Homework Set 5 **EECS 651** **Due: Wednesday, March 14, 2007**

There is no class on Wednesdqay, so bring your homework to my office. There is a grace period until Thursday 9 AM.

1. (a) A two-stage vector quantizer must be designed to quantize at rate 2 bits/sample a source that emits 1000 samples/sec. The device available for encoding can perform 10^6 arithmetic operations per second (floating point or otherwise) and has storage for 100,000 floating point numbers. Find the largest dimension that the two-stage VQ can use. (Choose the rates of each stage to obtain the largest possible dimension, subjet to the overall rate being 2 bits/sample. Don't worry about decoding.)

   (b) Compare the answer to that found in HW 4 Problem 8

2. Suppose a VQ with partition $S = \{S_1,...,S_M\}$ and codebook $C = \{\underline{w}_1,...,\underline{w}_M\}$ is used on a source vector $\underline{X}$ with pdf f. Show that the contribution to distortion due to the ith cell, satisfies the following relationship:

$$\int_{S_i} ||\underline{x}-\underline{w}_i||^2 f(\underline{x})\, d\underline{x} = \int_{S_i} ||\underline{x}-\underline{v}_i||^2 f(\underline{x})\, d\underline{x} + ||\underline{w}_i-\underline{v}_i||^2 \Pr(\underline{X}\in S_i)$$

   where $\underline{v}_i$ is the centroid of S_i with respect to f. (Notice that this equals the contribution to distortion if $\underline{w}_i$ were replaced by the centroid, which is the optimal codevector, plus the cell probability times the distance squared between $\underline{w}_i$ and the centroid.)

3. A VQ is needed for a (first-order) autoregressive, stationary Gaussian source with correlation coefficient $\rho = .95$. It must have rate 4 or less and signal-to-noise ratio 32.5 dB or more. Determine whether or not there exists a suitable VQ. If yes, estimate the smallest possible dimension.

4. CD recordings are made with a uniform scalar quantizer with fixed-length coding and rate 16 bits/sample. Assuming that music samples are Gaussian with variance σ^2 and assuming that the level spacing Δ is chosen optimally for this source model, in HW 4, Problem 2 we estimated the SNR to be 84.6 dB, and we found that SNR would be 94.8 dB with optimal scalar quantization with first-order variable-length coding.

   Now suppose further that the source is modelled as stationary, Gaussian and Markov with correlation coefficient 0.9. Estimate the SNR that would be attained by quantizers of the following types.

   (a) Optimal two-dimensional VQ with fixed-length coding.

   (b) Uniform scalar quantization with second-order variable-length coding.

   (c) Two-dimensional VQ optimized for first-order variable-length coding.

   (d) Eight dimensional fixed-length VQ.

   (e) The best possible lossy source coder of any type.

5. Derive the formula for the Zador factor β_k when $\underline{X}$ is a k-dimensional random vector with IID Laplacian components. Check your formula by comparing to the formula given in the notes.

6. Find an expression for the "oblongitis" suffered by scalar quantization for a Laplacian random variable. Check your answer by evaluating your expression and comparing it to the answer given in the notes.

7. Let X be a stationary Gaussian random process with power spectral density

$$S(\omega) = \begin{cases} A, & -W<\omega<W \\ 0, & \text{else} \end{cases}$$

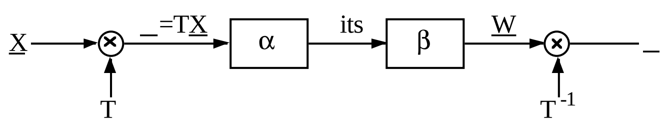
where $W \leq \pi$.

(a) Find $\mathcal{D}(R)$ the distortion-rate function for this source (which gives the smallest possible MSE of any lossy source code with rate R or less).

(b) Find Zador's formula for $Z(R)$ for this source.

(c) The answers to Parts (a) and (b) should be a little surprising. Explain what accounts for the behavior you find in (a) and (b).

8. In the transformed VQ system shown below



A k-dimensional VQ with fixed-length encoding has rate R, encoder α, decoder β, point density $\lambda(\underline{x})$ and inertial profile $m(\underline{x})$. The transform T is a $k \times k$ orthogonal matrix.

(a) Find the point density of the transformed VQ.

(b) Find the inertial profile of the transformed VQ.

(c) Find an approximate expression for the distortion of this system.

Extra Problems (won't be graded)

9. Suppose a two-dimensional quantizer with partition $S = \{S_1,...,S_M\}$, codebook $C = \{\underline{w}_1,...,\underline{w}_M\}$ and quantization rule Q is to be designed to minimize mean absolute error $D\, d(\underline{X},Q(\underline{X}))$ for a random vector $\underline{X} = (X_1,X_2)$ with pdf $f(\underline{x})$, where

$$d(\underline{x},\underline{y}) = \frac{1}{2}|x_1-y_1| + \frac{1}{2}|x_2-y_2|$$

(a) Find a condition that C must satisfy in order that it be optimal for S. Simplify as much as possible.

(b) Plot a best partition for the codebook $C_1 = \{ (0,0), (1,1) \}$

(c) Is this the only possiblebest partition for C_1? (Consider only partitions that differ substantially from that found in (b), i.e. not one that differs only on a set of measure zero.)

(d) Plot the best partition for the codebook $C_2 = \{ (0,0), (0, \sqrt{2}) \}$

(e) Is this the only possible best partition for C_2? (Consider only partitions that differ substantially from that found in (b), i.e. not one that differs only on a set of measure zero.)

(f) Note that C_2 is C_1 rotated by 45^o. Is there any optimal partition for C_2 that is a rotation of an optimal partition for C_1?

10. How do the VQ optimality conditions change if MSE $\frac{1}{k}\sum_{i=1}^{k}(x_i-y_i)^2$ is replaced by weighted MSE, defined as

$$\frac{1}{k}\sum_{i=1}^{k} w_i\,(x_i-y_i)^2\,, \text{ where } w_i > 0, \quad i = 1,...,k.$$

Weighted MSE might be used if believed that distortion in some components was more significant than in other components.