Homework Set 3 **EECS 651** **Due: Wed., Jan. 31, 2007**

Homework submission policy: same as before

1. A binary IID source has p(0) = .995 and p(1)=.005.

   (a) Find a fixed-to-variable length block lossless source code with rate no larger than 0.4 . (It should be as simple as possible.)

   (b) Is it possible to find a fixed-to-variable length block lossless source code with rate less than 0.1? If so, what can you say about how large its input block length must be?

2. Suppose we are told that a certain stationary source $\{X_k\}$ with alphabet $A = \{1,2,...,M\}$ has

   $$R_3^* = .95 \qquad R_4^* = .92$$

   where R_k^* denotes the least rate of any block to variable length prefix code with input blocklength k. What can you deduce about the values of H_∞ and H_k for k = 1,2,3, ... ? In other words, using the given information, find the tightest possible bounds to H_∞ and to each H_k .

3. Consider the "runlength" code shown below. This is a variable-length to fixed-length code, unlike the fixed-length to variable-length we have considered in class. Let the source be the IID source with p(0) = .995 and p(1)=.005

source sequence	run length	binary codeword
1	0	000
01	1	001
001	2	010
0001	3	011
00001	4	100
000001	5	101
0000001	6	110
0000000	7	111

   (a) Explain why this code is uniquely encodable and decodable.

   (b) Find the average length of the encoded source sequences.

   (c) Find the rate of this code.

   (d) Compare the rate and complexity of this code to that found in Problem 1a.

4. Let $\{Z_k\}$ be a binary IID source with alphabet $A_Z = \{0,1\}$ and $p_Z(1) = q$. Suppose $\{X_k\}$ is a stationary binary source with alphabet $A_X = \{0,1\}$, with $p_{X_k}(1) = 1/2$, $p_{X_k,X_{k-1}}(0,0) = p_{X_k,X_{k-1}}(0,1) = p_{X_k,X_{k-1}}(1,0) = p_{X_k,X_{k-1}}(1,1)$, and with

   $$X_k = X_{k-1} \oplus X_{k-2} \oplus Z_k ,$$

   where $\oplus$ denotes modulo 2 addition, and where Z_k is independent of $X_{k-1}, X_{k-2}, X_{k-3}, ...$.

   (a) Show that the $\{X_k\}$ is Nth-order Markov for some appropriately chosen N.

   (b) Find an expression for H_k and H_∞ .

   For the remaining parts assume q = 0.1 .

   (c) Make a sketch showing how H_k approaches H_∞ as k increases.

   (d) Find minimum rate FL-VL block prefix codes for block lengths 1, 2 and 3. Compute their rates and plot them vs. source length on the sketch of part (c).

   (e) Can you design a conditional code that is better than those in (d), taking into account rate and complexity?

5. Let X be a random variable with pdf $f_X(x)$. Consider a scalar quantizer for X with thresholds $\{t_0, t_1, ..., t_M\}$, levels $\{y_1, ..., y_M\}$, and MSE D_X. Let $Y = aX+b$, where a and b are constants. Consider quantizing Y with the quantizer with thresholds $\{at_0+b, at_1+b, ..., at_M+b\}$, levels $\{ay_1+b, ..., ay_M+b\}$ and MSE D_Y.

(a) Show that

$$D_Y = a^2 D_X$$

(b) Show that if the quantizer for X has smallest MSE of any quantizer for X with M levels, then the quantizer for Y has smallest MSE of any quantizer for Y with M levels, and vice versa.

(c) Show that the smallest possible MSE of any quantizer for Y with M levels is a^2 times the smallest possible MSE of any quantizer for X with M levels. (One may conclude that for two pdf's with the same shape, the minimal MSE is proportional to variance. This is a very important conclusion.)