Homework Set 4 **EECS 651** **Due: Friday, Feb. 16, 2007**

The homework is due Friday at 1:30 in class. No late homework accepted. Do not interrupt the lecture to turn in homework.

1. This problem should be done analytically without using a computer.

   (a) Find a scalar quantizer with three levels that satisfies the optimality critera for the Laplacian pdf

   $$p(x) = \frac{1}{2} e^{-|x|}.$$

   If you can find more than one, choose the one with smallest MSE.

   (b) Find the resulting MSE. and SNR (in dB).

2. (a) CD recordings are made with a uniform scalar quantizer with rate 16 bits/sample. Assuming that music samples are Gaussian with variance σ^2 and assuming that the level spacing Δ is chosen optimally for this source model, estimate the SNR of CD recordings.

   (b) Suppose instead that optimal nonuniform scalar quantization were used with the same rate (16 bits/sample). Estimate how much larger the SNR would be than in Part (a).

   (c) Suppose instead that optimal scalar quantization with variable-length coding is used with the saem rate. Estimate how much larger the SNR would be than in Part (a).

   (d) Suppose that the system of the type in part (c) were used with its rate chosen to give the same SNR as in part (a). Estimate how much more musiccould be stored on a CD. Express your answer in percent.

3. Compute β and η for a Laplacian density.

4. Suppose that to a zero-mean Gaussian random variable X with variance σ^2 and pdf $f(x)$, we apply a scalar quantizer with M cells, for some large M, reconstruction levels in the middle of the cells, and point density $\lambda(x) = c\, f^r(x)$, where $0<r<1/2$ is some constant greater than one, and c makes it integrate to one.

   (a) Find an approximate, though accurate, expression for the mean squared error of this quantizer. Your answer should be expressed in terms of σ^2, r, and the number of quantization levels M.

   (b) What happens to the expression when $r = 1/2$? Can you explain what is happening?

5. Assume high rate in this problem.

   Show that when variable-length coding is applied to a scalar quantizer that is optimized for fixed-length coding the SNR gain (in dB) over optimal SQ-FL at the same rate will be two-thirds of the SNR gain in dB of optimal SQ-VL over optimal SQ-FL at the same rate.

6. Let X_1 and X_2 be independent Gaussian random variables with means 0 and 1 and variances 1 and 8, respectively. Find a pair of scalar quantizers, one for X_1 and one for X_2, such that each has a number of levels that is a power of 2, the sum of their rates (with fixed-length encoding) is at most 4 bits/sample and MSE is as small as possible. Give the levels and thresholds, and distortion of each quantizer, as well as the overall distortion. You will need to use the table posted on the website that describes optimal quantizers for Gaussian random variables.

7. A two-dimensional random vector $\underline{X}$ is uniformly distributed on an equilateral triangle with sides of length 1.

   (a) Find a two-dimensional VQ with size M=2 that is locally optimal wrt MSE, i.e. it satisfies both optimality criteria. It is enough to draw a picture of the VQ.

   (b) Repeat for M=3.

   (c) Repeat for M=4.

8. A vector quantizer must be designed to quantize at rate 2 bits/sample a source that emits 1000 samples/sec. The device available for encoding can perform 10^6 arithmetic operations per second (floating point or otherwise) and has storage for 100,000 floating point numbers. Find the largest dimension that the VQ can use. (Don't worry about decoding.)

9. Consider a VQ with three two-dimensional codevectors that are not colinear. Show that the perpendicular bisectors between each pair of codevectors meet at a single point.

10. Show that Voronoi cells are always convex. That is, if $S = \{S_1,...,S_M\}$ is a Voronoi partition for a codebook $C = \{\underline{w}_1,...,\underline{w}_M\}$, then each cell S_i is convex, meaning that for any $\underline{x},\underline{y} \in S_i$ and $0 < \theta < 1$, $\theta\, \underline{x} + (1-\theta)\, \underline{y} \in S_i$.