Generalized Symmetries in Boolean Functions

Victor N. Kravets and Karem A. Sakallah

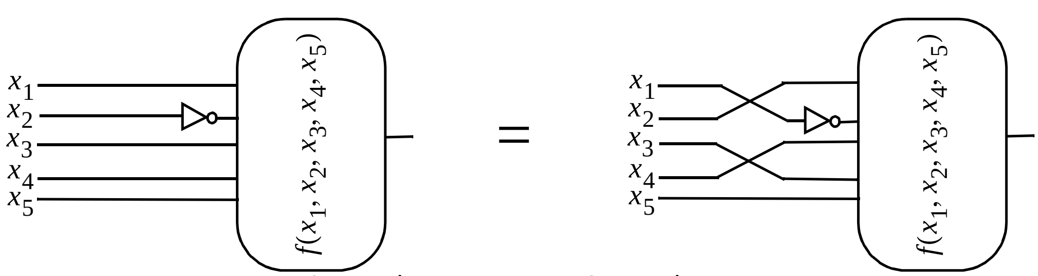
Advanced Computer Architecture Laboratory
Department of Electrical Engineering and Computer Science
The University of Michigan
Ann Arbor, MI 48109
{vkravets,karem}@eecs.umich.edu

Abstract

In this report we take a fresh look at the notion of symmetries in Boolean functions. In particular, we show that the classical characterization of symmetries based on invariance under variable swaps is a special case of a more general invariance based on unrestricted variable permutations. We propose a generalization of classical symmetry that allows for the simultaneous swap of groups of variables and show that it captures more of a function's invariant permutations without undue computational requirements. We apply the new symmetry definition to analyze a large set of benchmark circuits and provide extensive data showing the existence of substantial symmetries in those circuits. Specific case studies of several of these benchmarks reveal additional insights about their functional structure and how it might be related to their circuit structure.

I. Introduction

Symmetries usually refer to permutations of an object's parameters that leave it unchanged. They provide insights into the structure of the object that can be used to facilitate computations on it. They can also serve as a guide for preserving that structure when the object is transformed in some way. The object we study in this work is an *n*-variable Boolean function and the symmetries we explore are variable permutations, with possible complementation, that leave the function unchanged (see Fig. 1). The context for this work is logic synthesis which we view as a process that transforms an initial representation of the function (e.g. as a list of cubes or a BDD [2]) into a final implementation as a multi-level network of primitive cells from a given technology library. We contend, based on ample



$$f(x_1, x_2', x_3, x_4, x_5) = f(x_2, x_1', x_4, x_3, x_5)$$

Fig. 1. Illustration of function symmetry.

empirical evidence, that when guided by knowledge of functional symmetries, such a process yields more “natural” implementations of the function. The reader is referred to [9] for a description of our synthesis approach; in this report we focus exclusively on the study of functional symmetries with only occasional reference to their utility in logic synthesis.

The study of symmetries in Boolean functions dates back to Shannon [14] who recognized that symmetric functions have particularly efficient switch network implementations. Since then several attempts were made to devise synthesis procedures for symmetric functions [4, 8]. These efforts, however, failed to yield practical synthesis tools and are generally viewed as inapplicable to the types or sizes of functions typically encountered in today’s design automation environments. In recent years, the increasing use of BDDs for the manipulation of Boolean functions sparked renewed interest in the study of function symmetries. In [7], for example, the authors showed that the size of a BDD can be reduced by using a variable order that places symmetric variables contiguously. This observation led to the development of sifting procedures for dynamic BDD variable ordering based on function symmetries [12, 13]. Symmetries were also utilized to improve the efficiency of functional equivalence checking, especially for functions with unknown input correspondence [3].

Much of the existing literature on symmetry is based on function invariance under swaps of variable pairs in the function’s support; we’ll refer to this type of symmetry as *classical symmetry* to distinguish it from the more general symmetry we describe in this report. For completely-specified functions, classical symmetry can be represented as a partition on the set of variables: variables that belong to a given block of that partition are equivalent, i.e. symmetric, whereas variables that belong to different blocks are non-equivalent, i.e. non-symmetric. The blocks of such a partition are commonly referred to as the function’s *symmetry groups*, and variables within a symmetry group are equivalent in the sense that they can be permuted arbitrarily without changing the value of the function. The advent of BDDs led to the development of efficient symbolic methods for the identification of a function’s symmetry groups. The computational core in such algorithms is the check that determines the equivalence of a pair of variables; larger symmetry groups are then built incrementally using transitivity. Many of the recently-proposed techniques for symmetry identification achieve their efficiency through careful analysis of the structure of the BDD that represents the function [10, 12, 13]. In [15], the authors approach this problem by using the generalized Reed-Muller transform to speed-up computation of symmetries. A notable exception to the commonly-used definition of symmetry was proposed in [11]. Rather than invariance under swaps of variables, symmetry is defined in terms of equivalence among arbitrary subspaces (i.e. cofactors) of the function.

In this work we define symmetry as an invariance under *arbitrary* variable permutation rather than invariance under swaps of variable pairs. Under this broader definition, partitions on the set of variables fail to capture all the invariant input permutations. We explore the relation between symmetry groups and variable permutations in

Section II and highlight the inherent limitation of variable partitions as a means of representing arbitrary variable permutations. As an alternative to the explicit, and computationaly infeasible, listing of all invariant permutations, we propose an efficient hierarchical extension to the notion of symmetry groups—*a hierarchical partition*—that allows us to represent a larger (but not necessarily the complete) set of invariant variable permutations. These hierarchical partitions represent *higher-order* symmetries that arise from simultaneously swapping groups of, rather than single, variables. In Section III we formally state the conditions under which the classical *first-order* symmetries exist and provide computational procedures for the construction of the corresponding flat partition. In Section IV we generalize these conditions to define the higher-order symmetry and show how the corresponding hierarchical partition can be computed efficiently. In Section V we expand the notion of invariance to include the assignment of inversion phases to the function inputs. In Section VI we report on the results of applying hierarchical partitioning to a large set of benchmarks; we also provide detailed analyses of a few benchmarks to show that additional symmetries, missed by hierarchical partitioning or hidden through netlist flattening, can still be found. We conclude in Section VII by recapping the main contributions and suggesting several possible extensions.

II. Motivation

Consider the six-variable function $f(a, b, c, d, x, y) = abxy + cdxy$. It is relatively straightforward to show that its classical symmetry groups are $\{a, b\}$, $\{c, d\}$, and $\{x, y\}$ (the exact procedure for computing these groups is described in Section III.) To appreciate the need for a broader notion of symmetry it is useful to view these symmetry groups as an *implicit* representation of the variable permutations that leave the function unchanged. Specifically, a group G_i consisting of n_i variables corresponds to $n_i!$ permutations; the total number of permutations represented by all groups is the product of the number of permutations for each of the individual groups. Thus, the three-group partition on the variables of this function corresponds to the eight ($2! \times 2! \times 2!$) permutations:

$$\{\langle abcdxy\rangle, \langle abcdyx\rangle, \langle abdcxy\rangle, \langle abdcyx\rangle, \langle bacdxy\rangle, \langle bacdxy\rangle, \langle badcxy\rangle, \langle badcyx\rangle\} \quad (1)$$

Direct substitution of each of these permutations in the expression for the function confirms that they do indeed leave it unchanged. We will refer to such permutations as the function's *invariant permutations*. In addition, we will encode the "flat" partition that induced them by an ordered list of un-ordered groups:

$$\langle\{a, b\}, \{c, d\}, \{x, y\}\rangle \quad (2)$$

This encoding emphasizes the fact that the permutations in (1) are generated from variable swaps that are strictly *within*, and not across, groups.

Further examination of this function, however, reveals that it remains invariant under the following additional set of eight permutations

$$\{\langle cdabxy\rangle, \langle cdabyx\rangle, \langle dcabxy\rangle, \langle dcabyx\rangle, \langle cdbaxy\rangle, \langle cdbaxy\rangle, \langle dcbaxy\rangle, \langle dcbayx\rangle\} \quad (3)$$

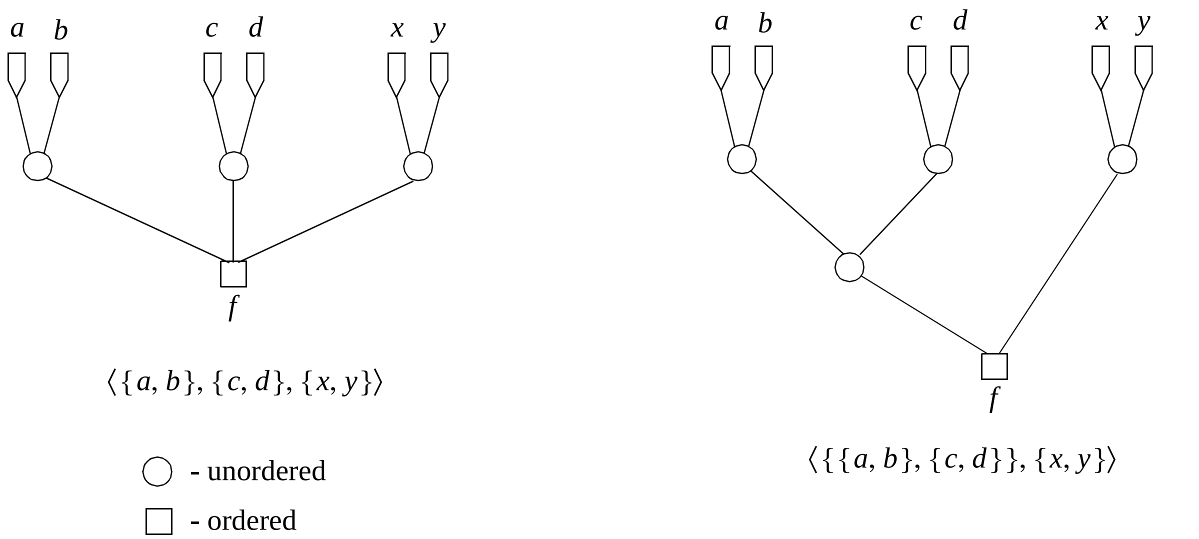


Fig. 2. Symmetry induced hierarchical partition of variables for function $f(a, b, c, d, x, y) = abxy + cdxy$.

which are not captured by the flat partition in (2). Note that each of these permutations can be derived from a corresponding permutation in (1) by swapping the groups $\{a, b\}$ and $\{c, d\}$. Thus, a suitable encoding that acts as an implicit representation for all sixteen permutations in (1) and (3) is the following *hierarchical* partition on the set of variables:

$$\langle \{ \{a, b\}, \{c, d\} \}, \{x, y\} \rangle \tag{4}$$

In both (2) and (4) we use angle brackets to indicate a fixed order (single permutation) and curly brackets to indicate all possible orders of the enclosed elements. A pictorial representation of the flat and hierarchical partitions in (2) and (4) is shown in Fig. 2.

This small example serves to illustrate several important points that motivate our desire for a fresh exploration of functional symmetries:

- Function invariance under unrestricted variable permutations expands the classical notion of symmetry by identifying more *structure* in functions than can be inferred from simple variable swaps.

- Functional structure may be specified by an explicit listing of all invariant permutations. However, such a listing may be infeasible due to the exponentially large number of such permutations. Compact implicit representations of this structure include flat and hierarchical partitions on the variable set that act as “stylized permutation generators.” The quality of an implicit representation of invariant permutations can be measured in terms of the number of permutations it generates: representation R_i is deemed superior to representation R_j if it corresponds to a larger number of invariant permutations; in some sense, R_i identifies more of a function’s structure than R_j . An ideal representation would identify the complete set of invariant permutations.

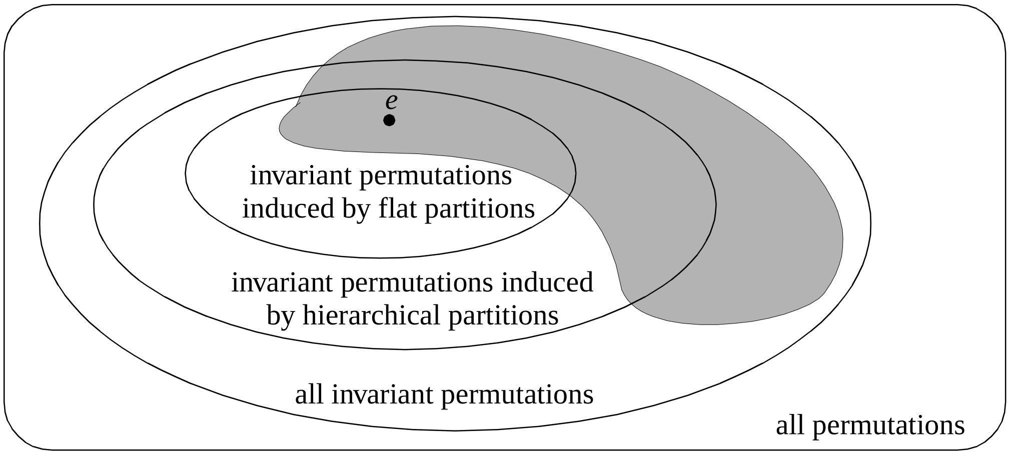


Fig. 3. Limitation of symmetry groups to represent all invariance permutations of a function.

- In addition to compactness, an implicit representation should be efficiently computable. Compact representations whose construction procedures require exponential run times are as infeasible as explicit listings of invariant permutations.

Fig. 3 depicts a Venn diagram that establishes the relation between the classical and new definitions of function symmetry. The universe is taken to be the entire set of variable permutations and *e* is used to denote the identity permutation, i.e. the normal variable order. The permutations induced by flat and hierarchical partitions of the variables are thus seen to be subsets of all invariant permutations. In addition, the permutations induced by hierarchical partitioning are clearly seen as a superset of those induced by flat partitioning. The shaded subset is meant to represent an alternative implicit representation of invariant permutations that is distinct from those based on partitions on the variable set.

In the remainder of this report we develop the concept of hierarchical partitions on the set of variables as a means of characterizing functional structure which extends the classical notion of symmetry. It is important, however, to keep in mind that, while provably superior to flat partitioning, hierarchical partitioning may still be too limited in its ability to capture a sizeable subset of invariant permutations. It does, however, serve as a catalyst for exploring other implicit representations of a function's invariant permutations, a point that we will allude to later when we analyze some of the benchmark circuits.

III. Classical First-Order Symmetries

First-order symmetries correspond to function invariance under swaps of variable pairs. Specifically, if variables *a* and *b* in the support of function *f* satisfy the condition:

```
P ← { };
while X ≠ ∅ do {
    select a ∈ X; X ← X\a ;
    G_a ← {a}
    for ∀b ∈ X do {
        if a and b are symmetric then
            G_a ← G_a ∪ {b};
    }
    X ← X\G_a ; P ← P ∪ {G_a};
```

Fig. 4. Construction of first-order symmetry partition P for a function f with support X

$$f(\ldots, a, \ldots, b, \ldots) = f(\ldots, b, \ldots, a, \ldots) \tag{5}$$

then we say that *f* has a first-order symmetry between variables *a* and *b*. These two variables are also said to form a symmetry group $\{a, b\}$. It is well known, as can be readily shown using Boole's expansion theorem [5], that condition (5) is equivalent to the following equality constraint on the function's cofactors [1, 4]:

$$f_{a'b} = f_{ab'} \tag{6}$$

Equation (6) serves as the computational check for first-order symmetry between variables *a* and *b* in function *f*. It also defines an equivalence relation on the set of variables that can be used to partition the set into its equivalence classes, i.e. symmetry groups, in quadratic time. A sketch of such a procedure, which is composed of two nested loops that iterate on the variables, is shown in Fig. 4. The procedure uses *X* to denote the set of variables, and *P* to represent the desired partition on *X*, i.e. the set of symmetry groups formed from *X*. In the outer while loop a variable *a* from *X* is chosen and used to *seed* a new symmetry group G_a. In the inner for loop, the symmetry of this variable to each other variable *b* in *X* is checked; if *b* is found to be symmetric to *a*, it is added to G_a. Before proceeding with the next pass of the outer loop, the variables collected in G_a are deleted from *X* ($X \backslash G_a$) and G_a is added to P. The procedure terminates when *X* becomes empty.

The extension of these classical results to higher-order symmetries is facilitated by adopting a matrix formulation of the symmetry check in (6). Let $m_i\langle x_1 x_2 \ldots x_n \rangle$ denote the *ith* minterm function on the specified ordered set of variables; for example, $m_2\langle ab \rangle = ab'$ and $m_9\langle cbda \rangle = cb'd'a = ab'cd'$. Cofactors of a function *f* with respect to variables *a* and *b* can now be expressed as the 2×2 matrix:

$$F_{\langle a \rangle, \langle b \rangle} = \begin{bmatrix} f_{m_0\langle a \rangle, m_0\langle b \rangle} & f_{m_0\langle a \rangle, m_1\langle b \rangle} \\ f_{m_1\langle a \rangle, m_0\langle b \rangle} & f_{m_1\langle a \rangle, m_1\langle b \rangle} \end{bmatrix} = \begin{bmatrix} f_{a'b'} & f_{a'b} \\ f_{ab'} & f_{ab} \end{bmatrix} \tag{7}$$

When clear from context, we will also adopt the following shorthand notation for this matrix:

$$F_{\langle a\rangle,\,\langle b\rangle} = \begin{bmatrix} f_{0,\,0} & f_{0,\,1} \\ f_{1,\,0} & f_{1,\,1} \end{bmatrix} \tag{8}$$

where $f_{i,\,j}$ is implicitly understood to stand for $f_{m_i\langle a\rangle,\,m_j\langle b\rangle}$.

Comparison of (7) or (8) with (6) immediately suggests that (6) is equivalent to requiring the cofactor matrix $F_{\langle a\rangle,\,\langle b\rangle}$ to be symmetric, i.e., that:

$$F^T_{\langle a\rangle,\,\langle b\rangle} = F_{\langle a\rangle,\,\langle b\rangle} \tag{9}$$

where the *T* superscript denotes matrix transpose. This result should not be too surprising since condition (6) expresses function invariance under the swap of two variables, which in the matrix formulation corresponds to interchanging rows and columns.

IV. Higher-Order Symmetries

Swaps of variable pairs can be extended in a straightforward manner to swaps of groups of ordered variables. For example, if variables *a*, *b*, *c,* and *d* in the support of function *f* satisfy the condition:

$$f(\ldots, a, \ldots, b, \ldots, c, \ldots, d, \ldots) = f(\ldots, c, \ldots, d, \ldots, a, \ldots, b, \ldots) \tag{10}$$

then we say that *f* has a second-order symmetry between ordered variable groups $\langle a, b\rangle$ and $\langle c, d\rangle$ This type of symmetry can be conveniently expressed by the symmetry group $\{\langle a, b\rangle, \langle c, d\rangle\}$. The invariance in (10) corresponds to the *simultaneous* swap of *a* with *c* and *b* with *d* and is readily shown to be equivalent to the following constraint on the function's cofactors:

$$F^T_{\langle ab\rangle,\,\langle cd\rangle} = F_{\langle ab\rangle,\,\langle cd\rangle} \tag{11}$$

where

$$F_{\langle ab\rangle,\,\langle cd\rangle} = \begin{bmatrix} f_{m_0\langle ab\rangle,\,m_0\langle cd\rangle} & f_{m_0\langle ab\rangle,\,m_1\langle cd\rangle} & f_{m_0\langle ab\rangle,\,m_2\langle cd\rangle} & f_{m_0\langle ab\rangle,\,m_3\langle cd\rangle} \\ f_{m_1\langle ab\rangle,\,m_0\langle cd\rangle} & f_{m_1\langle a\rangle b,\,m_1\langle cd\rangle} & f_{m_1\langle ab\rangle,\,m_2\langle cd\rangle} & f_{m_1\langle ab\rangle,\,m_3\langle cd\rangle} \\ f_{m_2\langle ab\rangle,\,m_0\langle cd\rangle} & f_{m_2\langle ab\rangle,\,m_1\langle cd\rangle} & f_{m_2\langle ab\rangle,\,m_2\langle cd\rangle} & f_{m_2\langle ab\rangle,\,m_3\langle cd\rangle} \\ f_{m_3\langle ab\rangle,\,m_0\langle cd\rangle} & f_{m_3\langle ab\rangle,\,m_1\langle cd\rangle} & f_{m_3\langle ab\rangle,\,m_2\langle cd\rangle} & f_{m_3\langle ab\rangle,\,m_3\langle cd\rangle} \end{bmatrix} \tag{12}$$

As an example, consider the six-variable function $f = a'c'x' + acx + abc'y + a'cdy + ab'c'y' + a'cd'y'$ whose cofactor matrix on $\langle a, b\rangle$ and $\langle c, d\rangle$ is:

$$F_{\langle ab\rangle,\langle cd\rangle} = \begin{bmatrix} x' & x' & y' & y \\ x' & x' & y' & y \\ y' & y' & x & x \\ y & y & x & x \end{bmatrix}$$

Since this matrix is symmetric, we can conclude that the function is invariant under the simultaneous swap of *a* with *c* and *b* with *d*. This is easily verified by direct substitution in the function expression. It is also instructive to examine the cofactor matrix on $\langle a\rangle$ and $\langle b\rangle$:

$$F_{\langle a\rangle,\langle b\rangle} = \begin{bmatrix} c'x' + cdy + cd'y' & cx + c'y' \\ c'x' + cdy + cd'y' & cx + c'y \end{bmatrix}$$

This matrix is clearly asymmetric; thus, while the function has a second-order symmetry between $\langle a, b\rangle$ and $\langle c, d\rangle$, it does not have a first-order symmetry between *a* and *b*. A similar check shows the function to lack first-order symmetry between *c* and *d*.

As another example, consider the function

$$g = x(ab(c+d) + cd(a+b)) + x'(ab)'(cd)' + y(abc'd' + a'b'cd)$$

which has the following cofactor matrix on $\langle a, b\rangle$ and $\langle c, d\rangle$:

$$G_{\langle ab\rangle,\langle cd\rangle} = \begin{bmatrix} x' & x' & x' & y \\ x' & x' & x' & x \\ x' & x' & x' & x \\ y & x & x & x \end{bmatrix}$$

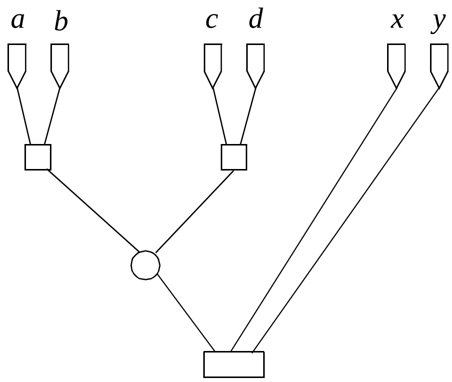
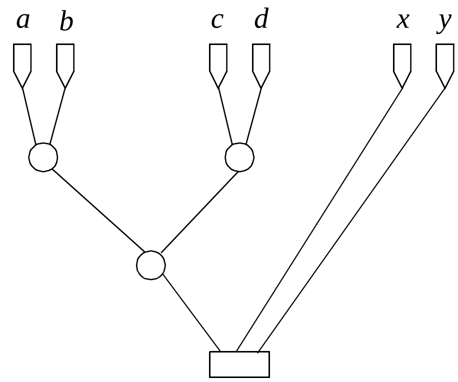
Thus, *g* has a second-order symmetry between $\langle a, b\rangle$ and $\langle c, d\rangle$. Unlike the previous function, however, *g* also has the first-order symmetries $\{a, b\}$ and $\{c, d\}$ since the cofactor matrices

$$G_{\langle a\rangle,\langle b\rangle} = \begin{bmatrix} x'(cd)' + y(cd) & x'(cd)' + x(cd) \\ x'(cd)' + x(cd) & x(c+d) + y(c+d)' \end{bmatrix} \text{ and } G_{\langle c\rangle,\langle d\rangle} = \begin{bmatrix} x'(ab)' + y(ab) & x'(ab)' + x(ab) \\ x'(ab)' + x(ab) & x(a+b) + y(a+b)' \end{bmatrix}$$

are both symmetric. This suggests that besides $\{\langle a, b\rangle, \langle c, d\rangle\}$, three additional second-order symmetry groups exist, namely $\{\langle a, b\rangle, \langle d, c\rangle\}$, $\{\langle b, a\rangle, \langle c, d\rangle\}$, and $\{\langle b, a\rangle, \langle d, c\rangle\}$. All four groups can, thus, be succinctly represented by the single second-order symmetry group on un-ordered variables $\{\{a, b\}, \{c, d\}\}$ which corresponds to eight invariant variable permutations.

