Chapter 9

Comparison of Coding and Modulation Schemes

We would like to compare coded modulation schemes is some fair way. The goal is to achieve a high value for the data rate to bandwidth ratio; that is the rate in bits/dimension or bits/sec/Hz while requiring a small amount of energy and yielding a small error probability. Clearly there is a tradeoff present here. We can however make fairly rough comparisons. We will assume the underlying modulation is BPSK. So the model of the channel is an additive white Gaussian noise channel with input $\pm\sqrt{E}$. The code has minimum Hamming distance d and rate R (in information bits/channel bit). It is easy to show that the Euclidean distance between codewords is given by

$$d_E^2 = 4Ed = 4E_b dR$$

Thus choosing codes with large Hamming distance will make the code have large Euclidean distance as well. (This is not true for other modulation schemes and hence careful attention needs to be paid to how "codewords are assigned signal points).

We will assume the error probability is dominated by errors made to the nearest neighbor codewords or signal points. The error probability is then approximated by

$$\begin{aligned} P_e &\approx w_d Q\left(\sqrt{\frac{2E_b dR}{N_0}}\right) \\ &\leq \frac{w_d}{2} \exp\{-(\frac{E_b dR}{N_0})\} \\ &= \frac{1}{2} \exp\{-(\frac{E_b dR}{N_0} - \ln(w_d))\} \end{aligned}$$

In order to achieve an error probability of 10^{-5} we need an exponent of 10.82. The baseline we will use to compare different coding and modulation schemes is uncoded BPSK. In this case w_d=1, $d = 1$ and $R = 1$. This needs a signal-to-noise ratio of 10.82=10.34dB to achieve the desired error probability. (Actually if we work with the exact expression for error probability the signal-to-noise ratio needed is 9.6dB. To be consistent we will just work with the bound on error probability.

Consider the rate 1/2 convolutional code. The distance for that code is 10. The number of nearest neighbors is 36. The signal-to-noise ratio needed for 10^{-5} bit error probability is then approximately $10.82 = 5E_b/N_0 - \ln(36)$ or $E_b/N_0 = 4.6$dB The coding gain is then 10.34-4.6=5.74dB. (The actual E_b/N_0 is 9.6dB uncoded and 4.0dB coded so the actual gain is 5.6dB. Below we list various codes and the approximate coding gain.

Modulation	Code	Rate	d_{min}	w_d	Approx. Gain
BPSK	None	1	1	1	0.0dB
BPSK	K=3 Conv.	0.5	5	1	3.98dB
BPSK	K=4 Conv.	0.5	6	2	4.50dB
BPSK	K=5 Conv.	0.5	7	4	4.91dB
BPSK	K=6 Conv.	0.5	8	2	5.75dB
BPSK	K=7 Conv.	0.5	10	36	5.74dB
BPSK	K=8 Conv.	0.5	10	2	6.72dB
BPSK	K=9 Conv.	0.5	12	33	7.04dB
BPSK	K=3 Conv.	1/3	8	3	3.83dB
BPSK	K=4 Conv.	1/3	10	6	5.04dB
BPSK	K=5 Conv.	1/3	12	12	5.12dB
BPSK	K=6 Conv.	1/3	13	1	6.37dB
BPSK	K=7 Conv.	1/3	15	7	6.27dB
BPSK	K=8 Conv.	1/3	16	1	7.27dB
BPSK	Golay Code	12/23	7	253	3.83dB
BPSK	(7,4) BCH Code	4/7	3	7	1.62dB
BPSK	(15,11) BCH Code	11/15	3	15	2.45dB
BPSK	(15,7) BCH Code	7/15	3	15	2.45dB
BPSK	(31,15) BCH Code	15/31	8	465	3.92dB

Note that the decoding algorithm for the block codes is assumed to be maximum likelihood decoding (soft decisions). This is fairly easy to implement for the Hamming codes and the first of the two BCH codes. However, for the second BCH code it is not trivial to implement soft decision decoding. For hard decision decoding we can subtract off roughly 2dB of the coding gain due throwing away the reliability of each decision.

The above table then compares different coding schemes for use with BPSK modulation. Because there is a direct relationship between Euclidean distance squared and Hamming distance designing good codes for use with BPSK modulation is equivalent to designing good codes for large Euclidean distance. This is not the case when we consider QAM modulation instead of BPSK modulation. For QAM modulation with more than 4 signal points special techniques need to be employed to get codes with large Euclidean distance.