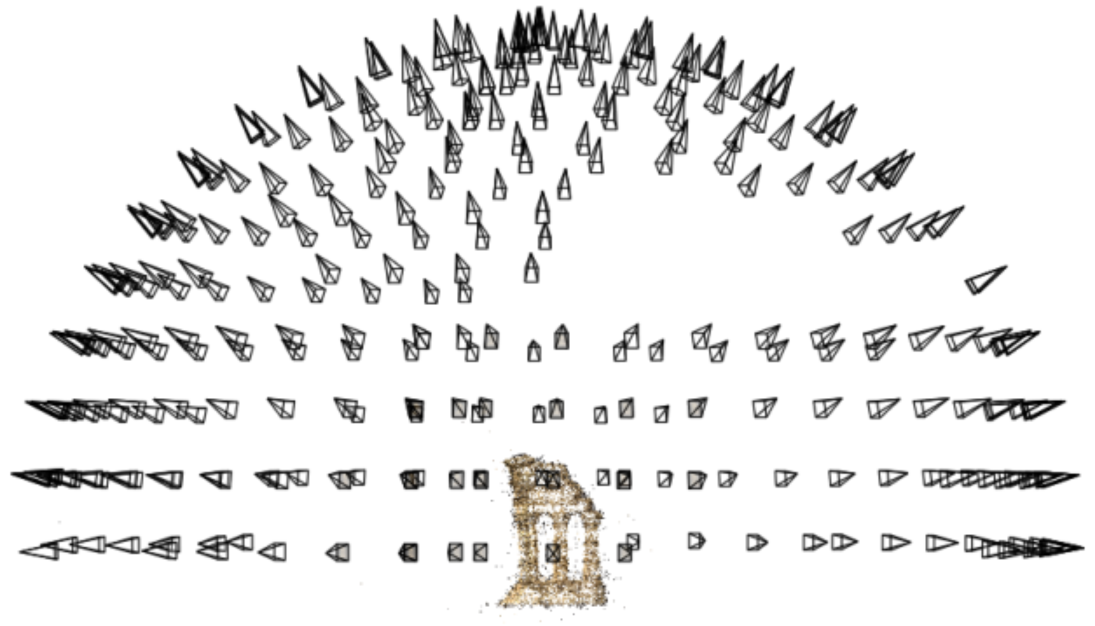
- Given many images, how can we…
  1. Figure out where they were all taken from?
  2. Build a 3D model of the scene?

This is the **structure from motion** problem

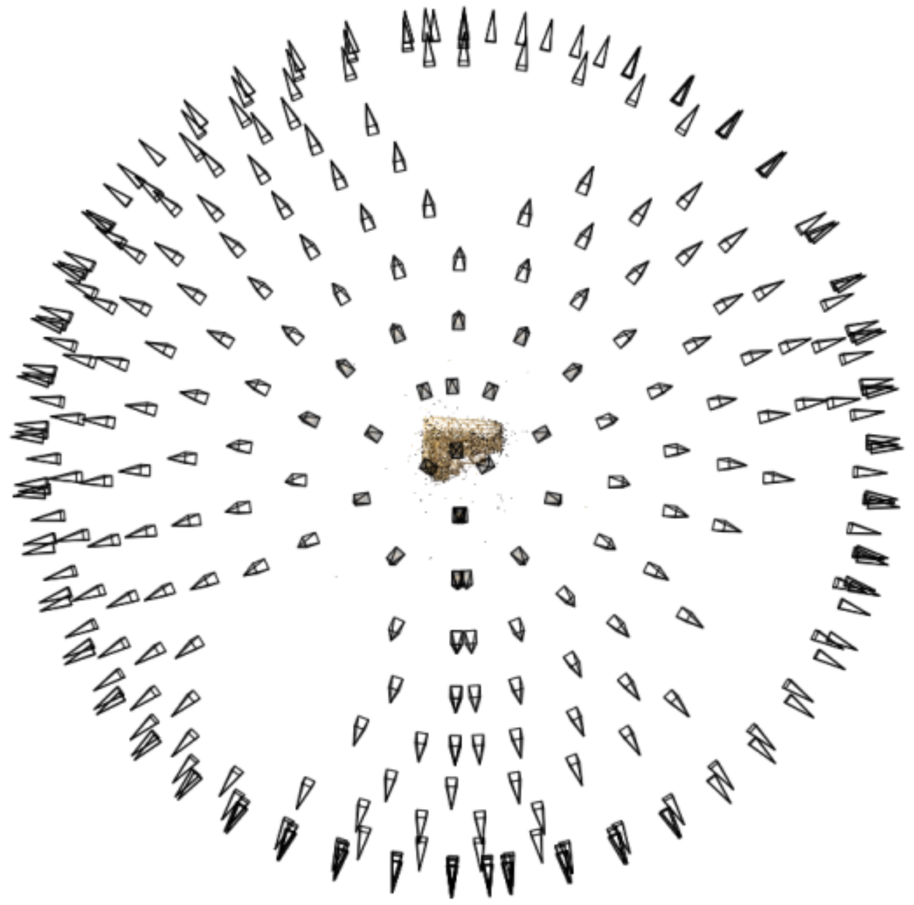


Source: N. Snavely

# Structure from motion



Reconstruction (side)



(top)

- Input: images with pixels in correspondence $p_{i,j} = (u_{i,j}, v_{i,j})$

- Output
  - **Structure:** 3D location $\mathbf{x}_i$ for each point $p_i$
  - **Motion:** camera parameters $\mathbf{R}_j$ , $\mathbf{t}_j$ possibly $\mathbf{K}_j$

- Objective function: minimize reprojection error



Source: N. Snavely

# Camera calibration & triangulation

- Suppose we know 3D points
  – And have matches between these points and an image
  – Computing camera parameters similar to homography estimation

- Suppose we have know camera parameters, each of which observes a point
  – We can solve for the 3D location

- Seems like a chicken-and-egg problem, but in SfM we can solve both at once



Source: N. Snavely

**Next class:** more 3D