The symmetry structures for these two functions are summarized in Fig. 5. The structures represent hierarchical partitions on the set of variables and can be generalized to higher orders using the following construction:

1. (*Basis*) Any variable x_i in the function's support is symmetric to itself and forms the symmetry structure $S_i \equiv \{x_i\}$.

	f	g
Function	$f\ =\ a'c'x' + acx + abc'y + a'cdy + ab'c'y' + a'cd'y'$	$g\ =\ x(ab(c+d) + cd(a+b)) + x'(a'+b')(c'+d') + y(abc'd' + a'b'cd)$
Hierarchical Partition — Graphical		
Hierarchical Partition — Symbolic	$\langle\{\langle a, b\rangle, \langle c, d\rangle\}, \{x\}, \{y\}\rangle$	$\langle\{\{a, b\}, \{c, d\}\}, \{x\}, \{y\}\rangle$
Invariant Permutations	$\{\langle abcdxy\rangle, \langle cdabxy\rangle\}$	$\left\{\begin{matrix}\langle abcdxy\rangle, \langle bacdxy\rangle, \langle abdcxy\rangle, \langle badcxy\rangle, \\ \langle cdabxy\rangle, \langle cdbaxy\rangle, \langle dcabxy\rangle, \langle dcbaxy\rangle\end{matrix}\right\}$

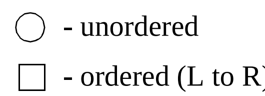


Fig. 5. Symmetry structures for two example functions.

2. (*Recursion*)

   a. If $S_1, \ldots, S_m$ are $m \geq 2$ symmetry structures that are pairwise symmetric, then $S \equiv \{S_1, \ldots, S_m\}$ is a symmetry structure.

   b. If $\langle S_1^1, \ldots, S_1^k\rangle, \ldots, \langle S_m^1, \ldots, S_m^k\rangle$ are $m \geq 2$ ordered lists of $k \geq 2$ symmetry structures that are pairwise symmetric, then $S \equiv \left\{\langle S_1^1, \ldots, S_1^k\rangle, \ldots, \langle S_m^1, \ldots, S_m^k\rangle\right\}$ is a symmetry structure.

3. (*Termination*) If $S_1, \ldots, S_m$ is a collection of $m \geq 2$ symmetry structures then $S \equiv \langle S_1, \ldots, S_m\rangle$ is a symmetry structure.

When applied to equal-sized variable groups that have disjoint support, the above construction induces a hierarchical partition that can be represented by a tree with two types of nodes:

1. Nodes, depicted as circles, that represent un-ordered sets $\{\ldots\}$

2. Nodes, depicted as rectangles, that represent ordered sets $\langle\ldots\rangle$

Let v be a node in such a tree, and let $|v|$ be the cardinality of v, i.e. the size of the set it represents; note that $|v|$ is equal to the number of v's immediate predecessors in the tree. The number of variable permutations corresponding to v, denoted by $\pi(v)$, can be computed according to the formula:

$$\pi(v) = \begin{cases} |v|! \times \prod_{u \in Pred(v)} \pi(u) & \text{if } v \text{ is un-ordered} \\ \prod_{u \in Pred(v)} \pi(u) & \text{if } v \text{ is ordered} \end{cases} \tag{13}$$

The symmetry check in the recursive step of the above construction can be performed by invoking a condition similar to (11) on representative variable permutations from each of the two symmetric structures being compared. Specifically, to check structures S_i and S_j for pairwise symmetry, let $\langle x_1 \ldots x_p \rangle$ and $\langle y_1 \ldots y_p \rangle$ be two variable permutations in their respective supports and $F_{\langle x_1 \ldots x_p \rangle, \langle y_1 \ldots y_p \rangle}$ be the corresponding cofactor matrix. Then, S_i and S_j are symmetric if and only if

$$F^T_{\langle x_1 \ldots x_p \rangle, \langle y_1 \ldots y_p \rangle} = F_{\langle x_1 \ldots x_p \rangle, \langle y_1 \ldots y_p \rangle} \tag{14}$$

Condition (14) can be verified by checking the equality of $2^{p-1}(2^p - 1)$ pairs of cofactors on $\langle x_1 \ldots x_p \rangle$ and $\langle y_1 \ldots y_p \rangle$. Clearly, such a check becomes quite expensive as the structures grow in size. Fortunately, the complexity of the check is reduced to $\frac{1}{2}p(p+1)$ if the structures being checked consist of pairwise symmetric sub-structures. This can be illustrated for $p = 2$ by noting that, when $\{a, b\}$ and $\{c, d\}$ are assumed to be first-order symmetry groups, the two middle columns (resp. rows) of the cofactor matrix in (12) become identical. This makes it possible to reduce the size of the matrix to 3×3 by merging rows and columns of equal minterm weight:

$$F_{\langle ab \rangle, \langle cd \rangle} = \begin{bmatrix} f_{m_0\langle ab \rangle, m_0\langle cd \rangle} & \{f_{m_0\langle ab \rangle, m_1\langle cd \rangle}, f_{m_0\langle ab \rangle, m_2\langle cd \rangle}\} & f_{m_0\langle ab \rangle, m_3\langle cd \rangle} \\ \begin{Bmatrix} f_{m_1\langle ab \rangle, m_0\langle cd \rangle}, \\ f_{m_2\langle ab \rangle, m_0\langle cd \rangle} \end{Bmatrix} & \begin{Bmatrix} f_{m_1\langle a \rangle b, m_1\langle cd \rangle}, f_{m_1\langle ab \rangle, m_2\langle cd \rangle}, \\ f_{m_2\langle ab \rangle, m_1\langle cd \rangle}, f_{m_2\langle ab \rangle, m_2\langle cd \rangle} \end{Bmatrix} & \begin{Bmatrix} f_{m_1\langle ab \rangle, m_3\langle cd \rangle}, \\ f_{m_2\langle ab \rangle, m_3\langle cd \rangle} \end{Bmatrix} \\ f_{m_3\langle ab \rangle, m_0\langle cd \rangle} & \{f_{m_3\langle ab \rangle, m_1\langle cd \rangle}, f_{m_3\langle ab \rangle, m_2\langle cd \rangle}\} & f_{m_3\langle ab \rangle, m_3\langle cd \rangle} \end{bmatrix} \tag{15}$$

For groups of p symmetric variables, the reduction yields a $(p+1) \times (p+1)$ matrix.

To further reduce the computational cost of constructing a function's hierarchical symmetry partition, we have developed the following necessary condition for two ordered groups of variables to be exchangeable:

Theorem 4.1 If two ordered disjoint variable groups, $G_1 = \langle x_1, \ldots, x_p \rangle$ and $G_2 = \langle y_1, \ldots, y_p \rangle$, are symmetric in function f, then variables x_1 and y_1 must be symmetric in function

$$f^* = \exists G_1 \backslash x_1, G_2 \backslash y_1 (f) \tag{16}$$

Proof: The theorem proof is done by contradiction. Suppose groups G_1 and G_2 are symmetric in f, and variables x_1 and y_1 are not symmetric in f^*. Symmetry between G_1 and G_2 implies that the cofactor matrix

$$F_{\langle G_1 \rangle, \langle G_2 \rangle} \equiv \begin{bmatrix} f_{0,0} & \ddots & \boxed{f_{i,j}} \\ \boxed{f_{j,i}} & \ddots & f_{n,n} \end{bmatrix}$$

must be symmetric, where $n = 2^p - 1$. Since in the symmetric matrix we have $\forall i, j [f_{i,j} = f_{j,i}]$ it must also be true that

$$\sum_{i<i} f_{i,j} = \sum_{i<i} f_{j,i}$$

Here the left and right hand sides correspond to the summation of cofactors in the upper-right and lower-left corners of matrix $F_{\langle G_1 \rangle, \langle G_2 \rangle}$. Since x_1 and y_1 occupy the most significant bit positions in their respective groups, it is should be clear that these sums define the cofactors of f^*:

$$f^*_{x_1' y_1} = \sum_{i<j} f_{i,j}$$

$$f^*_{x_1 y_1'} = \sum_{i<i} f_{j,i}$$

But this implies that $f^*_{x_1' y_1} = f^*_{x_1 y_1'}$ contradicting the assumption that x_1 and y_1 are not symmetric in f^*. Therefore x_1 and y_1 must be symmetric in f^*. It is easy to show that the same holds true for any corresponding pair of variables x_i and y_i. ■

To illustrate this theorem, consider the function $f = abc + xyz$. According to the theorem, symmetry between $\langle a, b, c \rangle$ and $\langle x, y, z \rangle$ requires that a and x be symmetric in $[\exists b, c, y, z(abc + xyz)] = a + x$ which, trivially, they are. To determine if these two groups are indeed symmetric, we need now to check the corresponding cofactor matrix for symmetry:

$$F_{\langle abc \rangle, \langle xyz \rangle} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

which it is.

To emphasize the fact that the condition in Theorem 4.1 is necessary but not sufficient, consider the function $f(a, b, c, d) = abd' + ab'd + bcd' + b'cd$ which has first-order symmetry groups $\{a, c\}$ and $\{b, d\}$. The cofactor matrix on groups $\langle a, c\rangle$ and $\langle b, d\rangle$ is

$$F_{\langle ac\rangle, \langle bd\rangle} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

and is clearly asymmetric. This fact, however, is not detected by the theorem's condition since *a* and *b* are trivially symmetric in $[\exists c, d(abd' + ab'd + bcd' + b'cd)] = 1$.

A final example illustrates the utility of the condition in the above theorem as a way to prune unnecessary symmetry checks on large sets of variables. It is easy to show that the function $f = ((a \oplus c)x + acy)(b \oplus d)$ has first-order symmetry groups $\{a, c\}$ and $\{b, d\}$. It is also evident, on the other hand, that *a* and *b* are not symmetric in the function $[\exists c, d(((a \oplus c)x + acy)(b \oplus d))] = x + ay$. This immediately implies that there are no second-order symmetries between $\{a, c\}$ and $\{b, d\}$ and obviates the need for the more expensive symmetry check implied by (11).

It is interesting to note that a variation on Theorem 4.1, in which the existential quantifier $\exists$ is replaced by the universal quantifier $\forall$, is possible. "Stronger" variations that abstract smaller subsets of variables are also possible and may provide useful trade-offs between runtime efficiency and the accuracy of estimating the function's symmetry. Finally, one can show that the theorem holds even when the variable groups are non-disjoint.

V. Symmetries Under Phase Assignment

The symmetry condition in (14) can be relaxed to admit more invariant permutations by allowing the variables being swapped to have selective inversions. Specifically, let $\langle \varphi_1 \ldots \varphi_p\rangle$ be a vector of binary phase assignment variables, and replace(14) with

$$\exists \varphi_1 \ldots \varphi_p (F^T_{\langle x_1 \ldots x_p\rangle \oplus \langle \varphi_1 \ldots \varphi_p\rangle, \langle y_1 \ldots y_p\rangle} = F_{\langle x_1 \ldots x_p\rangle \oplus \langle \varphi_1 \ldots \varphi_p\rangle, \langle y_1 \ldots y_p\rangle}) \tag{17}$$

where the exclusive OR is performed bit-wise.

For example, condition (14) applied to $f(a, b) = a + b'$ requires that

$$(F^T_{\langle a\rangle, \langle b\rangle} = F_{\langle a\rangle, \langle b\rangle}) \Leftrightarrow \left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right)$$

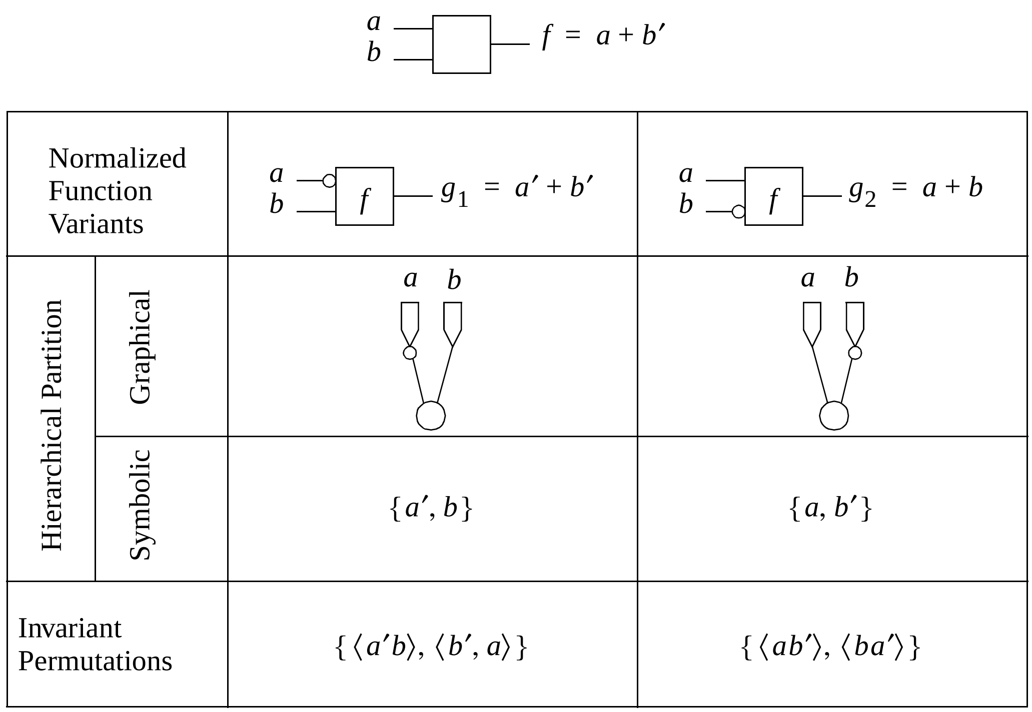


Fig. 6. Symmetry under phase assignment. $\{a', b\}$ and $\{a, b'\}$ represent two equivalent ways of denoting the symmetry of f with respect to a and b.

which obviously does not hold. On the other hand, applying (17) relaxes this requirement, replacing it instead with $(F^T_{\langle a \rangle, \langle b \rangle} = F_{\langle a \rangle, \langle b \rangle}) + (F^T_{\langle a' \rangle, \langle b \rangle} = F_{\langle a' \rangle, \langle b \rangle})$, i.e.

$$\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right) + \left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$$

whose second term is true implying the truth of the entire condition. Thus, the function can be said to have a first-order symmetry between a' and b, and that $\{a', b\}$ is a symmetry group.

The effect of introducing the phase assignment variables in the symmetry check can be seen as a “normalization” of the function that makes it insensitive to the inversion polarity of its inputs. In the matrix formulation of the symmetry check, different assignments to the phase variables correspond to different row orderings; if one or more such orderings yields a symmetric cofactor matrix, the function can be said to have symmetry under phase assignment. An alternative formulation of (17) in which the phase assignment variables are associated with the y instead of the x variables is possible and leads to an equivalent requirement on the function cofactors. In this case, however, different phase assignments correspond to different column orderings. For our simple example above, we would deduce that the function is symmetric in a and b' implying that $\{a, b'\}$ is a symmetry group. Fig. 6 illustrates the equivalence of this symmetry group with the one we identified earlier and pictorially shows the corresponding inversions on the function’s inputs, the resulting hierarchical partitions, and invariant permutations.

Care must be taken when applying (17) to check for higher-order symmetries. Specifically, the assignments available for the phase variables at any level of the partition hierarchy must necessarily be constrained by their assignments at earlier levels. The only flexibility in choosing phase assignments at higher levels of the hierarchy is to reverse the polarity of the support of a symmetry structure; this amounts to choosing one of the two alternative phase assignments propagated from earlier levels of the tree. Symbolically, let $\langle \hat{\varphi}_1 \ldots \hat{\varphi}_p \rangle$ be a phase assignment for which (17) held at some level of the hierarchy tree. The symmetry check at subsequent levels in the tree can then be simplified to:

$$\left(F^T_{\langle x_1 \ldots x_p \rangle \oplus \langle \hat{\varphi}_1 \ldots \hat{\varphi}_p \rangle,\, \langle y_1 \ldots y_p \rangle} = F_{\langle x_1 \ldots x_p \rangle \oplus \langle \hat{\varphi}_1 \ldots \hat{\varphi}_p \rangle,\, \langle y_1 \ldots y_p \rangle}\right) + \left(F^T_{\langle x_1 \ldots x_p \rangle \oplus \overline{\langle \hat{\varphi}_1 \ldots \hat{\varphi}_p \rangle},\, \langle y_1 \ldots y_p \rangle} = F_{\langle x_1 \ldots x_p \rangle \oplus \overline{\langle \hat{\varphi}_1 \ldots \hat{\varphi}_p \rangle},\, \langle y_1 \ldots y_p \rangle}\right) \qquad (18)$$

where complementation of the phase assignment is bit-wise.

As an example of high-order symmetry under phase assignment consider the function $f(a, b, c, d) = a'b + c'd$. It has first-order symmetries represented by the symmetry groups $\{a', b\}$ and $\{c, d'\}$. A check of second-order symmetry between these two groups entails the construction of the following two cofactor matrices:

$$F_{\langle a'b \rangle,\, \langle cd' \rangle} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix},\quad F_{\langle a'b \rangle,\, \langle c'd \rangle} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Since the second matrix is symmetric, we can infer that $\{\{a', b\}, \{c', d\}\}$ is a symmetry structure for this function.

VI. Characterization of Function Symmetry in Benchmark Circuits

We performed an extensive study of available benchmark circuits to determine their symmetry partitions based on the generalized symmetry model presented in this report. Specifically, we analyzed the 2812 output functions of the 101 logic synthesis and optimization benchmarks available from MCNC [17]. These circuits come from three suites:

- the multi-level MCNC benchmarks
- the multi-level ISCAS-85 benchmarks
- the two-level MCNC benchmarks

Benchmark suite	Total # of output functions	# of output functions with no symmetries	High-order symmetry statistics				
			# of output functions with high-order symmetries	Max order of symmetry	Max symmetry group size	High-to-first order permutation ratio	
						Min	Max
Multi-level MCNC	1560	76	41	4	64	2	10E13.32
Multi-level ISCAS85	703	85	16	3	12	2	10E6.08
Two-level MCNC	549	68	10	4	64	2	10E90.91

Fig. 7. Summary of symmetry characterization of benchmark circuits

The multi-level circuits were flattened before the symmetry partitions of their outputs were computed; thus, the reported symmetry partitions reflect the intrinsic functional structure of these outputs rather than any structural regularity in their circuit implementations. Detailed symmetry profiles for each of these circuits' outputs are tabulated in Appendix A. A summary of these results is shown in Fig. 7.

Several observations can be made from these data. The most striking is the relatively small number of output functions that do not exhibit any symmetries. Considering that fact that some of these functions were generated synthetically to stress synthesis algorithms, this suggests that the majority of functions one is likely to encounter in practical design situations will possess some degree of symmetry. The data also show that a small number of functions have higher order symmetries. In the majority of those cases, the order of symmetry was 2; several functions exhibited symmetries of order 3 and 4. As a measure of the additional symmetries found by hierarchical partitioning, we tabulate the ratio of the number of invariant permutations induced by the hierarchical partition to those induced by the first-order partition. This ratio ranged from a minimum of 2 to a maximum of 10^{91}. Finally, symmetry groups ranged in size from a minimum of 2 to a maximum of 64.

We should point out that the symmetry structures were computed under a restriction on the size of ordered groups as well as the variable order within those groups. Specifically, ordered groups chosen for symmetry checks were selected by partitioning the variables into equal-sized subsets using their netlist order. This was done for subset sizes from 2 to 10. This restriction was motivated by the desire to keep the computational effort reasonable, but is otherwise arbitrary.

In the remainder of this section we provide a closer examination of the symmetries discovered in four of the benchmark circuits: t481, C432, C499, and C6288.

Symmetry characterization of t481. This benchmark is interesting because its only output has a 4-level hierarchical symmetry partition (see Fig. 8.) The symmetry involves both ordered and un-ordered groups of variables and requires phase assignment to normalize the function. The number of invariant permutations induced by this partition is 8192 which is 16 times the number of invariant permutations induced by the flat partition of first-order sym-

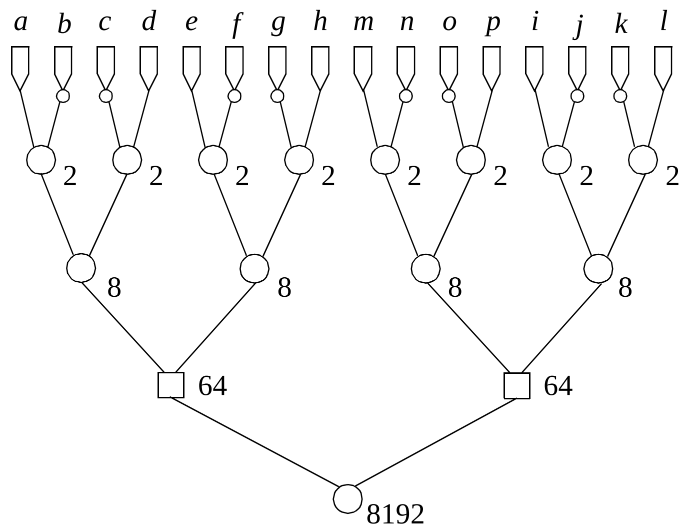


Fig. 8. Hierarchical symmetry partition of t481. Each subtree is annotated at its root with the number of variable permutations it represents.

metries. The multi-level netlist for this benchmark is quite irregular and large (over one thousand gates). This is at odds with the highly regular symmetry structure shown in Fig. 8 and suggests that other implementations that are more compact might be possible.

Symmetry characterization of C432. Of the seven output functions for this benchmark, only one (223GAT(84)) has first-order symmetry. The second-order symmetries exhibited by the other outputs are between ordered groups of variables that range in size from 3 to 6. These symmetries correspond to a substantial number of invariant permutations (up to $10^{5.6}$ for two of the outputs) that would have been overlooked by classical symmetry. The detailed hierarchical symmetry partitions for all seven outputs are given in Appendix B.

Symmetry characterization of C499. `C499` has 32 outputs none of which exhibits any symmetry in terms of its 41 inputs. Based on this, one may be led to believe, erroneously, that these functions lack any regularity. Closer examination, however, reveals that a significant amount of symmetry exists in this circuit when its high-level structure is recognized. This structure is depicted in Fig. 9. The circuit performs single-error-correction [6] and consists of two main modules *M*1 (syndrome generator) and *M*2 (error correction) that are quite regular. The circuit illustrates that completely asymmetric functions may result from the composition of highly symmetric functions. It also suggests that a suitable high-level decomposition might help uncover such latent symmetries. A characterization of the symmetry inherent in `C499` is detailed in Appendix C, where symmetry partitions are derived based on internal signals in addition to primary inputs.

