Today’s Class

- Cipher Modes
- Building a Secure Channel
- Implementations
  (BREAK)
- Diffie-Hellman Key Exchange
- RSA Encryption and Signing
- Establishing Trust
Cipher Modes

How do we encrypt more than one block?

Some definitions:

- $P_i$ – $i$-th plaintext block
- $C_i$ – $i$-th ciphertext block
- $E()$ – encryption function
- $D()$ – decryption function
- $K$ – encryption key

Cipher Modes: ECB

“Electronic codebook” (ECB) mode

$$C_i := E(K, P_i) \quad \text{for } i = 1, \ldots, k$$

- Most “natural” construction
- Never use ECB
What’s Wrong with ECB?

Same plaintext block always encrypts to same ciphertext block.
Don’t use ECB mode.

Cipher Modes: CBC

“Cipher-Block Chaining” (CBC) mode

\[ C_i := E(K, P_i \oplus C_{i-1}) \quad \text{for } i = 1, \ldots, k \]
Cipher Modes: CBC

“Cipher-Block Chaining” (CBC) mode

\[ C_i := E(K, P_i \oplus C_{i-1}) \quad \text{for } i = 1, \ldots, k \]

Is CBC appropriate for Encrypting an online video stream?

What if you reuse the IV? Bad.

Cipher Modes: CTR

“Counter” (CTR) mode

\[ K_i := E(K, \text{Nonce} || i) \quad \text{for } i = 1, \ldots, k \]
\[ C_i := P_i \oplus K_i \]

- Stream cipher construction – like OTP
- Plaintext never passes through \( E \)
- Don’t need to pad the message
- Must never reuse same \( K+\text{Nonce} \) (like OTP)
CBC vs. CTR?

- Advantages of CTR
  - Doesn’t require padding
  - Allows parallelization
  - Only need encryption function

- Advantages of CBC
  - Limits leak to first block if IV is reused
  - Can use random IV instead of unique nonce

Building a Secure Channel

// Initialization (Both Parties)

```
Separate keys for each function

KeySendEnc := HMAC-SHA256(K, "Enc A-to-B")
KeyRecvEnc := HMAC-SHA256(K, "Enc B-to-A")
KeySendAuth := HMAC-SHA256(K, "Auth A-to-B")
KeyRecvAuth := HMAC-SHA256(K, "Auth B-to-A")
```

if Bob then
  swap(KeySendEnc, KeyRecvEnc)
  swap(KeySendAuth, KeyRecvAuth)

MsgCntSend := 0
MsgCntRecv := 0

Session Key K
Never reuse

Session Key K
Never reuse
Building a Secure Channel

// Sender
MsgCntSend := MsgCntSend + 1
i := MsgCntSend
a := HMAC-SHA256(KeySendAuth, 
i || len(m) || m)
t := a || m
K := KeySendEnc
// to length of t :
MsgKey := E(K, i || 0) || E(K, i || 1) || ...
Transmit(t || (t ⊕ MsgKey))

// Receiver
i || t := Receive()
K := KeyRecvEnc
// to length of t :
MsgKey := E(K, i || 0) || E(K, i || 1) || ...
a || m := t ⊕ MsgKey
a' := HMAC-SHA256(KeyRecvAuth, 
i || len(m) || m)
Check(a' == a)
Check(i > MsgCntRecv)
MsgCntRecv := i

Encrypt First or Auth First?

HMAC(E(msg)) or E(HMAC(msg))?
Implementations: OpenSSL

- Try not to implement crypto functions. Use OpenSSL libraries if possible.
  - Open source implementation
  - SSL protocol plus general crypto functions
  - Very fast hand-tunes assembly language

OpenSSL on the Command Line

- Hashing (a.k.a. “message digest”)
  
  $ openssl dgst -sha256 myfile

- Encryption and decryption
  
  $ openssl enc -aes-256-cbc \\
  -in myfile -out myfile.enc
  $ openssl enc -d -aes-256-cbc \\
  -in myfile.enc -out myfile

- Performance tests
  
  $ openssl speed sha
  $ openssl speed aes
OpenSSL in C – Authentication

```c
#include <openssl/hmac.h>
#include <openssl/sha.h>
#include <openssl/evp.h>

unsigned char mac[SHA256_DIGEST_LENGTH];
mac = HMAC(
    EVP_sha256(), // use SHA-256 hash function
    (unsigned char*) key,
    (unsigned long ) keyNumBytes,
    (unsigned char*) data,
    (unsigned long ) dataNumBytes,
    NULL, NULL
);
```