Symmetry characterization of C6288. Another example that lacks first-order symmetry is the C6288 16-bit multiplier. In fact, this circuit is not included in Table 2 because it cannot be flattened due to the exponential memory requirements for its BDD. Smaller multipliers that could be flattened showed little first-order symmetry. However,

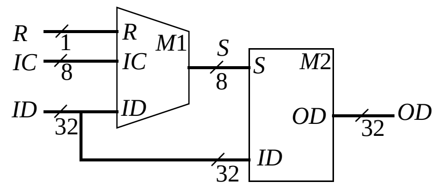


Fig. 9. High level structure of the C499 single-error-correcting circuit.

under a remapping of the multiplier's 32 inputs into the domain of its 16^2 partial products, a three-level structure that is rich with symmetries emerges. In fact, this structure can be readily obtained by a partial flattening of the circuit that stops at the first level of 256 AND gates whose output signals correspond to the partial products. Expressing the output functions of the circuit in terms of these signals we obtain functions which are highly symmetric. The detailed characterization of the remapped functions is given in Appendix D.

VII. Conclusions and Future Directions

We introduced a new definition of functional symmetry based on invariance under unrestricted variable permutations. This definition encompasses the classical notion of symmetry, based on variable swaps, as a special case. We also developed a hierarchical partitioning scheme that generalizes the flat partitioning implied by classical symmetry and yields more invariant permutations. The hierarchical partitioning algorithm is based on the symbolic detection of symmetry in specially-constructed cofactor matrices. The runtime efficiency of hierarchical partitioning was shown to be quite reasonable, aided in part by the application of a necessary condition that quickly detects asymmetry. Application of hierarchical partitioning to a large number of benchmark circuits revealed the existence of significant symmetries.

This work opens up several avenues of future exploration including:

- Examination of the symmetry structure in incompletely-specified functions. Such functions frequently arise during the synthesis of both combinational and sequential circuits.
- Identification of other symmetry forms. For example, single-output symmetries can be extended to multi-output symmetries: a function output exhibits multi-output invariance if permutations on its inputs either leave it invariant or transforms it to one of the other output functions.
- Development of systematic ways of "symmetrizing" a function by expressing it in a different domain. This is similar in spirit to the transformation of the multiplier functions, described earlier, which uncovered their "hidden" symmetries.

Finally, for reasons of efficiency, we would like to develop methods for symmetry identification that do not require netlist flattening.

Appendix A Symmetry profiles of benchmark circuits

The following three tables provide a detailed characterization of symmetry in the benchmark circuits. A sample table entry is shown below and has the following columns:

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Hier.	H Ratio	Flat	Hier.
foo	z	1: 2[9] 2[9] 2[9] 2: 1(2) 1(2) 1(2)	8	1.00	0.00	0.00

- Column 1 specifies the name of the benchmark circuit
- Column 2 indicates the primary output(s) whose functions are being characterized
- Column 3 gives the hierarchical partition for the indicated output(s) in the following stylized format:

$$1\colon n_{11}\|s_{11}\|, n_{12}\|s_{12}\|, \ldots$$
$$\ldots$$
$$l\colon n_{l1}\|s_{l1}\|, n_{l2}\|s_{l2}\|, \ldots$$

  where the number preceding the colon signifies the hierarchy level, and each entry $n_{ij}\|s_{ij}\|$ represents n_{ij} nodes in the hierarchy tree whose size (in-degree) is s_{ij}. The generic delimiters $\|.\|$ are replaced by $(.)$ or $[.]$ for nodes representing ordered and un-ordered groups, respectively. For flat partitions (that have no hierarchy), the level 1 designator is omitted. In addition, the level corresponding to a terminal “square” node is not shown and is understood to be implied.

- Columns 4 displays the number of invariant permutations induced by the hierarchical partition.
- Column 5 shows the ratio of the number of invariant permutations induced by hierarchical partitioning to that induced by flat partitioning. A ratio of 1 is equivalent to a hierarchical partition that has only one level, i.e. a flat partition.
- Columns 6 and 7 give the CPU time, in seconds, required to generate the flat and hierarchical partitions, respectively.

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 1 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
9symml	52	1(9)	10E5.56	1.00	0.00	0.00
alu2	k	8(1)	0	1.00	0.01	0.00
	l	10(1)	0	1.00	0.02	0.00
	m, n	1(2)	1	1.00	0.00	0.00
	o	10(1)	0	1.00	0.01	0.00
	p	1: 2(2) 2: 1(2)	7	2.00	0.00	0.01
alu4	o	8(1)	0	1.00	0.02	0.00
	p	10(1)	0	1.00	0.15	0.00
	q	12(1)	0	1.00	0.08	0.00
	r	14(1)	0	1.00	0.30	0.00
	s, t	1(2)	1	1.00	0.00	0.00
	u	14(1)	0	1.00	0.26	0.00
	v	1: 4(2) 2: 1(4)	383	24.00	0.00	0.01
apex6	SBUFF	1(2)	1	1.00	0.01	0.00
	STW_F	2(1) 1(3) 1(4)	143	1.00	0.00	0.00
	TD_P	16(1) 4(2)	15	1.00	0.10	0.03
	FSESR_P	2(1) 1(2)	1	1.00	0.00	0.00
	P1ZZZ0_P,...,P1ZZZ7_P	3(1) 1(2)	1	1.00	0.00	0.00
	P2ZZZ0_P,..., P2ZZZ7_P	3(1) 1(2)	1	1.00	0.00	0.00
	I1ZZZ0_P,..., I1ZZZ7_P	3(1) 1(2)	1	1.00	0.00	0.00
	I2ZZZ0_P,..., I2ZZZ7_P	3(1) 1(2)	1	1.00	0.00	0.00
	TXMESS_F, RYZ_P	1(1) 1(4)	23	1.00	0.00	0.00
	COMPPAR_P	14(1) 3(2)	7	1.00	0.07	0.02
	RPTWIN_P	10(1) 2(2) 1(6)	2879	1.00	0.10	0.01
	XZFR0_P	1(1) 1(2) 1(9)	10E5.86	1.00	0.00	0.00
	XZFR1_P	1(1) 1(2) 1(10)	10E6.861	1.00	0.00	0.00
	XZFS_P	1: 3(2) 2: 1(1) 1(2)	15	2.00	0.00	0.06
	RXZ0_P	11(1) 1(2) 1(6)	1439	1.00	0.02	0.00
	RXZ1_P	12(1) 1(2) 1(6)	1439	1.00	0.04	0.00
	OFS2_P, OFS1_P	1(2)	1	1.00	0.00	0.00
	A_P	6(1) 1(4)	23	1.00	0.00	0.00
	B_P	4(1) 1(2) 1(5)	239	1.00	0.01	0.00
	C_P	3(1) 1(3) 1(5)	719	1.00	0.01	0.00
	QPR0_P	2(1) 1(4)	23	1.00	0.00	0.00
	QPR1_P	3(1) 1(4)	23	1.00	0.00	0.00
	QPR2_P	2(1) 1(2) 1(4)	47	1.00	0.00	0.00
	QPR3_P	2(1) 1(3) 1(4)	143	1.00	0.00	0.00
	QPR4_P	2(1) 2(4)	575	1.00	0.00	0.01
	AXZ0_P	8(1) 1(3)	5	1.00	0.00	0.00
	AXZ1_P	7(1) 1(2) 1(3)	11	1.00	0.01	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 2 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
apex6 (Cont'd)	V1ZZZ0_P,..., V1ZZZ7_P	3(1) 1(2)	1	1.00	0.00	0.00
	V2ZZZ0_P,..., V2ZZZ7_P	3(1) 1(2)	1	1.00	0.00	0.00
	TXWRD0–P,...,TXWRD0–P14	10(1) 1(2)	1	1.00	0.01	0.00
	TXWRD15_P	9(1) 1(2)	1	1.00	0.01	0.00
	XZ320_P	1(3)	5	1.00	0.00	0.00
	XZ321_P	2(2)	3	1.00	0.00	0.01
	XZ322_P	1(1) 2(2)	3	1.00	0.00	0.00
	XZ323_P	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	XZ324_P	1(1) 1(2) 1(4)	47	1.00	0.01	0.00
	XZ160_F	1(1) 1(2) 1(5)	239	1.00	0.00	0.00
	XZ161_P	1(1) 1(2) 1(6)	1439	1.00	0.00	0.00
	XZ162_P	1(1) 1(2) 1(7)	10079	1.00	0.00	0.00
	XZ163_P	1(1) 1(2) 1(8)	80639	1.00	0.00	0.00
	ENWIN_P	1(1) 3(2)	7	1.00	0.00	0.01
apex7	SDO	1(1)	0	1.00	0.01	0.00
	LSD_P	13(1) 1(4)	23	1.00	0.13	0.00
	ACCRPY_P	12(1) 1(4)	23	1.00	0.18	0.00
	VERR_F	14(1) 1(3) 1(7)	30239	1.00	0.49	0.00
	RATR_P	4(1) 1(2)	1	1.00	0.00	0.00
	MARSSR_P	2(1) 1(8)	40319	1.00	0.00	0.01
	VLENESR_P	1(1) 1(2)	1	1.00	0.00	0.00
	VSUMESR_P	2(1) 1(2)	1	1.00	0.00	0.00
	PLUTO0_P,..., PLUTO5_P	4(1) 2(2) 1(3) 1(8)	10E5.98	1.00	0.02	0.00
	ORWD_F	10(1)	0	1.00	0.02	0.00
	OWL_F	1(3) 1(8)	10E5.38	1.00	0.00	0.00
	PY_P	4(1)	0	1.00	0.01	0.00
	END_P	12(1) 1(4)	23	1.00	0.09	0.00
	FBI_P	13(1) 1(3)	5	1.00	0.22	0.00
	WATCH_P	2(1) 1(2)	1	1.00	0.00	0.00
	OVACC_P	1(2)	1	1.00	0.00	0.00
	KBG_F	14(1) 1(3)	5	1.00	0.16	0.00
	DEL1_P	1(2)	1	1.00	0.00	0.00
	COMPPAR_P	2(1) 1(2)	1	1.00	0.00	0.00
	VST0_P, VST1_P	4(1)	0	1.00	0.00	0.00
	STAR0_P	12(1) 1(2)	1	1.00	0.07	0.00
	STAR1_P	13(1) 1(2)	1	1.00	0.12	0.00
	STAR2_P	12(1) 2(2)	3	1.00	0.10	0.01
	STAR3_P	12(1) 1(2) 1(3)	11	1.00	0.17	0.00
	BULL0_P	1(1) 1(2)	1	1.00	0.00	0.00
	BULL1_P	2(1) 1(2)	1	1.00	0.00	0.00
	BULL2_P	2(1) 1(3)	5	1.00	0.00	0.00
	BULL3_P	2(1) 1(4)	23	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 3 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
apex7 (Cont'd)	BULL4_P	2(1) 1(5)	119	1.00	0.00	0.00
	BULL5_P	2(1) 1(6)	719	1.00	0.00	0.00
	BULL6_P	2(1) 1(7)	5039	1.00	0.00	0.00
b1	d	1(1)	0	1.00	0.00	0.00
	e	1(2)	1	1.00	0.00	0.00
	f	1(3)	5	1.00	0.00	0.00
	g	1(1)	0	1.00	0.00	0.00
b9	p0	8(1) 1(3)	5	1.00	0.01	0.00
	q0	3(1) 2(2)	3	1.00	0.00	0.00
	r0	5(1) 2(2)	3	1.00	0.00	0.00
	s0, t0	3(1) 1(2)	1	1.00	0.01	0.00
	u0, v0	1(1) 1(2)	1	1.00	0.00	0.00
	w0	5(1) 2(2)	3	1.00	0.00	0.00
	x0	1(2)	1	1.00	0.00	0.00
	y0	1(4)	23	1.00	0.00	0.00
	z0	5(1) 4(2)	15	1.00	0.03	0.02
	a1	3(1) 2(2) 1(3)	23	1.00	0.00	0.01
	b1	6(1) 1(3)	5	1.00	0.00	0.00
	c1, d1	5(1) 1(2) 1(3) 1(4)	287	1.00	0.01	0.00
	e1	1(4)	23	1.00	0.00	0.00
	f1	1(3)	5	1.00	0.00	0.00
	g1	1(4)	23	1.00	0.00	0.00
	h1, i1	3(1) 2(2) 1(3)	23	1.00	0.00	0.00
	j1	4(1) 1(2) 1(3)	11	1.00	0.00	0.00
c8	pd0, pe0, pf0, pg0, ph0, pi0, pj0, pk0	3(1)	0	1.00	0.00	0.00
	pl0	1(1)	0	1.00	0.00	0.00
	pm0	4(1) 1(2)	1	1.00	0.00	0.00
	pn0	5(1) 1(2)	1	1.00	0.00	0.00
	po0	5(1) 1(3)	5	1.00	0.00	0.00
	pp0	5(1) 1(4)	23	1.00	0.00	0.00
	pq0	5(1) 1(5)	119	1.00	0.00	0.00
	pr0	5(1) 1(6)	719	1.00	0.01	0.00
	ps0	5(1) 1(7)	5039	1.00	0.00	0.00
	pt0	5(1) 1(8)	40319	1.00	0.01	0.00
	pu0	3(1) 1(8)	40319	1.00	0.00	0.00
cc	w	1(2)	1	1.00	0.00	0.00
	x	1(4)	23	1.00	0.00	0.00
	y	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	z	1(5)	119	1.00	0.00	0.00
	a0, b0, c0, d0, e0	1(1)	0	1.00	0.00	0.00
	f0, g0	1(2)	1	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 4 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
cc (Cont'd)	h0	1(1)	0	1.00	0.00	0.00
	i0	3(1) 1(3)	5	1.00	0.00	0.00
	j0	4(1) 1(2)	1	1.00	0.00	0.00
	k0	3(1) 1(2)	1	1.00	0.00	0.00
	l0	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	m0, n0, o0, p0	3(1) 1(3)	5	1.00	0.01	0.00
cm138a	g, h, i, j, k, l, m, n	1(6)	719	1.00	0.00	0.00
cm150a	v	1: 2[9] 2[9] 2[9] 2: 1(2) 1(2) 1(2)	3	4.00	0.04	10.99
cm151a	m, n	1: 2[5] 2[5] 2[5] 2: 1(2) 1(2) 1(2)	3	1.00	0.10	1.92
cm152a	l	1: 2[5] 2[5] 2[5] 2: 1(2) 1(2) 1(2)	3	4.00	0.00	14.08
cm162a	o	4(1) 2(2)	3	1.00	0.00	0.01
	p	5(1) 2(2)	3	1.00	0.01	0.00
	q	4(1) 3(2)	7	1.00	0.00	0.02
	r	4(1) 2(2) 1(3)	23	1.00	0.00	0.01
	s	1(3)	5	1.00	0.00	0.00
cm163a	q	4(1) 1(2)	1	1.00	0.00	0.00
	r	4(1) 1(3)	5	1.00	0.01	0.00
	s	4(1) 1(4)	23	1.00	0.00	0.00
	t	4(1) 1(5)	119	1.00	0.00	0.00
	u	1(5)	119	1.00	0.01	0.00
cm42a	e, f, g, h, i, j, k, l, m, n	1(4)	23	1.00	0.00	0.00
cm82a	f	1(3)	5	1.00	0.00	0.00
	g, h	1(2) 1(3)	11	1.00	0.00	0.00
cm85a	l	2(1) 4(2)	15	1.00	0.00	0.06
	m	1: 16(2) 2: 1(16)	383	24.00	0.00	0.07
	n	2(1) 4(2)	15	1.00	0.00	0.06
cmb	q, r, s, t	1(12)	10E8.68	1.00	0.00	0.00
comp	g0	16(2)	65535	1.00	0.34	0.28
	h0	1: 16(2) 2: 1(16)	10E18.13	10E13.32	0.39	0.06
	i0	16(2)	65535	1.00	0.34	0.26
cordic	d	1: 6(1) 3(3) 2(4) 2: 7(1) 2(2)	10E5.69	4.00	0.07	0.03
	dn	1: 4(1) 1(2) 3(3) 2(4) 2: 6(1) 2(2)	10E5.99	4.00	0.09	0.02
count	k0	3(1) 1(2)	1	1.00	0.00	0.00
	l0	4(1) 1(2)	1	1.00	0.00	0.00
	m0	4(1) 1(3)	5	1.00	0.00	0.00
	n0	4(1) 1(4)	23	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 5 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
count (Cont’d)	o0	4(1) 1(5)	119	1.00	0.00	0.00
	p0	4(1) 1(6)	719	1.00	0.00	0.00
	q0	4(1) 1(7)	5039	1.00	0.00	0.00
	r0	4(1) 1(8)	40319	1.00	0.01	0.00
	s0	4(1) 1(9)	10E5.56	1.00	0.00	0.00
	t0	4(1) 1(10)	10E6.55	1.00	0.00	0.00
	u0	4(1) 1(11)	10E7.60	1.00	0.01	0.00
	v0	4(1) 1(12)	10E8.68	1.00	0.00	0.00
	w0	4(1) 1(13)	10E9.79	1.00	0.01	0.00
	x0	4(1) 1(14)	10E10.94	1.00	0.01	0.00
	y0	4(1) 1(15)	10E12.11	1.00	0.01	0.00
	z0	4(1) 1(16)	10E13.32	1.00	0.01	0.00
cu	p,q	1(1) 1(3)	5	1.00	0.00	0.00
	r, s, t, u	1(7)	5039	1.00	0.00	0.00
	v	6(1) 2(2) 1(3)	23	1.00	0.01	0.00
	w	1(5)	119	1.00	0.00	0.00
	x	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	y	1(2)	1	1.00	0.00	0.00
	z	2(2)	3	1.00	0.00	0.01
des [[a]even indices; [b]odd indices]	inreg_new<1>,...,inreg_new<55>	3(1) 1(3)	5	1.00	0.00	0.00
	outreg_new<0>,..., outreg_new<54>[a]	4(1) 1(3)	5	1.00	0.00	0.00
	outreg_new<1>,..., outreg_new<55>[b]	4(1) 6(2) 1(3)	383	1.00	0.05	0.25
	outreg_new<56>,..., outreg_new<62>[a]	2(1) 1(4)	23	1.00	0.00	0.00
	outreg_new<57>,..., outreg_new<63>[b]	3(1) 6(2) 1(3)	383	1.00	0.03	0.26
	data_new<0>,..., data_new<31>	2(1) 1(4)	23	1.00	0.00	0.00
	data_new<32>,..., data_new<63>	2(1) 6(2) 1(4)	1535	1.00	0.06	0.13
	count_new<0>	1(2)	1	1.00	0.00	0.00
	count_new<1>	1(1) 1(2)	1	1.00	0.01	0.00
	count_new<2>	2(1) 1(3)	5	1.00	0.00	0.00
	C_new<0>,..., C_new<27>	11(1) 2(2)	3	1.00	0.04	0.02
	D_new<0>,..., D_new<27>	11(1) 2(2)	3	1.00	0.04	0.02
	encrypt_mode_new<0>	2(1) 1(4)	23	1.00	0.01	0.00
dalu	O15	10(1) 1(2) 2(15)	10E24.83	1.00	0.17	0.02
	O14	14(1) 1(2) 1(3) 2(14)	10EE23.26	1.00	0.66	0.01
	O13	17(1) 2(2) 2(13)	10E20.19	1.00	0.79	0.02
	O12	21(1) 1(2) 2(12)	10E16.66	1.00	1.16	0.02
	O11	23(1) 1(2) 2(7)	10E7.70	1.00	0.79	0.02

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 6 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
dalu (Cont'd)	O10	25(1) 1(2) 2(7)	10E7.70	1.00	0.98	0.02
	O9, O8, O7	27(1) 1(2) 2(7)	10E7.70	1.00	1.14	0.01
	O6	27(1) 1(2) 2(6)	10E6.01	1.00	1.48	0.01
	O5	27(1) 1(2) 2(5)	28799	1.00	1.25	0.02
	O4	27(1) 1(2) 2(4)	1151	1.00	1.02	0.01
	O3	27(1) 1(2) 2(3)	71	1.00	0.99	0.01
	O2	27(1) 3(2)	7	1.00	0.88	0.05
	O1	25(1) 3(2)	7	1.00	0.78	0.06
	O0	25(1) 1(2)	2	1.00	0.53	0.00
example2	i2	4(1) 1(3)	5	1.00	0.00	0.00
	j2	2(1) 1(2)	1	1.00	0.00	0.00
	k2	2(1) 2(2)	3	1.00	0.00	0.00
	l2	1(4)	23	1.00	0.00	0.00
	m2	1(1) 1(3)	5	1.00	0.00	0.00
	n2, o2, p2, q2, r2, s2, t2, u2, v2, w2, x2, y2, z2, a3, b3, c3, d3	3(1)	0	1.00	0.00	0.00
	e3	9(1) 1(2) 1(5)	239	1.00	0.03	0.00
	f3	1(2) 1(3)	11	1.00	0.00	0.00
	g3	1(1) 1(2) 1(4)	47	1.00	0.00	0.00
	h3, i3, j3, k3, l3, m3, n3, o3	1(3) 1(7)	30239	1.00	0.00	0.00
	p3	4(1) 1(3) 1(5)	719	1.00	0.01	0.00
	q3, r3, s3, t3, u3, v3, w3, x3, y3, z3, a4, b4, c4	1(2) 1(3) 1(6)	8639	1.00	0.00	0.00
	d4	5(1) 2(2) 1(3)	23	1.00	0.06	0.00
	e4, f4, g4, h4, i4, j4	6(1) 1(2) 1(3)	11	1.00	0.00	0.00
	k4	3(1) 1(5)	119	1.00	0.00	0.00
	l4	1(10)	10E6.56	1.00	0.00	0.00
	m4	2(1) 3(2)	7	1.00	0.00	0.02
	n4	1(4)	23	1.00	0.00	0.00
	o4, p4	7(1) 1(7)	5039	1.00	0.01	0.00
	q4	6(1) 1(2) 1(5)	239	1.00	0.05	0.00
	r4	1(1) 2(2) 1(3)	23	1.00	0.00	0.01
	s4	8(1) 1(5)	119	1.00	0.01	0.00
	t4	1(1) 2(2) 1(3)	23	1.00	0.00	0.01
	u4	6(1) 1(2)	1	1.00	0.01	0.00
	v4	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
f51m	44	8(1)	0	1.00	0.01	0.00
	45	7(1)	0	1.00	0.00	0.00
	46	6(1)	0	1.00	0.00	0.00
	47	3(1) 1(2)	1	1.00	0.00	0.00
	48	2(1) 1(2)	1	1.00	0.00	0.00
	49	1(1) 1(2)	1	1.00	0.00	0.00
	50	1(2)	1	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 7 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
f51m (Cont'd)	51	1(1)	0	1.00	0.00	0.00
frg2	o4	1(1)	0	1.00	0.00	0.00
	p4, q4, r4, s4	4(1) 1(2)	1	1.00	0.00	0.00
	t4, u4	1(1)	0	1.00	0.00	0.00
	v4, w4, x4, y4, z4, a5, b5, c5	3(1) 1(2)	1	1.00	0.00	0.00
	d5, e5, f5, g5, h5, i5, j5, k5, l5	1(2)	1	1.00	0.00	0.00
	m5	1(1)	0	1.00	0.00	0.00
	n5, o5, p5, q5, r5	1(3)	5	1.00	0.00	0.00
	s5, t5, u5, v5, w5	1(1) 1(2) 1(5)	239	1.00	0.00	0.00
	x5, y5, z5, a6, b6	1(3)	5	1.00	0.00	0.00
	c6, d6, e6, f6, g6	1(2)	1	1.00	0.00	0.00
	h6, i6, j6, k6, l6, m6	1(1)	0	1.00	0.00	0.00
	n6	1(2)	1	1.00	0.00	0.00
	o6	1(6)	719	1.00	0.00	0.00
	p6	1: 1(1) 7(2) 1(4) 2: 4(1) 1(5)	10E5.56	120.00	0.03	0.13
	q6, r6, s6, t6, u6	2(1) 1(2) 1(8)	80639	1.00	0.01	0.00
	v6	1: 2(1) 8(2) 1(4) 2: 6(1) 1(5)	10E5.86	120.00	0.04	0.32
	w6, x6, y6, z6, a7, b7, c7, d7, e7, f7,g7, h7, i7, j7, k7, l7, m7, n7, o7, p7, q7, r7, s7	4(1) 1(2) 2(6)	10E6.016	1.00	0.02	0.01
	t7, u7, v7, w7, x7, y7, z7, a8	5(1) 2(2) 1(6) 1(7)	10E7.16	1.00	0.03	0.01
	b8, c8, d8, e8, f8, g8, h8	4(1) 2(2) 1(6) 1(7)	10E7.16	1.00	0.02	0.01
	i8	3(1) 2(2) 1(6) 1(7)	10E7.16	1.00	0.01	0.01
	j8, k8, l8, m8, n8, o8, p8, q8, r8, s8, t8, u8, v8, w8, x8, y8, z8, a9, b9, c9, d9, e9	13(1) 1(2) 1(4)	47	1.00	0.04	0.00
	f9	17(1) 1(2) 1(4)	47	1.00	0.08	0.00
	g9	2(2) 1(5) 1(6)	10E5.53	1.00	0.01	0.01
	h9	1(1) 2(2) 1(4) 1(6)	69119	1.00	0.01	0.01
	i9	1(1) 1(2) 2(3) 1(6)	51839	1.00	0.01	0.07
	j9	2(1) 3(2) 1(4) 1(6)	10E5.14	1.00	0.02	0.03
	k9	3(1) 1(2) 1(5) 1(6)	10E5.23	1.00	0.01	0.00
	l9	2(1) 1(2) 2(6)	10E6.016	1.00	0.12	0.00
	m9	14(1) 1(2) 1(4)	47	1.00	0.07	0.00
	n9, o9	12(1) 1(2) 2(3)	71	1.00	0.05	0.01
	p9	15(1) 2(2) 1(4)	95	1.00	0.08	0.01
	q9	12(1) 2(2) 1(5)	479	1.00	0.06	0.01
	r9	1(2) 1(3) 1(6)	8639	1.00	0.01	0.00
	s9	6(1) 6(2) 1(3) 1(4)	9215	1.00	0.06	0.12
	t9	1(8)	40319	1.00	0.00	0.00
	u9	1(3) 1(6)	4319	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 8 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
frg2 (Cont'd)	v9	1: 1(1) 7(2) 1(4) 2: 4(1) 1(5)	10E5.56	120.00	0.03	0.12
	w9	1: 5(2) 1(3) 2: 1(1) 1(5)	23039	120.00	0.00	0.06
i1	V27_0	1(1)	0	1.00	0.00	0.00
	V27_1, V27_2	2(1) 1(2) 1(7)	10079	1.00	0.00	0.00
	V27_3	1(1)	0	1.00	0.00	0.00
	V27_4	1(2)	1	1.00	0.00	0.00
	V28_0	1(1) 1(9)	10E5.56	1.00	0.00	0.00
	V29_0	1(1)	0	1.00	0.00	0.00
	V30_0	1(2)	1	1.00	0.00	0.00
	V31_0	1(1)	0	1.00	0.00	0.00
	V32_0	1(2)	1	1.00	0.00	0.00
	V33_0, V34_0, V35_0, V36_0	1(3)	5	1.00	0.00	0.00
	V37_0	1(2)	1	1.00	0.00	0.00
	V38_0	1(4)	23	1.00	0.00	0.00
i2	V202(0)	13(1) 3(4) 3(16) 2(64)	10E222.3	1.00	6.43	0.28
i3	V134(0), V134(1)	1(2)	1	1.00	0.00	0.00
	V138(0), V138(1), V138(2), V138(3)	1: 16(2) 2: 1(16)	10E18.13	10E13.32	0.13	0.09
i4	V194(0), V194(1)	1(2)	1	1.00	0.00	0.00
	V198(0), V198(1), V198(2), V198(3)	11(1) 12(2) 4(3)	10E6.72	1.00	1.38	0.22
i5	V135(0)	11(1) 1(2)	1	1.00	0.01	0.00
	V135(1)	9(1) 1(2)	1	1.00	0.01	0.00
	V151(1)	17(1) 1(2)	1	1.00	0.04	0.00
	V151(2)	15(1) 1(2)	1	1.00	0.02	0.00
	V151(3)	13(1) 1(2)	1	1.00	0.01	0.00
	V151(5)	15(1) 1(2)	1	1.00	0.03	0.00
	V151(6)	13(1) 1(2)	1	1.00	0.01	0.01
	V151(7)	11(1) 1(2)	1	1.00	0.00	0.00
	V151(9)	13(1) 1(2)	1	1.00	0.02	0.00
	V151(10)	11(1) 1(2)	1	1.00	0.01	0.00
	V151(11)	9(1) 1(2)	1	1.00	0.00	0.00
	V151(13)	11(1) 1(2)	1	1.00	0.02	0.00
	V151(14)	9(1) 1(2)	1	1.00	0.01	0.00
	V151(15)	7(1) 1(2)	1	1.00	0.00	0.00
	V167(1)	15(1) 1(2)	1	1.00	0.03	0.00
	V167(2)	13(1) 1(2)	1	1.00	0.02	0.00
	V167(3)	11(1) 1(2)	1	1.00	0.01	0.00
	V167(5)	13(1) 1(2)	1	1.00	0.02	0.00
	V167(6)	11(1) 1(2)	1	1.00	0.02	0.00
	V167(7)	9(1) 1(2)	1	1.00	0.01	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 9 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
i5 (Cont'd)	V167(9)	11(1) 1(2)	1	1.00	0.01	0.00
	V167(10)	9(1) 1(2)	1	1.00	0.01	0.00
	V167(11)	7(1) 1(2)	1	1.00	0.00	0.00
	V167(13)	9(1) 1(2)	1	1.00	0.00	0.00
	V167(14)	7(1) 1(2)	1	1.00	0.00	0.00
	V167(15)	5(1) 1(2)	1	1.00	0.00	0.00
	V183(1)	13(1) 1(2)	1	1.00	0.02	0.00
	V183(2)	11(1) 1(2)	1	1.00	0.01	0.00
	V183(3)	9(1) 1(2)	1	1.00	0.01	0.00
	V183(5)	11(1) 1(2)	1	1.00	0.01	0.00
	V183(6)	9(1) 1(2)	1	1.00	0.00	0.00
	V183(7)	7(1) 1(2)	1	1.00	0.00	0.00
	V183(9)	9(1) 1(2)	1	1.00	0.01	0.00
	V183(10)	7(1) 1(2)	1	1.00	0.00	0.00
	V183(11)	5(1) 1(2)	1	1.00	0.00	0.00
	V183(13)	7(1) 1(2)	1	1.00	0.01	0.00
	V183(14)	5(1) 1(2)	1	1.00	0.00	0.00
	V183(15)	3(1) 1(2)	1	1.00	0.00	0.00
	V199(1)	11(1) 1(2)	1	1.00	0.02	0.00
	V199(2)	9(1) 1(2)	1	1.00	0.01	0.00
	V199(3)	7(1) 1(2)	1	1.00	0.01	0.00
	V199(5)	9(1) 1(2)	1	1.00	0.01	0.00
	V199(6)	7(1) 1(2)	1	1.00	0.00	0.00
	V199(7)	5(1) 1(2)	1	1.00	0.01	0.00
	V199(9)	7(1) 1(2)	1	1.00	0.01	0.00
	V199(10)	5(1) 1(2)	1	1.00	0.00	0.00
	V199(11)	3(1) 1(2)	1	1.00	0.00	0.00
	V199(13)	5(1) 1(2)	1	1.00	0.00	0.00
	V199(14)	3(1) 1(2)	1	1.00	0.00	0.00
	V199(15)	1(1) 1(2)	1	1.00	0.01	0.00
	V151(4)	11(1) 1(2)	1	1.00	0.01	0.00
	V151(8)	9(1) 1(2)	1	1.00	0.00	0.00
	V151(12)	7(1) 1(2)	1	1.00	0.00	0.00
	V167(4)	9(1) 1(2)	1	1.00	0.00	0.00
	V167(8)	7(1) 1(2)	1	1.00	0.01	0.00
	V167(12)	5(1) 1(2)	1	1.00	0.00	0.00
	V183(4)	7(1) 1(2)	1	1.00	0.00	0.00
	V183(8)	5(1) 1(2)	1	1.00	0.00	0.00
	V183(12)	3(1) 1(2)	1	1.00	0.00	0.00
	V199(4)	5(1) 1(2)	1	1.00	0.00	0.00
	V199(8)	3(1) 1(2)	1	1.00	0.00	0.00
	V199(12)	1(1) 1(2)	1	1.00	0.01	0.00

i5 (Cont'd)	V151(0)	7(1) 1(2)	1	1.00	0.00	0.00
	V167(0)	5(1) 1(2)	1	1.00	0.00	0.00
	V183(0)	3(1) 1(2)	1	1.00	0.00	0.00
	V199(0)	1(1) 1(2)	1	1.00	0.00	0.00
k2	t0	5(1) 2(2) 1(3)	23	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 11 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
lal	b0	1(2)	1	1.00	0.01	0.00
	c0	5(1) 2(2) 1(3)	23	1.00	0.00	0.00
	d0	1(1)	0	1.00	0.00	0.00
	e0	4(1) 1(2) 1(3)	11	1.00	0.00	0.00
	f0	1: 1(1) 4(2) 2: 1(1) 2(4)	383	24.00	0.01	0.01
	g0, h0, i0	1(2)	1	1.00	0.00	0.00
	j0	4(1) 1(2) 1(3)	11	1.00	0.00	0.00
	k0	1(4)	23	1.00	0.00	0.00
	l0	1(2) 1(3)	11	1.00	0.00	0.00
	m0	3(2)	7	1.00	0.00	0.01
	n0	1(1) 3(2)	7	1.00	0.00	0.00
	o0	1(1) 2(2) 1(3)	23	1.00	0.00	0.00
	p0	1(1) 2(2) 1(4)	95	1.00	0.01	0.00
	q0	1(1) 2(2) 1(5)	479	1.00	0.00	0.00
	r0	1(1) 2(2) 1(6)	2879	1.00	0.00	0.01
	s0	1(1) 2(2) 1(7)	20159	1.00	0.00	0.00
	t0	1(1) 2(2) 1(8)	10E5.21	1.00	0.00	0.00
majority	f	1(1) 1(4)	23	1.00	0.00	0.00
mux	l	1: 2[9] 2[9] 2[9] 2[9] 2: 1(2) 1(2) 1(2) 1(2)	4	5.00	0.04	11.36
my_adder	h0	15(2) 1(3)	10E5.29	1.00	0.69	0.17
	i0	14(2) 1(3)	98303	1.00	0.38	0.16
	j0	13(2) 1(3)	49151	1.00	0.23	0.17
	k0	12(2) 1(3)	24575	1.00	0.18	0.10
	l0	11(2) 1(3)	12287	1.00	0.14	0.08
	m0	10(2) 1(3)	6143	1.00	0.11	0.07
	n0	9(2) 1(3)	3071	1.00	0.09	0.05
	o0	8(2) 1(3)	1535	1.00	0.03	0.03
	p0	7(2) 1(3)	767	1.00	0.02	0.03
	q0	6(2) 1(3)	383	1.00	0.01	0.02
	r0	5(2) 1(3)	191	1.00	0.01	0.01
	s0	4(2) 1(3)	95	1.00	0.00	0.01
	t0	3(2) 1(3)	47	1.00	0.00	0.00
	u0	2(2) 1(3)	23	1.00	0.01	0.00
	v0	1(2) 1(3)	11	1.00	0.00	0.00
	w0	1(3)	5	1.00	0.00	0.00
	x0	15(2) 1(3)	10E5.29	1.00	0.45	0.23

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 12 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
pair	s5, t5, u5, v5	1: 11(2) 2: 1(11)	10E10.91	10E7.60	0.07	0.11
	w5	17(1) 5(2) 4(3) 1(5) 1(7)	10E10.39	1	2.25	0.19
	x5, y5, z5, a6	1: 11(2) 2: 1(11)	10E10.91	10E7.60	0.04	0.11
	b6	17(1) 6(2) 4(3) 1(5) 1(7)	10E9.33	1.00	2.37	0.25
	c6	6(1) 1(2) 2(3)	71	1.00	0.01	0.01
	d6	7(1) 1(2) 2(3)	71	1.00	0.02	0.01
	e6	8(1) 1(2) 2(3)	71	1.00	0.01	0.01
	f6	7(1) 2(2) 2(3)	143	1.00	0.03	0.03
	g6	7(1) 1(2) 3(3)	431	1.00	0.04	0.02
	h6	7(1) 1(2) 2(3) 1(4)	1727	1.00	0.17	0.01
	i6	7(1) 1(2) 2(3) 1(5)	8639	1.00	0.05	0.01
	j6	7(1) 1(2) 2(3) 1(6)	51839	1.00	0.05	0.01
	k6	7(1) 1(2) 2(3) 1(7)	10E5.56	1.00	0.15	0.01
	l6, m6	1(1)	0	1.00	0.00	0.00
	n6	1(1) 2(3)	35	1.00	0.00	0.01
	o6	2(1) 1(2) 1(3) 1(4)	287	1.00	0.00	0.00
	p6	2(1) 3(3)	215	1.00	0.00	0.03
	q6	4(1) 3(2) 1(3)	47	1.00	0.00	0.02
	r6	1(1)	0	1.00	0.00	0.00
	s6	1(1) 1(2) 2(3)	71	1.00	0.00	0.01
	t6	7(1) 1(2) 2(3) 1(7) 1(8)	10E9.33	1.00	0.08	0.01
	u6	2(1) 2(2) 1(3) 1(4)	575	1.00	0.01	0.01
	v6	1(1) 1(3)	5	1.00	0.00	0.00
	w6, x6, y6	7(1) 1(2)	1	1.00	0.01	0.00
	z6	10(1) 3(2) 1(3)	47	1.00	0.05	0.03
	a7	11(1) 3(2) 4(3)	10367	1.00	0.29	0.08
	b7	10(1) 4(2) 4(3)	20735	1.00	0.15	0.12
	c7	11(1) 3(2) 5(3)	62207	1.00	0.47	0.12
	d7	12(1) 3(2) 5(3)	62207	1.00	0.40	0.22
	e7	11(1) 4(2) 5(3)	10E5.09	1.00	0.39	0.15
	f7	11(1) 3(2) 4(3) 1(6)	10E6.87	1.00	0.45	0.09
	g7	12(1) 3(2) 4(3) 1(6)	10E6.87	1.00	0.60	0.08
	h7	11(1) 4(2) 4(3) 1(6)	10E7.17	1.00	0.37	0.12
	i7	1(2)	1	1.00	0.00	0.00
	j7	1(1) 1(2)	1	1.00	0.00	0.00
	k7	2(1) 1(2)	1	1.00	0.00	0.00
	l7	6(1) 1(2) 2(3)	71	1.00	0.02	0.01
	m7	7(1) 1(2) 2(3)	71	1.00	0.03	0.01
	n7	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	o7	8(1) 1(2) 2(3)	71	1.00	0.04	0.01
	p7	7(1) 2(2) 2(3)	143	1.00	0.02	0.02

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 13 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
pair (Cont'd)	q7	7(1) 1(2) 3(3)	431	1.00	0.03	0.04
	r7	9(1) 2(3) 1(4)	863	1.00	0.14	0.01
	s7	10(1) 2(3) 1(4)	863	1.00	0.05	0.01
	t7	9(1) 1(2) 2(3) 1(4)	1727	1.00	0.05	0.01
	u7	3(1) 2(2) 2(3) 1(8)	10E6.76	1.00	0.01	0.00
	v7	9(1) 3(3) 1(4)	5183	1.00	0.04	0.04
	w7	4(1) 1(2) 2(3)	71	1.00	0.01	0.01
	x7	8(1) 1(2) 2(3) 1(8)	10E6.46	1.00	0.11	0.01
	y7	11(1) 2(2) 1(3) 1(8)	10E5.99	1.00	0.10	0.00
	z7	10(1) 4(2) 1(3) 1(8)	10E6.59	1.00	0.25	0.05
	a8	14(1) 3(2) 1(3) 1(8)	10E6.29	1.00	0.32	0.03
	b8	2(1) 1(2) 3(3) 1(4)	10367	1.00	0.02	0.03
	c8	1(3) 1(5)	719	1.00	0.00	0.00
	d8	15(1) 3(2) 1(3) 1(8)	10E6.29	1.00	0.35	0.02
	e8	16(1) 3(2) 1(3) 1(8)	10E6.29	1.00	0.37	0.02
	f8	15(1) 4(2) 1(3) 1(8)	10E6.59	1.00	0.39	0.04
	g8	15(1) 3(2) 2(3) 1(8)	10E7.06	1.00	0.51	0.03
	h8	17(1) 3(2) 1(3) 1(8)	10E6.29	1.00	0.44	0.02
	i8	16(1) 4(2) 1(3) 1(8)	10E6.59	1.00	0.55	0.05
	j8	17(1) 4(2) 1(3) 1(8)	10E6.59	1.00	0.57	0.05
	k8	16(1) 5(2) 1(3) 1(8)	10E6.89	1.00	0.54	0.09
	l8	16(1) 4(2) 2(3) 1(8)	10E7.37	1.00	0.64	0.06
	m8	6(1) 3(2) 2(3) 1(8)	10E7.06	1.00	0.10	0.04
	n8	9(1) 4(2) 1(3) 1(8)	10E6.59	1.00	0.40	0.06
	o8	10(1) 5(2) 1(3) 1(8)	10E6.89	1.00	0.99	0.11
	p8	14(1) 4(2) 1(3) 1(8)	10E6.59	1.00	2.24	0.08
	q8	2(1) 1(2) 3(3) 1(4)	10367	1.00	0.01	0.03
	r8	1(3) 1(5)	719	1.00	0.00	0.00
	s8	15(1) 4(2) 1(3) 1(8)	10E6.59	1.00	1.76	0.20
	t8	16(1) 4(2) 1(3) 1(8)	10E6.59	1.00	2.29	0.08
	u8	15(1) 5(2) 1(3) 1(8)	10E6.89	1.00	2.55	0.12
	v8	15(1) 4(2) 2(3) 1(8)	10E7.37	1.00	2.83	0.24
	w8	17(1) 4(2) 1(3) 1(8)	10E6.59	1.00	3.19	0.09
	x8	16(1) 5(2) 1(3) 1(8)	10E6.89	1.00	2.38	0.11
	y8	17(1) 5(2) 1(3) 1(8)	10E6.89	1.00	2.78	0.12
	z8	16(1) 6(2) 1(3) 1(8)	10E7.19	1.00	3.18	0.18
	a9	16(1) 5(2) 2(3) 1(8)	10E7.67	1.00	3.25	0.15
	b9	10(1) 3(2) 1(3)	47	1.00	0.07	0.03
	c9	11(1) 3(2) 4(3)	10367	1.00	0.46	0.19
	d9	10(1) 4(2) 4(3)	20735	1.00	0.47	0.23
	e9	11(1) 3(2) 5(3)	62207	1.00	0.53	0.16
	f9	12(1) 3(2) 5(3)	62207	1.00	1.02	0.14

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 14 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
pair (Cont'd)	g9	11(1) 4(2) 5(3)	10E5.09	1.00	1.02	0.20
	h9	11(1) 3(2) 4(3) 1(6)	10E6.87	1.00	1.17	0.12
	i9	12(1) 3(2) 4(3) 1(6)	10E6.87	1.00	1.29	0.13
	j9	11(1) 4(2) 4(3) 1(6)	10E7.17	1.00	1.35	0.16
	k9	1(2)	1	1.00	0.00	0.00
	l9	1(1) 1(2)	1	1.00	0.00	0.00
	m9	2(1) 1(2)	1	1.00	0.01	0.00
	n9	6(1) 1(2) 2(3)	71	1.00	0.02	0.01
	o9	7(1) 1(2) 2(3)	71	1.00	0.03	0.01
	p9	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	q9	8(1) 1(2) 2(3)	71	1.00	0.04	0.01
	r9	7(1) 2(2) 2(3)	143	1.00	0.04	0.02
	s9	7(1) 1(2) 3(3)	431	1.00	0.12	0.03
	t9	9(1) 2(3) 1(4)	863	1.00	0.06	0.01
	u9	10(1) 2(3) 1(4)	863	1.00	0.06	0.01
	v9	9(1) 1(2) 2(3) 1(4)	1727	1.00	0.15	0.02
	w9	3(1) 2(2) 2(3) 1(8)	10E6.76	1.00	0.02	0.01
	x9	9(1) 3(3) 1(4)	5183	1.00	0.09	0.03
	y9	6(1) 1(2) 2(3)	71	1.00	0.02	0.01
	z9	7(1) 1(2) 2(3)	71	1.00	0.02	0.01
	a10	8(1) 1(2) 2(3)	71	1.00	0.03	0.01
	b10	7(1) 2(2) 2(3)	143	1.00	0.04	0.01
	c10	7(1) 1(2) 3(3)	431	1.00	0.05	0.03
	d10	7(1) 1(2) 2(3) 1(4)	1727	1.00	0.05	0.01
	e10	7(1) 1(2) 2(3) 1(5)	8639	1.00	0.06	0.01
	f10	7(1) 1(2) 2(3) 1(6)	51839	1.00	0.06	0.01
	g10	7(1) 1(2) 2(3) 1(7)	10E5.56	1.00	0.07	0.01
	h10, i10	1(1)	0	1.00	0.00	0.00
	j10	1(1) 2(3)	35	1.00	0.00	0.14
	k10	2(1) 1(2) 1(3) 1(4)	287	1.00	0.00	0.00
	l10	2(1) 3(3)	215	1.00	0.00	0.04
	m10, n10	5(1) 2(2) 2(3)	143	1.00	0.02	0.02
	o10	4(1) 3(2) 1(3)	47	1.00	0.01	0.03
	p10	1(1)	0	1.00	0.00	0.00
	q10	3(1) 1(3)	5	1.00	0.01	0.00
	r10	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	s10	1(1) 1(2) 1(3) 1(5)	1439	1.00	0.01	0.00
	t10	7(1) 1(2) 2(3) 1(7) 1(8)	10E9.33	1.00	0.06	0.01
	u10	2(1) 3(2) 1(5)	959	1.00	0.01	0.02
	v10	1(1) 1(3)	5	1.00	0.00	0.00
	w10, x10, y10	7(1) 1(2)	1	1.00	0.01	0.00
parity	q	1(16)	10E13.32	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 15 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
pcler8	b0	1(11)	10E7.60	1.00	0.00	0.00
	c0, d0, e0, f0, g0, h0, i0, j0	1(2)	1	1.00	0.00	0.00
	k0	3(1) 1(2) 1(8)	80639	1.00	0.00	0.00
	l0	2(1) 2(2) 1(7)	20159	1.00	0.00	0.01
	m0	3(1) 2(2) 1(6)	2879	1.00	0.00	0.00
	n0	3(1) 1(2) 1(3) 1(5)	1439	1.00	0.01	0.00
	o0	3(1) 1(2) 2(4)	1151	1.00	0.00	0.00
	p0	3(1) 1(2) 1(3) 1(5)	1439	1.00	0.00	0.00
	q0	3(1) 2(2) 1(6)	2879	1.00	0.01	0.00
	r0	4(1) 1(2) 1(7)	10079	1.00	0.00	0.00
pm1	r	1(3)	5	1.00	0.00	0.00
	s	1(2)	1	1.00	0.00	0.00
	t	1(7)	5039	1.00	0.00	0.00
	u	1(3) 1(4)	143	1.00	0.00	0.00
	v, w	1(1)	0	1.00	0.00	0.00
	x	2(2) 1(3)	23	1.00	0.00	0.00
	y	1(1)	0	1.00	0.00	0.00
	z	1(3) 1(5)	719	1.00	0.00	0.00
	a0	2(2)	3	1.00	0.00	0.00
	b0	1(3) 1(5)	719	1.00	0.00	0.00
	c0	1(1) 1(2) 2(3)	71	1.00	0.01	0.00
	d0	1(5)	119	1.00	0.00	0.00
pm4	r3	1(2) 1(3) 1(4)	287	1.00	0.00	0.00
	r4	1(2) 2(3) 1(4)	1727	1.00	0.01	0.00
	r0	1(1)	0	1.00	0.00	0.00
	r1	1(2)	1	1.00	0.00	0.00
	r2	1(2) 1(3)	11	1.00	0.00	0.00
	r5	2(2) 2(3) 1(4)	3455	1.00	0.02	0.00
	r6, r7	1(1) 2(2) 2(3) 1(4)	3455	1.00	0.02	0.00
rot	f4	2(1) 1(2)	1	1.00	0.00	0.00
	g4	3(1)	0	1.00	0.00	0.00
	h4	12(1) 1(2) 1(3)	11	1.00	0.02	0.00
	i4	15(1) 1(3)	5	1.00	0.08	0.00
	j4	14(1) 1(3)	5	1.00	0.11	0.00
	k4, l4, m4	1(1)	0	1.00	0.00	0.00
	n4	6(1) 1(2) 1(3)	11	1.00	0.00	0.00
	o4	1(4)	23	1.00	0.00	0.00
	p4	28(1) 7(2) 1(3)	767	1.00	6.33	0.28
	q4	2(1) 1(4)	23	1.00	0.00	0.00
	r4	6(1) 3(3)	215	1.00	0.03	0.01
	s4	17(1) 4(2) 1(3)	95	1.00	0.49	0.02
	t4	37(1) 5(2) 2(3)	1151	1.00	32.41	0.50