OpenSSL in C – Encryption

```c
#include <openssl/evp.h>
// 256-bit AES in CBC mode with padding
void AesEncrypt(unsigned char key[32], unsigned char iv[16])
{
    unsigned char inData[16], outData[16];
    Int inLen, outLen;
    EVP_CIPHER_CTX ctx;
    EVP_CIPHER_CTX_init(&ctx);
    EVP_EncryptInit_ex(&ctx, EVP_aes_256_cbc(), NULL,
        (unsigned char *)key, (unsigned char *)iv);
    while ((inLen = fread(inData, 1, 16, stdin)) > 0) {
        EVP_EncryptUpdate(&ctx, outData, &outLen, inData, inLen);
        fwrite(outData, 1, outLen, stdout);
    }
    EVP_EncryptFinal_ex(&ctx, outData, &outLen);
    fwrite(outData, 1, outLen, stdout);
    EVP_CIPHER_CTX_cleanup(&ctx); // zeroize the key
}
Try OpenSSL at Home

- Install OpenSSL or use try it on a cluster
  - Sign and encrypt a message
  - Compare the speed of various functions
  - Think... How does the AES implementation compare to the speed of your Internet connection? You hard disk? You RAM?
- Use C, Python, or Perl and the OpenSSL library to implement our secure message passing protocol

Summary of Practical Advice

- Don’t use MD5; avoid hash function pitfalls
- Don’t use DES; avoid ECB mode
- Don’t use rand() and its ilk

- For a hash/MAC, use **HMAC-SHA256**
- For a block cipher, use **AES-256**
- For randomness, use the **OS’s CPRNG**
- For implementations, use **OpenSSL**
Related Research Problems

- **Cryptanalysis**: Ongoing work to break crypto functions... rapid progress on hash collisions
- **Cryptographic function design**: We desperately need better hash functions... NIST competition now to replace SHA
- **Attacks**: Only beginning to understand implications of MD5 breaks – likely enables many major attacks
Public-Key Cryptography

- **Problem**: With symmetric ciphers, every sender-receiver pair must share a secret key.
- **Question**: What if we could use *different keys* for encryption and decryption?

Diffie-Hellman Key Exchange

- Whitfield Diffie and Martin Hellman, 1976
- Let's Alice and Bob establish a shared secret even if Eve is listening in.
**Diffie-Hellman Key Exchange**

Agree on a large prime $p$ and primitive element $g$ that generates the group $\mathbb{Z}_p^*$. ($p, g$ can be public)

**Alice**
- Chooses random $x < p$ (secret)
- Calculates $a := g^x \mod p$

**Bob**
- Chooses random $y < p$ (secret)
- Calculates $b := g^y \mod p$

**Why this works:**
$$b^x = (g^y)^x = (g^x)^y = a^y \mod p$$

**Difficulty?**

- **Diffie-Hellman (DH) problem:**
  Compute $g^{xy}$ given $g^x$ and $g^y$ (mod $p$)

- Best *known* approach: Compute $x$ from $g^x$
  - Called the discrete logarithm (DL) problem
  - No *known* efficient algorithm

- Modular exponentiation believed to be a one-way function
  - Easy to compute
  - Hard to invert
# Attacking Diffie-Hellman

Mallory chooses random $x < p$

$g^x$

Mallory chooses random $y < p$

$g^y$

Mallory chooses random $v < p$

$g^v$

Mallory chooses random $w < p$

$g^w$

$k := (g^x)^y$

$k' := (g^y)^v$

$k'' := (g^w)^x$

# RSA

- Rivest, Shamir, Ln Adleman (1977)
- Used for encryption and signatures
- Based on a trapdoor function
  - Easy to compute
  - Hard to invert without special information
- Based on apparent difficulty of factoring large numbers
## RSA in One Slide

- **\( p, q \)**: large random primes
- **\( n := pq \)**: modulus
- **\( t := (p-1)(q-1) \)**: ensures \( x^t = 1 \pmod{n} \)
- **\( e := \text{[small odd value]} \)**: public exponent
- **\( d := \frac{1}{e} \pmod{t} \)**: private exponent

Public key: \((n, e)\)

Private key: any of \(p, q, t, d\)

Encryption: \(c := m^e \pmod{n}\)

Decryption: \(m := c^d \pmod{n}\)

**Why?** \((m^e)^d = m^{ed} = m^{kt+1} = (m^t)^k m = 1^k m = m \pmod{n}\)

## RSA for Encryption

- **Publish:** \((n, e)\)
  - Store secretly: \(d\)
- **Encryption of** \(m\)
  - Choose random \(k\) same size as \(n\)
  - \(c := k^e \pmod{n}\)
  - Send \(c\), encrypt \(m\) with AES using \(k\)
- **Decryption**
  - \(k := c^d \pmod{n}\); decrypt \(m\) with AES using \(k\)
RSA for Signatures

- Publish: \((n, e)\)
  Store secretly: \(d\)

- Signing \(m\)
  Seed a CPRNG with \(m\) and calculate pseudorandom string \(s\) same size as \(n\)
  \(\sigma := s^d \mod n\)

- Verifying a signature on \(m\)
  Recalculate \(s\) from \(m\)
  Check \(s = \sigma^e \mod n\)

D-H with Authentication

- Chooses random \(x < p\) (secret)
- Mallory
- \(a := g^x \mod p\)
- \(\text{Sign}_{\text{Alice}}(a)\)
- \(b := g^y \mod p\)
- \(\text{Sign}_{\text{Bob}}(a, b)\)
- Chooses random \(y < p\) (secret)
- Verifies signature
- Calculates \(k := b^y \mod p\)
- Verifies signature
- Calculates \(k := a^y \mod p\)
Establishing Trust

How do Alice and Bob learn each others’ signature verification keys?

- Web of Trust
  - Transitive trust among associates (e.g. PGP)
- Public Key Infrastructure (PKI)
  - Trusted third-party Certificate Authority (CA)
    binds keys-identities (e.g. SSL)

Tuesday: Crypto Attacks (I)

- Optional Background Reading
  - Introducing SSL and Certificates using SSLeay
- Required Reading – Responses Due Before Class
  - MD5 To Be Considered Harmful Someday
  - MD5 Considered Harmful Today
Paper Responses

- **Brief** written response to each required paper (must be < 350 words/paper):
  1. state the problem the paper is trying to solve
  2. summarize its main contributions
  3. evaluate its strengths and weaknesses
  4. suggest at least two interesting open problems on related topics
  5. tell me if anything was too difficult to understand

- Due by email before class
  - Graded “check”/“check−”
  - Put “[reading588]” in subject line

Talk This Afternoon

- **Alessandro Acquisti** (CMU)
  “The Best of Strangers: Behavioral economics, Malleable privacy valuations, and Context-dependent willingness to divulge personal information”

  4-5:30PM, 1202 SI North