rot (Cont'd)	u4	37(1) 4(2) 2(3)	575	1.00	25.47	0.25
	v4	31(1) 6(2) 2(3)	2303	1.00	7.55	0.25
	w4	30(1) 7(2) 2(3)	4607	1.00	12.29	0.79
	x4	31(1) 7(2) 2(3)	4607	1.00	12.83	0.48
	y4	1(1) 1(2)	1	1.00	0.00	0.00
	z4	1(2)	1	1.00	0.00	0.00
	a5	1(1)	0	1.00	0.00	0.00
	b5	1(2)	1	1.00	0.00	0.00
	c5	1(1) 1(2)	1	1.00	0.00	0.00
	d5	1(1) 2(3)	35	1.00	0.00	0.01
	e5, f5	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	g5	1(1) 1(4)	23	1.00	0.00	0.00
	h5	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	i5, j5	1(1) 1(4)	23	1.00	0.00	0.00
	k5	1(1)	0	1.00	0.00	0.00
	l5	1(4)	23	1.00	0.00	0.00
	m5	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	n5	1(1) 1(4)	23	1.00	0.00	0.00
	o5	2(2)	3	1.00	0.00	0.01
	p5	2(1) 1(2)	1	1.00	0.00	0.00
	q5, r5	35(1) 7(2) 2(3)	4607	1.00	23.80	0.35
	s5	6(1) 2(2)	3	1.00	0.00	0.01
	t5	2(1) 1(3) 1(4)	143	1.00	0.01	0.00
	u5	4(1) 4(2) 1(3)	95	1.00	0.02	0.02
	v5	3(1) 1(2) 1(3) 2(4)	6911	1.00	0.02	0.00
	w5	8(1) 1(2) 1(4)	47	1.00	0.01	0.00
	x5	1: 2(1) 2(2) 2(3) 2: 4(1) 1(2) 6(1)7(5411)-30242.5(0.0)11(0)-2362.5(0.0)132(()-2-1.566 .0)61459.8(				
		1p5	2(1) 2(	1	1.0(	

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 17 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
rot (Cont'd)	m6	12(1) 3(2) 1(4)	191	1.00	0.22	0.01
	n6	10(1) 3(2) 1(4)	191	1.00	0.05	0.01
	o6	13(1) 2(2) 1(4)	95	1.00	0.08	0.01
	p6	15(1) 2(2) 1(4)	95	1.00	0.26	0.01
	q6	1(2)	1	1.00	0.00	0.00
	r6	1(1) 3(2) 1(5)	959	1.00	0.00	0.01
	s6	3(1)	0	1.00	0.00	0.00
	t6	6(1) 1(5)	119	1.00	0.01	0.00
	u6	1: 1(1) 3(2) 2: 2(1) 2(2)	15	2.00	0.01	0.02
	v6	1(2)	1	1.00	0.00	0.00
	w6	1(1)	0	1.00	0.00	0.00
	x6	1(2)	1	1.00	0.01	0.00
	y6	1(1)	0	1.00	0.00	0.00
	z6	1(2)	1	1.00	0.00	0.00
	a7, b7	1(1)	0	1.00	0.00	0.00
	c7	4(1) 2(2) 1(3) 1(4)	575	1.00	0.02	0.00
	d7	4(1) 4(2) 1(4)	383	1.00	0.02	0.03
	e7	1(2) 1(4)	47	1.00	0.00	0.00
	f7	1(1) 1(3)	5	1.00	0.00	0.00
	g7	4(1) 2(2)	3	1.00	0.01	0.00
	h7	3(1) 3(2)	7	1.00	0.00	0.01
	i7	1(3)	5	1.00	0.00	0.00
	j7	12(1) 1(2) 1(3)	11	1.00	0.03	0.00
	k7	1(1) 1(3)	5	1.00	0.00	0.00
	l7	28(1) 6(2) 1(3)	383	1.00	4.13	0.13
	m7	17(1) 7(2) 2(3)	4607	1.00	0.61	0.12
	n7	17(1) 7(2) 1(3)	767	1.00	0.56	0.10
	o7	17(1) 7(2) 2(3)	4607	1.00	0.65	0.13
	p7	36(1) 9(2) 2(3)	18431	1.00	10.51	0.43
	q7	15(1) 1(2) 1(3)	11	1.00	0.07	0.00
	r7	14(1) 2(2) 1(3)	23	1.00	0.08	0.00
	s7, t7, u7, v7, w7	1(1)	0	1.00	0.00	0.00
	x7	1(1) 2(2)	3	1.00	0.00	0.01
	y7	1(2)	1	1.00	0.00	0.00
	z7	3(1)	0	1.00	0.00	0.00
	a8	14(1) 1(2) 1(3)	11	1.00	0.42	0.00
	b8	12(1) 1(6)	719	1.00	0.04	0.00
	c8	2(1) 1(3)	5	1.00	0.00	0.00
	d8	11(1) 1(2) 1(3)	11	1.00	0.02	0.00
	e8	1(2)	1	1.00	0.00	0.00
	f8, g8	1(1)	0	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 18 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
rot (Cont'd	h8	1(1) 1(4)	23	1.00	0.00	0.00
sct	t	3(1)	0	1.00	0.00	0.00
	u	2(1) 1(2)	1	1.00	0.00	0.00
	v	1(1) 2(2)	3	1.00	0.00	0.00
	w	2(1) 2(2)	3	1.00	0.00	0.00
	x	3(1) 2(2)	3	1.00	0.00	0.00
	y	3(1) 1(2) 1(3)	11	1.00	0.01	0.00
	z	3(1) 1(2) 1(4)	47	1.00	0.00	0.00
	a0	3(1) 1(2) 1(5)	239	1.00	0.00	0.00
	b0	3(1) 1(2) 1(6)	1439	1.00	0.01	0.00
	c0	3(1) 1(2) 1(7)	10079	1.00	0.00	0.00
	d0	4(1) 1(2) 2(4)	1151	1.00	0.02	0.01
	e0	1(1)	0	1.00	0.00	0.00
	f0	2(1) 1(2)	1	1.00	0.00	0.00
	g0	1(1)	0	1.00	0.00	0.00
	h0	1(2)	1	1.00	0.00	0.00
t481	v16.0	1: 8(2) 2: 4(2) 3: 2[2] 4: 1(2)	8191	16.00	0.02	0.45
term1	j0	1(1)	0	1.00	0.00	0.00
	k0	2(1) 1(2)	1	1.00	0.00	0.00
	l0	5(1) 2(2)	3	1.00	0.00	0.03
	m0	1: 6(2) 1(4) 2: 2(1) 1(5)	10E5.26	120.00	0.02	0.28
	n0	1: 7(2) 1(3) 2: 3(1) 1(5)	92159	120.00	0.03	0.38
	o0	1: 1(1) 7(2) 1(3) 2: 4(1) 1(5)	92159	120.00	0.19	0.35
	p0	1: 1(1) 6(2) 2(3) 2: 4(1) 1(5)	10E5.44	120.00	0.04	0.33
	q0	1: 1(1) 6(2) 1(3) 1(4) 2: 4(1) 1(5)	10E6.04	120.00	0.05	0.28
	r0	14(1) 2(2)	3	1.00	0.19	0.03
	s0	12(1) 3(2)	7	1.00	0.32	0.06
too_large	n0	1: 27(1) 3(2) 1(3) 2: 29(1) 1(2)	95	2.00	2.63	0.03
	o0	1: 25(1) 4(2) 1(3) 2: 28(1) 1(2)	191	2.00	1.31	0.01
	p0	1: 24(1) 4(2) 1(3) 2: 27(1) 1(2)	191	2.00	1.81	0.04
ttt2	z	6(1)	0	1.00	0.00	0.00
	a0	8(1)	0	1.00	0.00	0.00
	b0	5(1) 1(2)	1	1.00	0.01	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 19 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
ttt2 (Cont'd)	c0	8(1)	0	1.00	0.00	0.00
	d0, e0	6(1)	0	1.00	0.00	0.00
	f0	1(2)	1	1.00	0.00	0.00
	g0	1(1) 2(2)	3	1.00	0.00	0.00
	h0	2(1) 1(2)	1	1.00	0.00	0.00
	i0	3(1) 1(2)	1	1.00	0.00	0.00
	j0	3(1) 1(4)	23	1.00	0.00	0.00
	k0	4(1) 1(2) 1(4)	47	1.00	0.00	0.00
	l0	3(1) 1(2) 1(4)	47	1.00	0.01	0.00
	m0	4(1) 1(2) 1(4)	47	1.00	0.01	0.00
	n0	3(1) 2(4)	575	1.00	0.00	0.01
	o0	3(1) 1(2) 1(4) 1(5)	5759	1.00	0.00	0.00
	p0	3(1) 1(4) 1(6)	17279	1.00	0.00	0.00
	q0	3(1) 1(2) 1(4) 1(5)	5759	1.00	0.01	0.00
	r0	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	s0, t0	1(1) 1(2)	1	1.00	0.00	0.00
unreg	l0, m0, n0, o0, p0, q0, r0, s0, t0, u0, v0, w0, x0, y0, z0, a1	6(1)	0	1.00	0.00	0.00
vda	r	11(1) 2(2)	3	1.00	0.02	0.01
	s	11(1) 1(2)	1	1.00	0.02	0.00
	t	11(1) 2(2)	3	1.00	0.10	0.01
	u	6(1)	0	1.00	0.00	0.00
	v	10(1)	0	1.00	0.01	0.00
	w, x	11(1) 2(2)	3	1.00	0.01	0.01
	y	10(1)	0	1.00	0.01	0.00
	z	1(5)	119	1.00	0.00	0.00
	a0	6(1) 1(2)	1	1.00	0.00	0.00
	b0	1(7)	5039	1.00	0.00	0.00
	c0	1(6)	719	1.00	0.00	0.00
	d0	15(1)	0	1.00	0.03	0.00
	e0	14(1) 1(2)	1	1.00	0.12	0.00
	f0	17(1)	0	1.00	0.03	0.00
	g0	13(1) 1(2)	1	1.00	0.03	0.00
	h0	17(1)	0	1.00	0.12	0.00
	i0	16(1)	0	1.00	0.03	0.00
	j0	11(1) 1(2)	1	1.00	0.02	0.00
	k0	15(1)	0	1.00	0.10	0.00
	l0, m0	10(1) 1(2)	1	1.00	0.01	0.00
	n0	9(1) 2(2)	3	1.00	0.01	0.00
	o0	16(1)	0	1.00	0.11	0.00
	p0	7(1) 1(2) 1(3)	11	1.00	0.02	0.00
	q0	10(1) 1(2) 1(3)	11	1.00	0.02	0.00

Table 1: **Symmetry characterization of the multi-level MCNC benchmarks (Sheet 20 of 22).**

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
vda (Cont'd)	r0	10(1) 2(2)	3	1.00	0.01	0.01
	s0	10(1) 1(2) 1(3)	11	1.00	0.01	0.00
	t0	13(1) 2(2)	3	1.00	0.11	0.00
	u0	9(1) 1(2) 1(3)	11	1.00	0.02	0.00
	v0	8(1) 2(2)	3	1.00	0.01	0.01
	w0	7(1) 2(2)	3	1.00	0.00	0.00
	x0, y0	9(1) 1(2) 1(3)	11	1.00	0.00	0.00
	z0, a1	1(6)	719	1.00	0.00	0.00
	b1	1(8)	40319	1.00	0.00	0.00
	c1, d1	1(6)	719	1.00	0.00	0.00
x1	a1	2(1) 1(2) 2(3)	71	1.00	0.00	0.00
	b1	3(1) 1(2) 2(3) 1(4)	1727	1.00	0.02	0.00
	c1	1(1) 1(2)	1	1.00	0.00	0.00
	d1	3(1) 1(2) 1(4)	47	1.00	0.00	0.00
	e1	2(1) 4(2) 1(3)	95	1.00	0.01	0.01
	f1	3(1) 1(2) 1(5)	239	1.00	0.00	0.00
	g1	6(1) 5(2) 1(3)	191	1.00	0.16	0.03
	h1	1: 3(2) 1(5) 2: 2(1) 1(2)	1919	2.00	0.00	0.00
	i1, j1	1(1)	0	1.00	0.00	0.00
	k1	8(1) 6(2) 1(5)	7679	1.00	0.25	0.04
	l1	12(1) 4(2)	15	1.00	0.23	0.02
	m1	1(2) 1(10)	10E6.86	1.00	0.00	0.00
	n1	14(1) 3(2) 1(3)	47	1.00	0.38	0.03
	o1	4(1) 2(2) 1(3)	23	1.00	0.01	0.00
	p1, q1	1(1)	0	1.00	0.00	0.00
	r1	4(1) 2(3) 1(6)	25919	1.00	0.01	0.00
	s1	1(3)	5	1.00	0.00	0.00
	t1, u1	1(1)	0	1.00	0.00	0.00
	v1	6(1) 1(3)	5	1.00	0.01	0.00
	w1	2(2)	3	1.00	0.00	0.00
	x1, y1	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	z1, a2	1(2)	1	1.00	0.00	0.00
	b2	2(2) 1(7)	40319	2.00	0.00	0.01
	c2	1(1)	0	1.00	0.00	0.00
	d2, e2	4(1) 1(2) 1(3) 1(4)	287	1.00	0.01	0.00
	f2, g2	3(1) 5(2) 1(3) 1(4)	36863	6.00	0.06	0.01
	h2, i2	5(1) 3(2) 2(5)	10E5.06	1.00	0.14	0.01
x2	k	1(3)	5	1.00	0.00	0.00
	l	1(1) 1(2)	1	1.00	0.00	0.00
	m	1(3)	5	1.00	0.00	0.00
	n	1(6)	719	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 21 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
x2 (Cont'd)	o	2(2)	3	1.00	0.00	0.00
	p, q	5(1) 1(2) 1(3)	11	1.00	0.01	0.00
x3	i4	1(2)	1	1.00	0.00	0.00
	j4	2(1) 1(3) 1(4)	143	1.00	0.00	0.00
	k4	16(1) 4(2)	15	1.00	4.12	0.33
	l4	2(1) 1(2)	1	1.00	0.00	0.00
	m4, n4, o4, p4, q4, r4, s4, t4, u4, v4, w4, x4, y4, z4, a5, b5, c5, d5, e5, f5, g5, h5, i5, j5, k5, l5, m5, n5, o5, p5, q5, r5	3(1) 1(2)	1	1.00	0.00	0.00
	s5, t5	1(1) 1(4)	23	1.00	0.00	0.00
	u5	14(1) 3(2)	7	1.00	1.04	0.11
	v5	10(1) 2(2) 1(6)	2879	1.00	0.14	0.01
	w5	1(1) 1(2) 1(9)	10E5.86	1.00	0.00	0.00
	x5	1(1) 1(2) 1(10)	10E6.86	1.00	0.00	0.00
	y5	3(2)	15	2.00	0.00	0.03
	z5	11(1) 1(2) 1(6)	1439	1.00	0.19	0.00
	a6	12(1) 1(2) 1(6)	1439	1.00	0.20	0.00
	b6, c6	1(2)	1	1.00	0.00	0.00
	d6	6(1) 1(4)	23	1.00	0.01	0.00
	e6	4(1) 1(2) 1(5)	239	1.00	0.00	0.00
	f6	3(1) 1(3) 1(5)	719	1.00	0.00	0.00
	g6	2(1) 1(4)	23	1.00	0.00	0.00
	h6	3(1) 1(4)	23	1.00	0.00	0.00
	i6	2(1) 1(2) 1(4)	47	1.00	0.00	0.00
	j6	2(1) 1(3) 1(4)	143	1.00	0.00	0.00
	k6	2(1) 2(4)	575	1.00	0.01	0.01
	l6	8(1) 1(3)	5	1.00	0.01	0.00
	m6	7(1) 1(2) 1(3)	11	1.00	0.00	0.00
	n6, o6, p6, q6, r6, s6, t6, u6, v6, w6, x6, y6, z6, a7, b7, c7	3(1) 1(2)	1	1.00	0.00	0.00
	d7, e7, f7, g7, h7, i7, j7, k7, l7, m7, n7, o7, p7, q7, r7	10(1) 1(2)	1	1.00	0.01	0.00
	s7	9(1) 1(2)	1	1.00	0.01	0.00
	t7	1(3)	5	1.00	0.00	0.00
	u7	2(2)	3	1.00	0.00	0.00
	v7	1(1) 2(2)	3	1.00	0.00	0.00
	w7	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	x7	1(1) 1(2) 1(4)	47	1.00	0.00	0.00
	y7	1(1) 1(2) 1(5)	239	1.00	0.01	0.00
	z7	1(1) 1(2) 1(6)	1439	1.00	0.00	0.00
	a8	1(1) 1(2) 1(7)	10079	1.00	0.01	0.00
	b8	1(1) 1(2) 1(8)	80639	1.00	0.00	0.00

Table 1: Symmetry characterization of the multi-level MCNC benchmarks (Sheet 22 of 22).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU Time, s	
			Max	H Ratio	Flat	Hier.
x3 (Cont'd)	c8	1(1) 3(2)	7	1.00	0.00	0.02
x4	w2, x2, y2, z2, a3, b3	1(1)	0	1.00	0.00	0.00
	c3	2(1) 2(2) 1(5)	479	1.00	0.00	0.01
	d3	6(1) 1(4)	23	1.00	0.00	0.00
	e3	2(1) 1(6)	719	1.00	0.01	0.00
	f3	3(1) 1(2) 1(4)	47	1.00	0.00	0.00
	g3	1(1) 1(2) 1(6)	1439	1.00	0.00	0.00
	h3, i3, j3, k3, l3, m3	1(3)	5	1.00	0.00	0.00
	n3, o3, p3, q3, r3, s3, t3, u3	1(2)	1	1.00	0.01	0.00
	v3	7(1) 1(3) 1(5)	719	1.00	0.02	0.00
	w3	2(4)	575	1.00	0.00	0.01
	x3	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	y3, z3, a4, b4, c4, d4	3(1) 1(6)	719	1.00	0.00	0.00
	e4	2(1) 1(6)	719	1.00	0.00	0.00
	f4	2(1) 1(2) 1(5)	239	1.00	0.00	0.00
	g4	4(1) 1(2) 2(3)	71	1.00	0.01	0.01
	h4	3(1) 1(2) 1(3) 1(4)	287	1.00	0.00	0.00
	i4, j4, k4, l4, m4, n4, o4, p4, q4, r4, s4, t4, u4, v4, w4, x4, y4, z4, a5, b5	4(1) 1(2) 1(3) 1(4)	287	1.00	0.01	0.00
	c5	3(1) 1(2) 1(3) 1(4)	287	1.00	0.00	0.00
	d5	3(1) 1(2)	1	1.00	0.01	0.00
	e5, f5	5(1) 1(2)	1	1.00	0.00	0.00
	g5	2(1) 1(2) 1(3)	11	1.00	0.01	0.00
	h5	3(1) 1(2) 1(3)	11	1.00	0.00	0.00
	i5	2(1) 2(2) 1(3)	23	1.00	0.00	0.01
	j5	2(1) 1(2) 2(3)	71	1.00	0.00	0.02
	k5	2(1) 1(2) 1(3) 1(4)	287	1.00	0.00	0.00
	l5	3(1) 1(2)	1	1.00	0.00	0.00
	m5	2(1) 1(2)	1	1.00	0.00	0.00
	n5	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	o5	4(1) 1(2) 1(5)	239	1.00	0.01	0.00
z4ml	24, 25	2(2) 1(3)	23	1.00	0.00	0.01
	26	1(2) 1(3)	11	1.00	0.00	0.00
	27	1(3)	5	1.00	0.00	0.00

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 1 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C1355 C499	1324GAT(583), 1325GAT(579), 1326GAT(575), 1327GAT(571), 1328GAT(584), 1329GAT(580), 1330GAT(576), 1331GAT(572), 1332GAT(585), 1333GAT(581), 1334GAT(577), 1335GAT(573), 1336GAT(586), 1337GAT(582), 1338GAT(578), 1339GAT(574), 1340GAT(567), 1341GAT(563), 1342GAT(559), 1343GAT(555), 1344GAT(568), 1345GAT(564), 1346GAT(560), 1347GAT(556), 1348GAT(569), 1349GAT(565), 1350GAT(561), 1351GAT(557), 1352GAT(570), 1353GAT(566), 1354GAT(562), 1355GAT(558)	41(1)	0	1.00	36.08	0.00
C17	22	4(1)	0	1.00	0.00	0.00
	23	1: 2(2) 2: 1(2)	3	2.00	0.00	0.00
C1908	3(865), 6(864), 9(863), 12(862), 30(856), 45(851), 48(850), 15(861), 18(860), 21(859), 24(858), 27(857), 33(855), 36(854), 39(853), 42(852)	32(1)	0	1.00	5.40	0.00
	75(866)	31(1)	0	1.00	7.16	0.00
	51(899)	8(1) 2(2) 1(4) 1(12)	10E10.66	1.00	0.16	0.02
	54(900)	8(1) 2(2) 1(4) 1(11)	10E9.58	1.00	0.12	0.00
	60(901)	12(1) 1(3) 1(8)	10E5.38	1.00	0.24	0.00
	63(902)	9(1) 1(2) 1(4) 1(6)	34559	1.00	0.08	0.00
	66(903)	12(1) 1(3) 1(7)	30239	1.00	0.12	0.00
	69(908), 72(909)	32(1)	0	1.00	6.19	0.00
	57(912)	8(1) 1(3) 1(4) 1(10)	10E8.71	1.00	0.11	0.00
C3540	353(405)	1(4)	23	1.00	0.00	0.00
	355(399)	1(1) 1(2)	1	1.00	0.00	0.00
	361(940)	1: 13(1) 3(2) 2: 14(1) 1(2)	15	2.00	0.05	0.13
	358(1161), 351(1247)	1(8)	40319	1.00	0.00	0.00
	372(1243), 369(1321)	35(1)	0	1.00	11.79	0.00
	399(1428)	24(1) 1(2)	1	1.00	1.48	0.00
	364(1484)	21(1) 2(2)	3	1.00	0.79	0.02
	396(1504)	33(1) 1(2)	1	1.00	0.81	0.00
	384(1553)	37(1) 1(2)	1	1.00	6.20	0.00
	367(1585)	38(1)	0	1.00	15.85	0.00
	387(1616)	36(1) 1(2)	1	1.00	4.10	0.00

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 2 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C3540 (Cont'd)	393(1605)	34(1) 1(2) 1(4)	47	1.00	2.38	0.00
	390(1603)	35(1) 1(2)	1	1.00	2.67	0.00
	378(1597)	42(1)	0	1.00	16.06	0.00
	375(1624)	44(1)	0	1.00	20.99	0.00
	381(1626)	38(1) 1(2)	1	1.00	11.17	0.00
	407(1657), 409(1670)	49(1)	0	1.00	34.83	0.00
	405(1717)	50(1)	0	1.00	63.94	0.00
	402(1718)	49(1)	0	1.00	49.44	0.00
C432	223GAT(84)	1: 9(2) 2: 1(9)	10E8.26	10E5.55	0.02	0.03
	329GAT(133)	1: 9[3] 2: 1(9)	10E5.55	10E5.55	0.69	5.13
	370GAT(163)	1: 9[3] 9(1) 2: 1(9)	10E5.55	10E5.55	3.79	40.69
	421GAT(188),	1: 15(1) 7[3] 2: 1(7)	5039	5040	2.68	27.72
	430GAT(193)	1: 9(1) 4[3] 5[3] 2: 1(4) 1(5)	2879	2880	5.12	43.94
	431GAT(194)	1: 15(1) 2[2] 2[2] 3[3] 2: 2(2) 1(3)	23	24	5.65	43.94
	432GAT(195)	1: 2[3] 2[6] 2: 2(2)	3	4	6.30	103.7
C5315	144(354), 298(299), 973(202), 594(224), 599(269), 600(259)	1(1)	0	1.00	0.00	0.00
	601(220)	1(2)	1	1.00	0.01	0.00
	602(222), 603(225), 604(223), 611(275), 612(263)	1(1)	0	1.00	0.00	0.00
	810(356)	1(2)	1	1.00	0.01	0.00
	848(330), 849(219), 850(217), 851(218)	1(1)	0	1.00	0.00	0.00
	634(665), 815(627), 845(845), 847(465)	1(2)	1	1.00	0.00	0.00
	926(624), 923(619), 921(664), 892(408), 887(528), 606(407)	1(1)	0	1.00	0.00	0.00
	656(621)	1(3)	5	1.00	0.00	0.00
	809(655)	1(2)	1	1.00	0.00	0.00
	993(850), 978(851), 949(852), 939(853), 889(734), 593(733)	1(1)	0	1.00	0.00	0.00
	636(1280), 704(1281), 717(1282)	3(1) 1(2)	1	1.00	0.00	0.00
	820(1283)	1(3)	5	1.00	0.00	0.00
	639(1275), 673(1276), 707(1277), 715(1278)	4(1) 1(2)	1	1.00	0.01	0.00
	598(1623)	20(1) 1(2)	1	1.00	0.49	0.00
	610(1519)	26(1)	0	1.00	0.97	0.00

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 3 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C5315 (Cont'd)	588(1696)	28(1)	0	1.00	0.35	0.00
	615(1750), 626(1752)	18(1) 2(3)	35	1.00	0.08	0.02
	632(1692)	28(1)	0	1.00	0.38	0.00
	1002(1920)	1(9)	10E5.55	1.00	0.00	0.00
	1004(1977)	1(10)	10E15.10	1.00	0.01	0.00
	591(1894)	28(1)	0	1.00	0.50	0.00
	618(1925)	16(1) 4(2)	15	1.00	0.10	0.12
	621(1893)	28(1)	0	1.00	0.39	0.00
	629(1926)	16(1) 4(2)	15	1.00	0.09	0.12
	822(1933)	9(1)	0	1.00	0.01	0.00
	838(2064)	14(1)	0	1.00	0.02	0.00
	861(2070)	12(1)	0	1.00	0.02	0.00
	623(2152)	25(1)	0	1.00	0.24	0.00
	722(2131)	23(1)	0	1.00	0.14	0.00
	832(2133)	22(1)	0	1.00	0.06	0.00
	834(2123), 836(2128)	17(1)	0	1.00	0.11	0.00
	859(2132)	23(1)	0	1.00	0.13	0.00
	871(2127)	22(1) 1(2)	1	1.00	0.15	0.00
	873(2124)	19(1) 1(2)	1	1.00	0.05	0.00
	875(2125)	16(1) 1(2)	1	1.00	0.11	0.00
	877(2126)	13(1) 1(2)	1	1.00	0.02	0.00
	998(2163)	1(1) 1(9) 1(10)	10E12.11	1.00	0.01	0.00
	1000(2168)	1(1) 2(10)	10E13.11	1.00	0.01	0.03
	575(2240)	27(1) 1(2)	1	1.00	0.60	0.00
	585(2236)	21(1) 2(2)	3	1.00	0.13	0.01
	661(2178), 693(2179)	24(1)	0	1.00	0.06	0.00
	747(2187)	42(1) 1(2)	1	1.00	0.61	0.00
	752(2189)	36(1) 1(2)	1	1.00	0.39	0.00
	757(2190)	31(1) 1(2)	1	1.00	0.30	0.00
	762(2184)	25(1) 1(2)	1	1.00	0.22	0.00
	787(2186)	42(1) 1(2)	1	1.00	0.59	0.00
	792(2188)	36(1) 1(2)	1	1.00	0.39	0.00
	797(2191)	31(1) 1(2)	1	1.00	0.30	0.00
	802(2183)	25(1) 1(2)	1	1.00	0.23	0.00
	642(2222)	43(1) 1(2)	1	1.00	0.63	0.00
	664(2223)	26(1) 1(2)	1	1.00	0.17	0.00
	667(2224)	32(1) 1(2)	1	1.00	0.32	0.00
	670(2225)	37(1) 1(2)	1	1.00	0.49	0.00
	676(2229)	43(1) 1(2)	1	1.00	0.73	0.00
	696(2226)	26(1) 1(2)	1	1.00	0.17	0.00
	699(2227)	32(1) 1(2)	1	1.00	0.32	0.00
	702(2228)	37(1) 1(2)	1	1.00	0.49	0.00

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 4 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C5315 (Cont'd)	818(2273)	31(1) 1(2)	1	1.00	0.34	0.00
	813(2260)	24(1)	0	1.00	0.22	0.00
	824(2274)	30(1)	0	1.00	0.36	0.00
	826(2275), 828(2233)	28(1)	0	1.00	0.25	0.00
	830(2182)	25(1)	0	1.00	0.18	0.00
	854(2268)	2(1) 1(6) 1(9) 3(10)	10E28.09	1.00	0.81	0.09
	863(2276)	34(1) 1(2)	1	1.00	0.56	0.00
	865(2277)	31(1) 1(2)	1	1.00	0.43	0.00
	867(2237)	28(1) 1(2)	1	1.00	0.25	0.00
	869(2181)	25(1) 1(2)	1	1.00	0.28	0.00
	712(2297), 727(2298)	64(1) 1(2)	1	1.00	2.12	0.00
	732(2300)	61(1) 1(2)	1	1.00	1.80	0.00
	737(2279)	58(1) 1(2)	1	1.00	1.59	0.00
	742(2238)	52(1) 1(2)	1	1.00	1.07	0.00
	772(2299)	61(1) 1(2)	1	1.00	1.80	0.00
	777(2278)	58(1) 1(2)	1	1.00	1.47	0.00
	782(2239)	52(1) 1(2)	1	1.00	1.17	0.00
	645(2271)	53(1) 1(2)	1	1.00	1.13	0.00
	648(2295)	59(1) 1(2)	1	1.00	1.65	0.00
	651(2314)	62(1) 1(2)	1	1.00	1.99	0.00
	654(2315)	65(1) 1(2)	1	1.00	2.19	0.00
	679(2272)	53(1) 1(2)	1	1.00	1.25	0.00
	682(2296)	59(1) 1(2)	1	1.00	1.66	0.00
	685(2316)	62(1) 1(2)	1	1.00	1.88	0.00
	688(2317)	65(1) 1(2)	1	1.00	2.32	0.00
	843(2455)	32(1)	0	1.00	1.05	0.00
	882(2456)	36(1)	0	1.00	2.20	0.00
	767(2479), 807(2480)	66(1)	0	1.00	19.03	0.00
	658(2483), 690(2484)	67(1)	0	1.00	20.53	0.00
C880	388GAT(133), 389GAT(132), 390GAT(131)	1(3)	5	1.00	0.00	0.00
	391GAT(124)	1(2)	1	1.00	0.00	0.00
	418GAT(168)	1(4)	23	1.00	0.00	0.00
	419GAT(164)	1(3) 1(4)	143	1.00	0.00	0.00
	420GAT(158), 421GAT(162), 422GAT(161)	1(3)	5	1.00	0.00	0.00
	423GAT(155)	1(1) 1(2)	1	1.00	0.00	0.00
	446GAT(183)	1(7)	5039	1.00	0.00	0.00
	447GAT(182)	1(3)	5	1.00	0.00	0.00
	448GAT(179)	1(6)	719	1.00	0.00	0.00
	449GAT(176)	1(7)	5039	1.00	0.00	0.00
	450GAT(173)	1(1) 1(2)	1	1.00	0.00	0.00
	767GAT(349), 768GAT(334)	1(10)	10E15.10	1.00	0.00	0.01

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 5 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C880 (Cont'd)	850GAT(404)	1: 18(1) 2(2) 1(3) 1(4) 2: 2(1) 1(2)	1151	2.00	0.56	0.03
	863GAT(424)	25(1) 2(2) 1(3) 1(4)	575	1.00	4.16	0.02
	864GAT(423)	1: 22(1) 3(2) 1(3) 1(4) 2: 25(1) 1(2)	2303	2.00	1.91	0.05
	865GAT(422)	1: 19(1) 3(2) 1(3) 1(4) 2: 22(1) 1(2)	2302	2.00	1.02	0.04
	866GAT(426)	31(1) 1(2) 1(3)	11	1.00	9.02	0.00
	874GAT(433)	27(1) 3(2) 1(3) 1(4)	1151	1.00	10.57	0.06
	878GAT(442)	37(1) 2(2) 1(4)	95	1.00	61.31	0.18
	879GAT(441)	33(1) 2(2) 1(3) 1(4)	575	1.00	44.37	0.13
	880GAT(440)	31(1) 2(2) 1(3) 1(4)	575	1.00	21.68	0.06
C7552	2(313), 3(312), 450(288), 448(284), 444(282), 442(280), 440(277), 438(274), 496(271), 494(267), 492(265), 490(263), 488(260), 486(258), 484(256), 482(253), 480(250), 560(248), 542(246), 558(244), 556(242), 554(240), 552(238), 550(236), 548(234), 546(232), 544(230), 540(227), 538(224), 536(222), 534(220), 532(218), 530(216), 528(214), 526(212), 524(210), 279(304), 436(286), 478(269), 522(226)	1(1)	0	1.00	0.00	0.00
	402(395)	1(2)	1	1.00	0.00	0.00
	404(390), 406(388), 408(385), 410(387)	1(4)	23	1.00	0.00	0.00
	432(428), 446(393)	1(1)	0	1.00	0.01	0.00
	284(384)	1(2)	1	1.00	0.00	0.00
	286(419)	1(1)	0	1.00	0.00	0.00
	289(383)	1(2)	1	1.00	0.01	0.00
	292(392)	1(3)	5	1.00	0.01	0.00
	341(420)	1(1)	0	1.00	0.00	0.00
	281(547)	1(3)	5	1.00	0.00	0.00
	453(596)	1(1)	0	1.00	0.00	0.00
	278(536)	1(2)	1	1.00	0.01	0.00
	373(2994)	2(1) 1(2)	1	1.00	0.01	0.00
	246(3110)	87(1) 3(2)	7	1.00	24.60	0.11
	258(3122), 264(3121)	23(1) 45(2) 1(10)	10E20.10	1.00	35.45	29.16
	270(3109)	87(1) 3(2)	7	1.00	24.77	0.31
	388(3093)	14(1) 1(2)	1	1.00	0.04	0.00
	391(3094)	11(1) 1(2)	1	1.00	0.02	0.00
	394(3095)	8(1) 1(2)	1	1.00	0.01	0.00
	397(3097)	5(1) 1(2)	1	1.00	0.01	0.00

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 6 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C7552 (Cont'd)	376(3206)	26(1) 1(2)	1	1.00	0.34	0.00
	379(3207)	23(1) 1(2)	1	1.00	0.24	0.00
	382(3148)	20(1) 1(2)	1	1.00	0.11	0.00
	385(3151)	17(1) 1(2)	1	1.00	0.16	0.00
	412(3369)	1(1) 1(2) 3(5) 1(7) 4(10)	10E37.08	4.00	0.39	1.87
	414(3338)	1: 1(1) 2(8) 6(10) 2: 3(1) 2(3) 3: 3(1) 1(2)	10E36.94	72.00	0.66	12.82
	416(3368)	1: 1(1) 1(2) 3(5) 1(7) 4(10) 2: 8(1) 1(2)	10E34.40	4.00	0.34	1.75
	249(3418)	23(1) 45(2) 1(10)	10E9.33	1.00	35.60	29.42
	295(3352)	56(1) 1(2)	1	1.00	3.65	0.00
	324(3363)	77(1) 2(2)	3	1.00	13.02	0.03
	252(3450)	2(1) 35(2)	10E9.33	1.00	5.21	17.19
	276(3401)	87(1) 3(2)	7	1.00	25.18	0.10
	310(3393)	67(1) 2(2)	3	1.00	7.83	0.03
	313(3396)	65(1) 1(2)	1	1.00	5.96	0.00
	316(3397)	62(1) 1(2)	1	1.00	5.09	0.00
	319(3398)	59(1) 1(2)	1	1.00	4.29	0.00
	327(3408)	85(1) 2(2)	3	1.00	19.95	0.03
	330(3411)	83(1) 2(2)	3	1.00	17.67	0.04
	333(3416)	81(1) 2(2)	3	1.00	15.21	0.03
	336(3412)	79(1) 2(2)	3	1.00	14.31	0.03
	418(3449)	1: 1(1) 1(2) 5(5) 2(7) 2(8) 12(10) 1(16) 2: 10(1) 1(2) 1(5) 1(7)	10E125.4	10E6.08	65.95	9.58
	273(3402)	87(1) 3(2)	7	1.00	24.68	0.12
	298(3387)	75(1) 2(2)	3	1.00	11.46	0.03
	301(3388)	73(1) 2(2)	3	1.00	10.34	0.04
	304(3390)	71(1) 2(2)	3	1.00	9.56	0.04
	307(3389)	69(1) 2(2)	3	1.00	8.48	0.04
	344(3382)	29(1) 1(2)	1	1.00	0.43	0.00
	422(3451), 469(3452)	89(1) 2(2)	3	1.00	24.71	0.03
	419(3444), 471(3445)	86(1) 3(2)	7	1.00	22.72	0.12
	359(3426)	41(1) 1(2)	1	1.00	1.25	0.00
	362(3429)	38(1) 1(2)	1	1.00	1.01	0.00
	365(3430)	35(1) 1(2)	1	1.00	0.76	0.00
	368(3431)	32(1) 1(2)	1	1.00	0.63	0.00
	347(3420)	53(1) 1(2)	1	1.00	2.90	0.00
	350(3421)	50(1) 1(2)	1	1.00	2.51	0.00
	353(3425)	47(1) 1(2)	1	1.00	2.07	0.00
	356(3424)	44(1) 1(2)	1	1.00	1.65	0.00
	321(3715)	75(1) 2(2)	3	1.00	12.26	0.03

Table 2: Symmetry characterization of the multi-level ISCAS85 benchmarks (Sheet 7 of 7).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
C7552 (Cont'd)	338(3716)	89(1) 2(2)	3	1.00	35.11	0.03
	370(3718)	53(1) 1(2)	1	1.00	7.83	0.00
	399(3717)	26(1) 1(2)	1	1.00	0.58	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 1 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
9sym	1	1(9)	10E5.56	1.00	0.00	0.00
5xp1	1	5(1) 1(2)	1	1.00	0.00	0.00
	2, 3	7(1)	0	1.00	0.01	0.00
	4	6(1)	0	1.00	0.00	0.00
	5	3(1) 1(2)	1	1.00	0.00	0.00
	6	2(1) 1(2)	1	1.00	0.01	0.00
	7	1(1) 1(2)	1	1.00	0.00	0.00
	8	1(2)	1	1.00	0.00	0.00
	9	1(1)	0	1.00	0.00	0.00
	10	2(2) 1(3)	23	1.00	0.00	0.00
apex1	1	5(1) 2(2) 1(3)	23	1.00	0.01	0.00
	2	1(2) 1(6)	1439	1.00	0.00	0.00
	3	Trivial output: no inputs				
	4	1(1) 1(2) 1(6)	1439	1.00	0.00	0.00
	5	4(1) 1(2) 1(4)	47	1.00	0.00	0.00
	6	5(1) 1(4)	23	1.00	0.00	0.00
	7	17(1) 1(2)	1	1.00	0.72	0.00
	8	1(1) 2(3) 1(5)	4319	1.00	0.01	0.00
	9	13(1) 3(2)	7	1.00	0.28	0.01
	10	6(1) 3(2)	7	1.00	0.00	0.00
	11	16(1) 1(2) 2(3)	71	1.00	0.17	0.01
	12	16(1) 1(8)	40319	1.00	0.12	0.00
	13	17(1) 1(6)	719	1.00	0.41	0.00
	14	23(1) 2(2) 1(3)	23	1.00	3.89	0.05
	15	25(1)	0	1.00	0.95	0.00
	16	24(1) 3(2) 1(8)	10E5.51	1.00	16.16	0.51
	17	27(1) 1(2) 2(3)	71	1.00	40.87	0.62
	18	20(1) 3(2) 2(4)	4607	1.00	3.30	0.07
	19	21(1) 1(3) 1(4)	143	1.00	2.69	0.00
	20	21(1) 2(2) 1(8)	10E5.21	1.00	2.25	0.06
	21	1(9)	10E5.56	1.00	0.00	0.00
	22	1(8)	40319	1.00	0.00	0.00
	23	28(1) 1(5) 1(6)	86399	1.00	30.01	0.00
	24	1(2) 1(6)	1439	1.00	0.00	0.00
	25	1(7)	5039	1.00	0.00	0.00
	26	15(1) 2(2)	3	1.00	0.15	0.00
	27	6(1) 1(4)	23	1.00	0.00	0.00
	28	15(1)	0	1.00	0.08	0.00
	29	18(1) 1(2) 1(8)	80639	1.00	0.72	0.00
	30	18(1) 1(2)	1	1.00	0.23	0.00
	31	6(1) 3(2) 1(3)	47	1.00	0.01	0.00
	32	4(1) 4(2)	15	1.00	0.00	0.01

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 2 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
apex1 (Cont'd)	33	1(8)	40319	1.00	0.00	0.00
	34	4(1) 2(2) 1(4)	95	1.00	0.01	0.01
	35	22(1) 3(2)	7	1.00	1.73	0.08
	36	23(1) 1(4)	23	1.00	3.46	0.00
	37	30(1) 2(2)	3	1.00	61.11	0.72
	38	26(1) 1(7)	5039	1.00	24.66	0.00
	39	21(1)	0	1.00	1.75	0.00
	40	16(1) 1(2)	1	1.00	0.32	0.00
	41	9(1) 2(2)	3	1.00	0.01	0.00
	42	1(11)	10E7.60	1.00	0.00	0.00
	43	Trivial output: no inputs				
	44	1(2) 1(6)	1439	1.00	0.00	0.00
	45	1(7)	5039	1.00	0.00	0.00
apex5	1	1(1)	0	1.00	0.00	0.00
	2	1(8)	40319	1.00	0.00	0.00
	3	2(1) 1(3)	5	1.00	0.00	0.00
	4, 5, 6, 7, 8, 9	14(1) 1(3)	5	1.00	0.04	0.00
	10, 11	12(1) 1(4)	23	1.00	0.08	0.00
	12, 13, 14, 15, 16, 17, 18, 19	3(1) 1(8)	40319	1.00	0.01	0.00
	20, 21	1(1)	0	1.00	0.00	0.00
	22, 23, 24, 25, 26, 27	3(1) 1(8)	40319	1.00	0.00	0.00
	28, 29	1(2)	1	1.00	0.00	0.00
	30, 31, 32	Trivial outputs: no inputs				
	33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48	6(1) 1(7)	5039	1.00	0.00	0.00
	49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64	3(1) 1(8)	40319	1.00	0.00	0.00
	65, 66	9(1) 2(5)	14399	1.00	0.12	0.01
	67, 68	5(1) 1(2) 1(5) 1(6)	10E5.24	1.00	0.02	0.00
	69, 70, 71	9(1) 2(5)	14399	1.00	0.06	0.01
	72	5(1) 1(2) 1(5) 1(6)	10E5.24	1.00	0.02	0.00
	73	3(1) 1(6) 1(15)	10E14.97	1.00	0.02	0.00
	74	1(1) 1(2) 1(7) 1(14)	10E14.94	1.00	0.02	0.00
	75	1(1) 1(2) 1(8) 1(13)	10E14.70	1.00	0.00	0.00
	76	1(1) 1(2) 1(9) 1(12)	10E14.54	1.00	0.02	0.00
	77	4(1) 1(4) 1(6) 1(11)	10E11.83	1.00	0.04	0.00
	78	2(1) 1(2) 1(4) 1(7) 1(10)	10E11.94	1.00	0.03	0.00
	79, 80	2(1) 1(2) 1(4) 1(8) 1(9)	10E10.94	1.00	0.03	0.00
	81	4(1) 1(6) 1(7) 1(8)	10E11.16	1.00	0.05	0.00
	82	2(1) 1(2) 1(6) 1(7) 1(8)	10E11.46	1.00	0.03	0.00
	83	2(1) 1(2) 1(5) 2(8)	10E6.98	1.00	0.03	0.00
	84	2(1) 1(2) 1(4) 1(8) 1(9)	10E9.33	1.00	0.01	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 3 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
apex5 (Cont'd)	85	4(1) 1(3) 1(6) 1(12)	10E12.31	1.00	0.13	0.00
	86	2(1) 2(2) 1(7) 1(12)	10E12.68	1.00	0.02	0.01
	87	3(1) 1(2) 1(8) 1(12)	10E13.58	1.00	0.02	0.00
	88	2(1) 1(2) 1(9) 1(12)	10E14.54	1.00	0.02	0.00
b12	v15.0	6(1)	0	1.00	0.00	0.00
	v15.1	5(1) 1(2)	1	1.00	0.00	0.00
	v15.2	8(1)	0	1.00	0.01	0.00
	v15.3	1: 2(2) 2: 1(2)	7	2.00	0.00	0.01
	v15.4	2(1) 1(3)	5	1.00	0.00	0.00
	v15.5	1(5)	119	1.00	0.00	0.00
	v15.6	3(1) 3(2)	7	1.00	0.00	0.01
	v15.7	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	v15.8	2(1) 1(2) 1(4)	47	1.00	0.01	0.00
bw	1	5(1)	0	1.00	0.00	0.00
	2, 3	3(1) 1(2)	1	1.00	0.00	0.00
	4	2(1) 1(2)	1	1.00	0.00	0.00
	5, 6	5(1)	0	1.00	0.00	0.00
	7, 8, 9, 10	3(1) 1(2)	1	1.00	0.00	0.00
	11	1(2) 1(3)	11	1.00	0.00	0.00
	12	5(1)	0	1.00	0.00	0.00
	13	3(1) 1(2)	1	1.00	0.01	0.00
	14	5(1)	0	1.00	0.00	0.00
	15	1(1) 2(2)	3	1.00	0.00	0.00
	16	5(1)	0	1.00	0.00	0.00
	17	3(1) 1(2)	1	1.00	0.00	0.00
	18	1(1) 2(2)	3	1.00	0.00	0.00
	19, 20	3(1) 1(2)	1	1.00	0.00	0.00
	21	2(1) 1(2)	1	1.00	0.00	0.00
	22	1(5)	119	1.00	0.00	0.00
	23, 24	3(1) 1(2)	1	1.00	0.00	0.00
	25	5(1)	0	1.00	0.00	0.00
	26	3(1) 1(2)	1	1.00	0.00	0.00
	27	1(1) 2(2)	3	1.00	0.00	0.00
	28	1(5)	119	1.00	0.01	0.00
clip	1	6(1) 1(3)	5	1.00	0.00	0.00
	2, 3	7(1) 1(2)	1	1.00	0.01	0.00
	4	9(1)	0	1.00	0.02	0.00
	5	7(1) 1(2)	1	1.00	0.00	0.00
con1	f0	6(1)	0	1.00	0.00	0.00
	f1	5(1)	0	1.00	0.00	0.00
cordic	d	1: 6(1) 3(3) 2(4) 2: 2(2) 7(1)	10E5.69	4.00	0.06	0.04

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 4 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
cordic (Cont'd)	dn	1: 4(1) 1(2) 3(3) 2(4) 2: 2(2) 6(1)	10E5.99	4.00	0.11	0.04
cps	1	12(1) 1(2) 1(3) 1(5)	1439	1.00	0.10	0.00
	2	14(1) 1(4)	23	1.00	0.17	0.00
	3	16(1) 1(2) 1(4)	47	1.00	0.13	0.00
	4, 5, 6, 7	1(7)	5039	1.00	0.00	0.00
	8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23	1(18)	10E15.80	1.00	0.01	0.00
	24	5(1) 1(2) 1(11)	10E7.90	1.00	0.00	0.00
	25	15(1) 1(4)	23	1.00	0.21	0.00
	26	2(2) 1(17)	10E14.85	1.00	0.01	0.02
	27	1(1) 1(2) 1(17)	10E14.85	1.00	0.01	0.00
	28	13(1) 1(2) 1(4)	47	1.00	0.05	0.00
	29	7(1)	0	1.00	0.00	0.00
	30	5(1) 1(2)	1	1.00	0.00	0.00
	31	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	32	11(1) 1(2) 1(5)	239	1.00	0.04	0.00
	33	1(17)	10E14.55	1.00	0.00	0.00
	34	14(1) 1(2) 1(4)	47	1.00	0.18	0.00
	35	1(1) 1(5) 1(13)	10E11.87	1.00	0.00	0.00
	36	7(1) 3(2) 1(3) 1(5)	5759	1.00	0.06	0.02
	37	2(1) 2(2) 3(4)	55295	1.00	0.01	0.04
	38, 39	18(1) 1(4)	23	1.00	0.24	0.00
	40	17(1) 1(4)	23	1.00	0.14	0.00
	41	18(1) 1(4)	23	1.00	0.28	0.00
	42	13(1) 2(2) 1(4)	95	1.00	0.09	0.00
	43	14(1) 1(4)	23	1.00	0.05	0.00
	44	2(2) 1(3) 1(11)	10E8.98	1.00	0.00	0.01
	45	12(1) 1(2) 1(4)	47	1.00	0.17	0.00
	46	6(1) 1(2) 1(3) 1(7)	60479	1.00	0.02	0.00
	47	1(2) 1(5) 1(11)	10E9.98	1.00	0.00	0.00
	48	6(1) 1(2) 1(8)	80639	1.00	0.01	0.00
	49, 50	1(17)	10E14.55	1.00	0.00	0.00
	51	9(1) 1(2) 1(6)	1439	1.00	0.02	0.00
	52	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
	53	6(1) 2(2) 1(3) 1(5)	2879	1.00	0.02	0.01
	54	11(1) 1(6)	719	1.00	0.02	0.00
	55	8(1) 1(2) 1(3) 1(5)	1439	1.00	0.03	0.00
	56	5(1) 2(3) 1(7)	10E5.26	1.00	0.01	0.01
	57	4(1) 1(14)	10E10.94	1.00	0.01	0.00
	58	5(1) 1(2) 1(3) 1(8)	10E5.68	1.00	0.10	0.00
	59	1(17)	10E14.55	1.00	0.01	0.00
	60	10(1) 2(4)	575	1.00	0.02	0.02

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 5 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
cps (Cont'd)	61, 62	1(2) 1(16)	10E13.32	1.00	0.00	0.00
	63	1(17)	10E14.55	1.00	0.00	0.00
	64, 65	1(2) 1(16)	10E13.62	1.00	0.00	0.00
	66	8(1) 1(2) 1(7)	10079	1.00	0.02	0.00
	67	14(1) 1(4)	23	1.00	0.03	0.00
	68	1(2) 1(15)	10E12.41	1.00	0.01	0.00
	69	1(2) 1(4) 1(12)	10E10.36	1.00	0.00	0.00
	70	3(2) 1(12)	10E9.58	1.00	0.01	0.02
	71	2(1) 1(2) 2(3) 1(7)	10E5.56	1.00	0.00	0.01
	72	1(16)	10E13.32	1.00	0.00	0.00
	73	6(1) 3(2) 1(6)	5759	1.00	0.01	0.02
	74	14(1) 1(4)	23	1.00	0.11	0.00
	75	1(1) 1(2) 1(3) 1(4) 1(8)	10E7.06	1.00	0.01	0.00
	76	5(1) 1(2) 1(3) 1(8)	10E5.68	1.00	0.02	0.00
	77	1(17)	10E14.55	1.00	0.00	0.00
	78	3(1) 2(2) 1(3) 1(8)	10E5.99	1.00	0.02	0.01
	79	5(1) 2(2) 1(3) 1(6)	17279	1.00	0.02	0.01
	80	2(1) 2(2) 1(3) 1(9)	10E6.93	1.00	0.01	0.01
	81	4(1) 1(2) 2(3) 1(6)	51839	1.00	0.01	0.01
	82	3(1) 2(2) 1(3) 1(8)	10E5.99	1.00	0.02	0.00
	83	8(1) 1(3) 1(7)	30239	1.00	0.03	0.00
	84	4(1) 2(2) 1(3) 1(7)	10E5.08	1.00	0.01	0.01
	85	3(1) 2(2) 1(11)	10E8.20	1.00	0.01	0.02
	86	1(6)	719	1.00	0.00	0.00
	87	3(1) 3(2) 1(9)	10E6.46	1.00	0.02	0.02
	88	5(1) 3(2) 1(7)	40319	1.00	0.02	0.02
	89	1(1) 3(2) 1(3) 1(8)	10E6.29	1.00	0.01	0.01
	90	1(1) 1(2) 1(4)	47	1.00	0.00	0.00
	91	1(2) 1(5) 1(11)	10E9.68	1.00	0.00	0.00
	92	1(17)	10E14.55	1.00	0.11	0.00
	93	1(6)	719	1.00	0.01	0.00
	94	1(1) 2(3)	35	1.00	0.00	0.01
	95	1(1)	0	1.00	0.00	0.00
	96	1(5)	119	1.00	0.00	0.00
	97, 98	1(7)	5039	1.00	0.00	0.00
	99	1(1) 1(2) 1(4)	47	1.00	0.00	0.00
	100	1(7)	5039	1.00	0.00	0.00
	101	1(2) 1(4)	47	1.00	0.00	0.00
	102	1(17)	10E14.55	1.00	0.00	0.00
	103, 104, 105, 106, 107, 108, 109	Trivial outputs: no inputs				
duke2	o1	5(1) 1(2) 1(4)	47	1.00	0.00	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 6 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
duke2 (Cont'd)	o2	14(1) 1(2)	1	1.00	0.04	0.00
	o3	15(1)	0	1.00	0.17	0.00
	o4	5(1) 1(2)	1	1.00	0.00	0.00
	o5	1(2)	1	1.00	0.00	0.00
	o6	6(1) 2(2) 1(7)	20159	1.00	0.02	0.01
	o7	16(1) 1(2)	1	1.00	0.28	0.00
	o8	1: 2(1) 3(2) 2: 1(2) 4(1)	7	2.00	0.00	0.02
	o9	1(2)	1	1.00	0.00	0.00
	o10	17(1)	0	1.00	0.07	0.00
	o11	6(1) 1(2)	1	1.00	0.01	0.00
	o12	1(7)	5039	1.00	0.00	0.00
	o13	15(1) 1(2)	1	1.00	0.22	0.00
	o14	1(2) 1(3)	11	1.00	0.00	0.00
	o15	13(1) 1(2)	1	1.00	0.04	0.00
	o16	1(6)	719	1.00	0.00	0.00
	o17	13(1) 1(2) 1(3)	11	1.00	0.15	0.00
	o18	13(1) 1(2)	1	1.00	0.04	0.00
	o19	11(1) 1(2) 1(5)	239	1.00	0.14	0.00
	o20	1(6)	719	1.00	0.00	0.00
	o21	14(1) 1(2)	1	1.00	0.04	0.00
	o22, o23	6(1) 1(2)	1	1.00	0.01	0.00
	o24	1(7)	5039	1.00	0.00	0.00
	o25	12(1) 1(2)	1	1.00	0.03	0.00
	o26	1(3) 1(4)	143	1.00	0.00	0.00
	o27	1(1) 1(2)	1	1.00	0.00	0.00
	o28	14(1) 1(2)	1	1.00	0.18	0.00
	o29	17(1)	0	1.00	0.08	0.00
e64	1	1(45)	10E56.07	1.00	0.02	0.00
	2	1(44)	10E54.42	1.00	0.01	0.00
	3	1(65)	10E90.91	1.00	0.03	0.00
	4	1(64)	10E89.10	1.00	0.03	0.00
	5	1(2)	1	1.00	0.00	0.00
	6	1(1)	0	1.00	0.00	0.00
	7	1(6)	719	1.00	0.00	0.00
	8	1(5)	119	1.00	0.00	0.00
	9	1(63)	10E87.29	1.00	0.03	0.00
	10	1(62)	10E85.49	1.00	0.03	0.00
	11	1(4)	23	1.00	0.00	0.00
	12	1(3)	11	1.00	0.00	0.00
	13	1(26)	10E26.60	1.00	0.01	0.00
	14	1(25)	10E25.19	1.00	0.00	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 7 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
e64 (Cont'd)	15	1(9)	10E5.55	1.00	0.01	0.00
	16	1(8)	10E40319	1.00	0.00	0.00
	17	1(24)	10E23.79	1.00	0.00	0.00
	18	1(23)	10E22.41	1.00	0.01	0.00
	19	1(22)	10E21.05	1.00	0.00	0.00
	20	1(51)	10E66.19	1.00	0.02	0.00
	21	1(30)	10E32.42	1.00	0.00	0.00
	22	1(29)	10E30.94	1.00	0.00	0.00
	23	1(54)	10E71.36	1.00	0.02	0.00
	24	1(57)	10E76.60	1.00	0.03	0.00
	25	1(16)	10E13.32	1.00	0.00	0.00
	26	1(15)	10E12.11	1.00	0.00	0.00
	27	1(60)	10E81.19	1.00	0.03	0.00
	28	1(59)	10E80.14	1.00	0.02	0.00
	29	1(14)	10E10.94	1.00	0.00	0.00
	30	1(13)	10E9.79	1.00	0.01	0.00
	31	1(56)	10E74.85	1.00	0.02	0.00
	32	1(55)	10E73.10	1.00	0.11	0.00
	33	1(12)	10E8.68	1.00	0.00	0.00
	34	1(11)	10E7.60	1.00	0.00	0.00
	35	1(7)	5039	1.00	0.00	0.00
	36	1(34)	10E38.47	1.00	0.01	0.00
	37	1(33)	10E36.93	1.00	0.00	0.00
	38	1(61)	10E83.70	1.00	0.02	0.00
	39	1(36)	10E41.47	1.00	0.01	0.00
	40	1(35)	10E40.01	1.00	0.01	0.00
	41	1(41)	10E49.52	1.00	0.02	0.00
	42	1(40)	10E47.91	1.00	0.01	0.00
	43	1(48)	10E61.09	1.00	0.02	0.00
	44	1(47)	10E59.41	1.00	0.02	0.00
	45	1(46)	10E57.74	1.00	0.01	0.00
	46	1(39)	10E46.30	1.00	0.01	0.00
	47	1(38)	10E44.71	1.00	0.01	0.00
	48	1(37)	10E43.13	1.00	0.01	0.00
	49	1(43)	10E52.78	1.00	0.02	0.00
	50	1(42)	10E51.14	1.00	0.00	0.00
	51	1(50)	10E64.48	1.00	0.02	0.00
	52	1(49)	10E62.78	1.00	0.01	0.01
	53	1(53)	10E69.63	1.00	0.02	0.00
	54	1(52)	10E67.90	1.00	0.01	0.00
	55	1(32)	10E35.42	1.00	0.00	0.00
	56	1(31)	10E33.91	1.00	0.01	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 8 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
e64 (Cont'd)	57	1(58)	10E78.37	1.00	0.02	0.00
	58	1(28)	10E29.48	1.00	0.00	0.00
	59	1(27)	10E28.03	1.00	0.01	0.00
	60	1(10)	10E6.55	1.00	0.00	0.00
	61	1(21)	10E19.70	1.00	0.01	0.00
	62	1(20)	10E18.38	1.00	0.00	0.00
	63	1(19)	10E17.08	1.00	0.01	0.00
	64	1(18)	10E15.80	1.00	0.00	0.00
	65	1(17)	10E6.55	1.00	0.00	0.00
ex2	1	1(2) 1(3)	11	1.00	0.00	0.00
ex3	1	2(1) 1(2)	1	1.00	0.00	0.00
ex4	1, 2, 3, 4, 5	5(1) 2(2)	3	1.00	0.00	0.00
	6, 7	12(1) 2(2)	3	1.00	0.57	0.01
	8	Trivial output: no inputs				
	9	12(1) 2(2)	3	1.00	0.14	0.01
	10, 11, 12, 13	5(1) 2(2)	3	1.00	0.01	0.01
	14, 15	12(1) 2(2)	3	1.00	0.20	0.01
	16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28	Trivial outputs: no inputs				
ex5	1, 2	1(3)	5	1.00	0.00	0.00
	3, 4, 5	1(5)	119	1.00	0.00	0.00
	6	1(4)	23	1.00	0.00	0.00
	7	1(7)	5039	1.00	0.00	0.00
	8, 9, 10	1(8)	40319	1.00	0.01	0.00
	11	1(7)	5039	1.00	0.00	0.00
	12, 13, 14, 15, 16, 17	1(8)	40319	1.00	0.00	0.00
	18	1(5)	119	1.00	0.00	0.00
	19	1(6)	719	1.00	0.00	0.00
	20	1(5)	119	1.00	0.00	0.00
	21, 22, 23	1(7)	5039	1.00	0.00	0.00
	24	1(5)	119	1.00	0.00	0.00
	25, 26	1(8)	40319	1.00	0.00	0.00
	27	1(7)	5039	1.00	0.00	0.00
	28, 29, 30, 31	1(8)	40319	1.00	0.00	0.00
	32	5(1) 1(2)	1	1.00	0.00	0.00
	33	8(1)	0	1.00	0.01	0.00
	34	1(1) 1(2) 1(5)	239	1.00	0.00	0.00
	35	6(1) 1(2)	1	1.00	0.01	0.00
	36	4(1) 1(2)	1	1.00	0.00	0.00
	37	1(2) 2(3)	71	1.00	0.00	0.01
	38	3(1) 1(2) 1(3)	11	1.00	0.00	0.00
	39	8(1)	0	1.00	0.01	0.00

Table 3: **Symmetry characterization of the two-level MCNC benchmarks (Sheet 9 of 13).**

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
ex5 (Cont'd)	40	2(1) 2(3)	35	1.00	0.00	0.01
	41	8(1)	0	1.00	0.00	0.00
	42	1(1) 2(2)	3	1.00	0.00	0.00
	43	1(1) 2(2) 1(3)	23	1.00	0.00	0.00
	44	8(1)	0	1.00	0.01	0.00
	45	2(2) 1(3)	23	1.00	0.00	0.01
	46	5(1) 1(2)	1	1.00	0.00	0.00
	47	2(1) 1(2) 1(4)	47	1.00	0.00	0.00
	48	6(1) 1(2)	1	1.00	0.04	0.00
	49	2(1) 1(2) 1(4)	47	1.00	0.00	0.00
	50	2(1) 1(2) 1(3)	11	1.00	0.01	0.00
	51	1(1) 1(2) 1(5)	239	1.00	0.00	0.00
	52	4(1) 2(2)	3	1.00	0.00	0.01
	53	5(1) 1(2)	1	1.00	0.00	0.00
	54, 55, 56	6(1) 1(2)	1	1.00	0.01	0.00
	57, 58	8(1)	0	1.00	0.00	0.00
	59, 60	6(1) 1(2)	1	1.00	0.00	0.00
	61	2(1) 2(3)	35	1.00	0.00	0.01
	62	1(1) 1(2) 1(4)	47	1.00	0.00	0.00
	63	1(1) 1(2) 1(3)	11	1.00	0.00	0.00
ex1010	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	10(1)	0	1.00	0.05	0.00
inc	1, 2	6(1)	0	1.00	0.01	0.00
	3, 4, 5	7(1)	0	1.00	0.00	0.00
	6	4(1) 1(2)	1	1.00	0.00	0.00
	7	2(1) 1(2) 1(3)	11	1.00	0.00	0.00
	8	4(1) 1(2)	1	1.00	0.00	0.00
	9	2(2)	3	1.00	0.00	0.00
misex1	dmnst3B	1(1) 1(3)	5	1.00	0.00	0.00
	dmnst2B	6(1)	0	1.00	0.00	0.00
	dmnst1B	7(1)	0	1.00	0.00	0.00
	dmnst0B	5(1) 1(2)	1	1.00	0.00	0.00
	adctlp2B	2(1) 1(2)	1	1.00	0.00	0.00
	adctlp1B, adctlp0B	6(1)	0	1.00	0.01	0.00
misex2	z, a1, b1	1(8)	40319	1.00	0.00	0.00
	c1	2(3) 1(4)	863	1.00	0.00	0.01
	d1	1(7)	5039	1.00	0.00	0.00
	e1	1(5)	119	1.00	0.00	0.00
	f1	1(6)	719	1.00	0.00	0.00
	g1	1(1) 1(2) 1(4)	47	1.00	0.01	0.00
	h1	1(12)	0	1.00	0.00	0.00
	i1	2(2) 1(3) 1(7)	10E5.08	1.00	0.00	0.01
	j1	1(1) 1(2) 1(4) 1(7)	10E5.38	1.00	0.01	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 10 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
misex2 (Cont'd)	k1	2(2) 1(3) 1(7)	10E5.08	1.00	0.00	0.00
	l1	5(1) 2(2)	3	1.00	0.00	0.01
	m1	1(5)	119	1.00	0.00	0.00
	n1	1(8)	40319	1.00	0.00	0.00
	o1	5(1)	0	1.00	0.00	0.00
	p1	1(2)	1	1.00	0.00	0.00
	q1	1(3)	5	1.00	0.00	0.00
misex3	r2, s2, t2, u2, n2	14(1)	0	1.00	0.10	0.00
	o2	13(1)	0	1.00	0.11	0.00
	p2, q2	14(1)	0	1.00	0.13	0.00
	h2	12(1) 1(2)	1	1.00	0.04	0.00
	i2, j2	14(1)	0	1.00	0.14	0.00
	k2	12(1) 1(2)	1	1.00	0.04	0.00
	m2, l2	14(1)	0	1.00	0.10	0.00
o64	1	1: 65(2) 2: 1(65)	10E110.48	10E90.91	23.68	0.58
pdc	1, 2	2(1) 5(2) 1(3)	191	1.00	0.01	0.01
	3	2(1) 4(2) 2(3)	575	1.00	0.01	0.01
	4	16(1)	0	1.00	0.10	0.00
	5, 6	10(1) 1(2) 1(4)	47	1.00	0.04	0.00
	7	1(3)	5	1.00	0.00	0.00
	8	2(1) 4(2) 2(3)	575	1.00	0.02	0.01
	9	16(1)	0	1.00	0.06	0.00
	10	10(1) 1(2) 1(4)	47	1.00	0.03	0.00
	11	2(2) 1(4) 1(6)	69119	1.00	0.00	0.00
	12	1(4) 1(8)	10E5.99	1.00	0.00	0.00
	13, 14	1(8)	40319	1.00	0.03	0.00
	15	1(2)	1	1.00	0.00	0.00
	16	1(1) 1(2)	1	1.00	0.00	0.00
	17, 18	11(1) 1(5)	119	1.00	0.03	0.00
	19, 20	9(1) 1(2) 1(5)	239	1.00	0.03	0.00
	21	1(1) 3(2) 1(5)	959	1.00	0.01	0.00
	22	1(1) 1(2) 1(7)	10079	1.00	0.01	0.00
	23	1(2) 1(5) 1(7)	10E6.08	1.00	0.00	0.00
	24	4(1) 2(2) 1(3) 1(5)	2879	1.00	0.05	0.00
	25	1(1) 1(2) 2(3) 1(7)	10E5.56	1.00	0.01	0.00
	26	1(7)	5039	1.00	0.01	0.00
	27	4(1) 2(2) 1(8)	10E5.21	1.00	0.01	0.00
	28	1(8)	40319	1.00	0.00	0.00
	29, 30	7(1) 3(2) 1(3)	47	1.00	0.03	0.00
	31	6(1) 2(2) 2(3)	143	1.00	0.03	0.01
	32	4(1) 1(3) 1(4)	143	1.00	0.00	0.00

Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 11 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
pdc (Cont'd)	33	5(1) 1(3)	5	1.00	0.00	0.00
	34	5(1) 3(2) 1(5)	959	1.00	0.02	0.00
	35	9(1) 1(2) 1(5)	239	1.00	0.03	0.00
	36	4(1) 1(2) 2(5)	28799	1.00	0.02	0.00
	37	9(1) 2(2) 1(3)	23	1.00	0.03	0.01
	38	7(1) 3(2) 1(3)	47	1.00	0.05	0.00
	39, 40	1(7)	5039	1.00	0.00	0.00
rd53	1, 2, 3	1(5)	119	1.00	0.00	0.00
rd73	1, 2, 3	1(7)	5039	1.00	0.00	0.00
rd84	1, 2, 3, 4	1(8)	40319	1.00	0.00	0.00
sao2	1, 2	8(1) 1(2)	1	1.00	0.01	0.00
	3	10(1)	0	1.00	0.02	0.00
	4	8(1) 1(2)	1	1.00	0.01	0.00
seq	1	25(1) 3(2) 2(3)	287	1.00	0.46	0.01
	2	18(1) 1(2) 1(3) 1(4)	287	1.00	0.15	0.00
	3	19(1) 5(2) 1(3) 1(4)	4607	1.00	0.30	0.03
	4	31(1) 1(2) 1(3)	11	1.00	0.50	0.00
	5	21(1) 6(2) 1(3)	383	1.00	0.40	0.03
	6	25(1) 5(2) 1(3)	191	1.00	0.61	0.03
	7	21(1) 3(2) 2(3)	287	1.00	0.29	0.02
	8	22(1) 1(2) 1(3)	11	1.00	0.22	0.00
	9	2(1) 1(2) 1(3) 1(5)	1439	1.00	0.00	0.00
	10	8(1) 4(2) 1(3) 2(4)	55295	1.00	0.11	0.02
	11	1: 6(1) 3(2) 2(4) 2: 9(1) 1(2)	9216	2.00	0.03	0.02
	12	11(1) 2(2) 2(3) 1(4)	3455	1.00	0.07	0.00
	13	1(5) 1(7)	10E5.78	1.00	0.00	0.00
	14	6(1) 1(4)	23	1.00	0.00	0.00
	15	7(1) 1(4)	23	1.00	0.01	0.00
	16	1(2)	1	1.00	0.00	0.00
	17	7(1) 2(2) 2(3)	143	1.00	0.09	0.01
	18	23(1) 3(2) 1(3)	47	1.00	0.30	0.01
	19	21(1) 5(2) 1(3)	191	1.00	0.41	0.03
	20	10(1) 2(3)	35	1.00	0.02	0.00
	21	16(1) 1(2) 1(3)	11	1.00	0.17	0.00
	22	13(1)	0	1.00	0.02	0.00
	23	16(1) 2(2)	3	1.00	0.09	0.00
	24	20(1) 1(2)	1	1.00	0.17	0.00
	25	24(1) 2(2) 2(3)	143	1.00	0.44	0.01
	26	19(1) 2(2) 2(3) 1(5)	17279	1.00	0.25	0.00
	27	8(1) 2(2) 1(3)	23	1.00	0.03	0.06
	28	19(1) 1(2) 1(3)	11	1.00	0.14	0.00

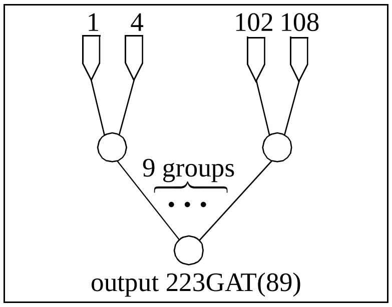
Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 12 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
seq (Cont'd)	29	3(2) 2(3) 1(5)	34559	1.00	0.01	0.01
	30	8(1) 2(2) 3(3)	863	1.00	0.10	0.01
	31	11(1) 3(2) 2(3)	287	1.00	0.08	0.01
	32	9(1) 4(2) 2(3) 1(5)	69119	1.00	0.13	0.01
	33	1(10)	10E6.55	1.00	0.00	0.00
	34	26(1) 4(2) 1(3)	95	1.00	0.49	0.01
	35	21(1) 2(2)	3	1.00	0.22	0.01
spla	1	2(1) 1(14)	10E10.94	1.00	0.00	0.00
	2	1(1) 1(2) 1(13)	10E9.79	1.00	0.01	0.00
	3	1(15)	10E12.11	1.00	0.00	0.00
	4, 5	2(1) 5(2) 1(3)	191	1.00	0.01	0.02
	6	16(1)	0	1.00	0.10	0.00
	7	2(1) 4(2) 2(3)	575	1.00	0.01	0.01
	8	10(1) 1(2) 1(4)	47	1.00	0.04	0.00
	9	1(3)	5	1.00	0.00	0.00
	10	12(1) 1(4)	23	1.00	0.07	0.00
	11	16(1)	0	1.00	0.06	0.00
	12	2(1) 4(2) 2(3)	575	1.00	0.02	0.00
	13	12(1) 1(4)	23	1.00	0.08	0.00
	14	1(2) 1(3) 1(5)	1439	1.00	0.00	0.00
	15	1(8)	40319	1.00	0.00	0.00
	16	2(2) 1(8)	10E5.21	1.00	0.00	0.00
	17	1(4) 1(8)	10E5.99	1.00	0.01	0.00
	18, 19	1(8)	40319	1.00	0.00	0.00
	20, 21	1(16)	10E13.32	1.00	0.00	0.00
	22, 23	1(1) 1(2) 1(3) 1(4) 1(6)	10E5.32	1.00	0.01	0.00
	24	6(1) 2(2) 2(3)	143	1.00	0.02	0.01
	25, 26	1(7)	5039	1.00	0.00	0.00
	27, 28	6(1) 1(2) 1(3) 1(5)	1439	1.00	0.01	0.00
	29	5(1) 2(2) 1(7)	20159	1.00	0.04	0.01
	30	4(1) 1(2) 1(10)	10E6.86	1.00	0.00	0.00
	31	8(1) 1(2) 1(6)	1439	1.00	0.02	0.00
	32	5(1) 1(3)	5	1.00	0.01	0.00
	33	8(1) 1(2) 2(3)	71	1.00	0.02	0.00
	34	4(1) 3(2) 2(3)	287	1.00	0.01	0.01
	35	1(7)	5039	1.00	0.00	0.00
	36, 37	11(1) 1(5)	119	1.00	0.01	0.00
	38, 39	1: 5(1) 3(2) 1(5) 2: 1(2) 7(1)	959	2.00	0.02	0.01
	40	4(1) 1(2) 1(8)	80639	1.00	0.00	0.00
	41	1(1) 1(2) 2(3) 1(7)	10E5.56	1.00	0.00	0.01
	42	1(12)	10E8.68	1.00	0.00	0.00

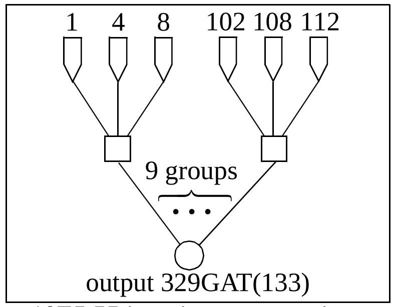
Table 3: Symmetry characterization of the two-level MCNC benchmarks (Sheet 13 of 13).

Benchmark	Primary Outputs	Hierarchical Partition	Permutation Count		CPU time, s	
			Max	H Ratio	Flat	Hier.
spla (Cont'd)	43	1: 2(2) 1(6) 2: 1(2) 1(6)	2879	2.00	0.00	0.02
	44	2(1) 2(2) 1(6)	2879	1.00	0.00	0.00
	45	3(1) 2(2) 1(5)	479	1.00	0.01	0.00
	46	4(1) 2(2) 1(3) 1(5)	2879	1.00	0.02	0.00
t481	v16.0	1: 8(2) 2: 4(2) 3: 2[2] 4: 1(2)	8191	16.00	0.02	0.45
vg2	1	3(1) 1(2) 1(3) 1(6)	8639	1.00	0.01	0.00
	2	7(1) 2(2) 1(14)	10E10.94	1.00	0.07	0.00
	3	3(1) 1(2) 1(3) 1(6)	8639	1.00	0.01	0.00
	4	7(1) 2(2) 1(7)	20159	1.00	0.03	0.00
	5	12(1) 3(2)	7	1.00	0.56	0.01
	6	3(1) 1(2) 1(3)	11	1.00	0.00	0.00
	7	10(1) 3(2)	7	1.00	0.15	0.01
	8	3(1) 1(2) 1(3)	11	1.00	0.01	0.00
xor5	xor5	1(5)	119	1.00	0.00	0.00
Z5xp1	1	2(2) 1(3)	23	1.00	0.00	0.00
	2	5(1) 1(2)	1	1.00	0.01	0.00
	3, 4	7(1)	0	1.00	0.00	0.00
	5	6(1)	0	1.00	0.01	0.00
	6	3(1) 1(2)	1	1.00	0.00	0.00
	7	2(1) 1(2)	1	1.00	0.00	0.00
	8	1(1) 1(2)	1	1.00	0.00	0.00
	9	1(2)	1	1.00	0.00	0.00
	10	1(1)	0	1.00	0.00	0.00
Z9sym	1	6(1) 1(3)	5	1.00	0.00	0.00

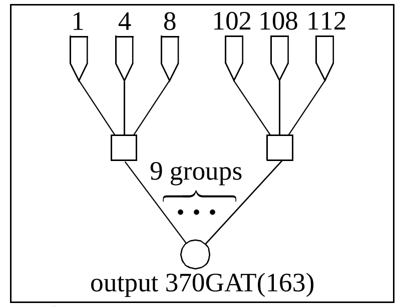
Appendix B Symmetry structures for C432



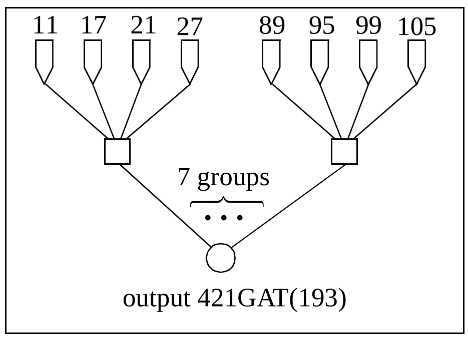
10E8.26 invariant permutations



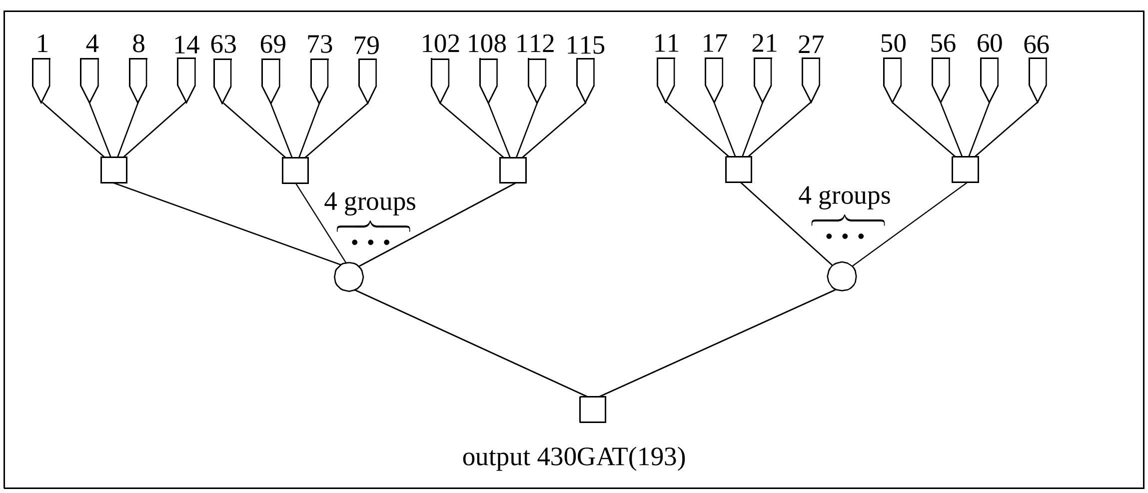
10E5.55 invariant permutations



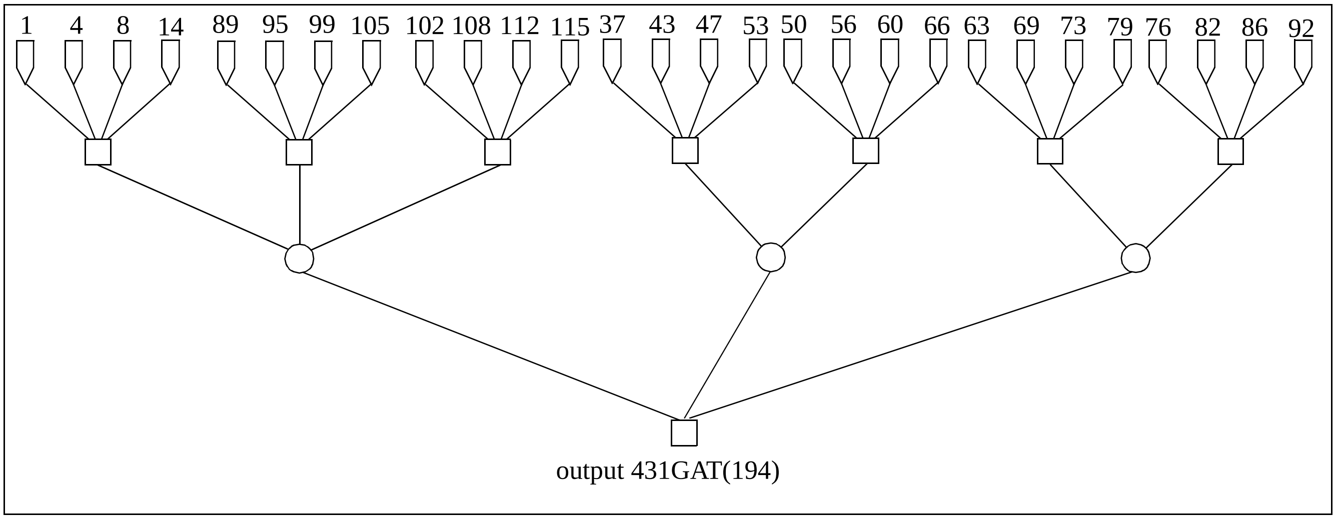
10E5.55 invariant permutations



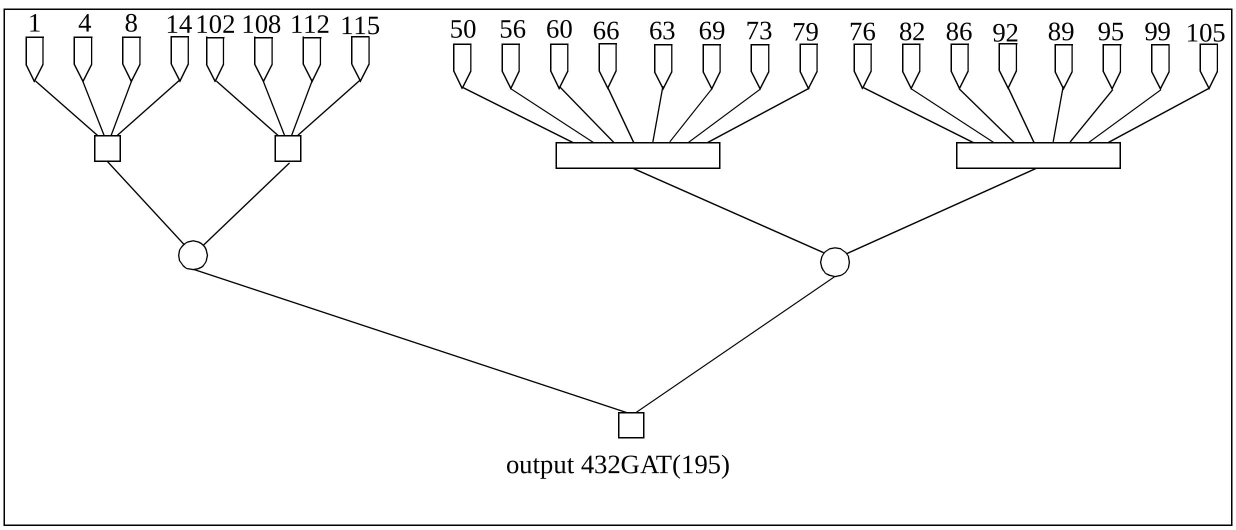
5039 invariant permutations



2879 invariant permutations



23 invariant permutations



3 invariant permutations

Appendix C Symmetry structures for C499 based on its high-level decomposition

Symmetries of the circuit are described relying on the signal namings from [16].

The $M1$ module implements syndrome equations. We first give symmetries for some of its internal signals:

$$D_0:\{ID_{00}, ID_{04}, ID_{08}, ID_{12}, ID_{16}, ID_{17}, ID_{18}, ID_{19}, ID_{20}, ID_{21}, ID_{22}, ID_{23}\}$$
$$D_1:\{ID_{01}, ID_{05}, ID_{09}, ID_{13}, ID_{24}, ID_{25}, ID_{26}, ID_{27}, ID_{28}, ID_{29}, ID_{30}, ID_{31}\}$$
$$D_2:\{ID_{02}, ID_{06}, ID_{10}, ID_{14}, ID_{16}, ID_{17}, ID_{18}, ID_{19}, ID_{24}, ID_{25}, ID_{26}, ID_{27}\}$$
$$D_3:\{ID_{03}, ID_{07}, ID_{11}, ID_{15}, ID_{20}, ID_{21}, ID_{22}, ID_{23}, ID_{28}, ID_{29}, ID_{30}, ID_{31}\}$$
$$D_4:\{ID_{16}, ID_{20}, ID_{24}, ID_{28}, ID_{00}, ID_{01}, ID_{02}, ID_{03}, ID_{04}, ID_{05}, ID_{06}, ID_{07}\}$$
$$D_5:\{ID_{17}, ID_{21}, ID_{25}, ID_{29}, ID_{08}, ID_{09}, ID_{10}, ID_{11}, ID_{12}, ID_{13}, ID_{14}, ID_{15}\}$$
$$D_6:\{ID_{18}, ID_{22}, ID_{26}, ID_{30}, ID_{00}, ID_{01}, ID_{02}, ID_{03}, ID_{08}, ID_{09}, ID_{10}, ID_{11}\}$$
$$D_7:\{ID_{19}, ID_{23}, ID_{27}, ID_{31}, ID_{04}, ID_{05}, ID_{06}, ID_{07}, ID_{12}, ID_{13}, ID_{14}, ID_{15}\}$$

These are multi-phase symmetries (i.e. symmetries which hold under any phase assignments to the group variables). The remaining inputs to this module have the following symmetries:

$$C_0:\{R, IC_0\} \quad C_2:\{R, IC_2\} \quad C_4:\{R, IC_4\} \quad C_6:\{R, IC_6\}$$
$$C_1:\{R, IC_1\} \quad C_3:\{R, IC_3\} \quad C_5:\{R, IC_5\} \quad C_7:\{R, IC_7\}$$

D_i and C_i form multi-phase symmetries:

$$S_0:\{D_0, C_0\} \quad S_2:\{D_2, C_2\} \quad S_4:\{D_4, C_4\} \quad S_6:\{D_6, C_6\}$$
$$S_1:\{D_1, C_1\} \quad S_3:\{D_3, C_3\} \quad S_5:\{D_5, C_5\} \quad S_7:\{D_7, C_7\}$$

The S_i syndrome signals have the following symmetries in the $M2$ module:

$$N_{00}:\{S_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, S_4, \bar{S}_5, S_6, \bar{S}_7\} \qquad N_{16}:\{S_0, \bar{S}_1, S_2, \bar{S}_3, S_4, \bar{S}_5, \bar{S}_6, \bar{S}_7\}$$
$$N_{01}:\{\bar{S}_0, S_1, \bar{S}_2, \bar{S}_3, S_4, \bar{S}_5, S_6, \bar{S}_7\} \qquad N_{17}:\{S_0, \bar{S}_1, S_2, \bar{S}_3, \bar{S}_4, S_5, \bar{S}_6, \bar{S}_7\}$$
$$N_{02}:\{\bar{S}_0, \bar{S}_1, S_2, \bar{S}_3, S_4, \bar{S}_5, S_6, \bar{S}_7\} \qquad N_{18}:\{S_0, \bar{S}_1, S_2, \bar{S}_3, \bar{S}_4, \bar{S}_5, S_6, \bar{S}_7\}$$
$$N_{03}:\{\bar{S}_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, S_4, \bar{S}_5, S_6, \bar{S}_7\} \qquad N_{19}:\{S_0, \bar{S}_1, S_2, \bar{S}_3, \bar{S}_4, \bar{S}_5, \bar{S}_6, \bar{S}_7\}$$
$$N_{04}:\{S_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, S_4, \bar{S}_5, \bar{S}_6, S_7\} \qquad N_{20}:\{S_0, \bar{S}_1, \bar{S}_2, S_3, S_4, \bar{S}_5, \bar{S}_6, \bar{S}_7\}$$
$$N_{05}:\{\bar{S}_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, S_4, \bar{S}_5, \bar{S}_6, S_7\} \qquad N_{21}:\{S_0, \bar{S}_1, \bar{S}_2, S_3, \bar{S}_4, S_5, \bar{S}_6, \bar{S}_7\}$$
$$N_{06}:\{\bar{S}_0, \bar{S}_1, S_2, \bar{S}_3, S_4, \bar{S}_5, \bar{S}_6, \bar{S}_7\} \qquad N_{22}:\{S_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, \bar{S}_4, \bar{S}_5, S_6, \bar{S}_7\}$$
$$N_{07}:\{\bar{S}_0, \bar{S}_1, \bar{S}_2, S_3, S_4, \bar{S}_5, \bar{S}_6, S_7\} \qquad N_{23}:\{S_0, \bar{S}_1, \bar{S}_2, S_3, \bar{S}_4, \bar{S}_5, \bar{S}_6, S_7\}$$

$N_{08}:\{S_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, \bar{S}_4, S_5, S_6, S_7\}$	$N_{24}:\{\bar{S}_0, S_1, S_2, \bar{S}_3, S_4, \bar{S}_5, \bar{S}_6, \bar{S}_7\}$
$N_{09}:\{\bar{S}_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, \bar{S}_4, S_5, S_6, \bar{S}_7\}$	$N_{25}:\{\bar{S}_0, S_1, S_2, \bar{S}_3, \bar{S}_4, S_5, \bar{S}_6, \bar{S}_7\}$
$N_{10}:\{\bar{S}_0, \bar{S}_1, S_2, \bar{S}_3, \bar{S}_4, S_5, S_6, \bar{S}_7\}$	$N_{26}:\{\bar{S}_0, S_1, S_2, \bar{S}_3, \bar{S}_4, \bar{S}_5, S_6, \bar{S}_7\}$
$N_{11}:\{\bar{S}_0, \bar{S}_1, \bar{S}_2, S_3, \bar{S}_4, S_5, S_6, \bar{S}_7\}$	$N_{27}:\{\bar{S}_0, S_1, S_2, \bar{S}_3, \bar{S}_4, \bar{S}_5, \bar{S}_6, S_7\}$
$N_{12}:\{S_0, \bar{S}_1, \bar{S}_2, \bar{S}_3, \bar{S}_4, S_5, \bar{S}_6, S_7\}$	$N_{28}:\{\bar{S}_0, S_1, \bar{S}_2, S_3, S_4, \bar{S}_5, \bar{S}_6, \bar{S}_7\}$
$N_{13}:\{\bar{S}_0, S_1, \bar{S}_2, \bar{S}_3, \bar{S}_4, S_5, \bar{S}_6, S_7\}$	$N_{29}:\{\bar{S}_0, S_1, \bar{S}_2, S_3, \bar{S}_4, S_5, \bar{S}_6, \bar{S}_7\}$
$N_{14}:\{\bar{S}_0, \bar{S}_1, S_2, \bar{S}_3, \bar{S}_4, S_5, \bar{S}_6, S_7\}$	$N_{30}:\{\bar{S}_0, S_1, \bar{S}_2, S_3, \bar{S}_4, \bar{S}_5, S_6, \bar{S}_7\}$
$N_{15}:\{\bar{S}_0, \bar{S}_1, \bar{S}_2, S_3, \bar{S}_4, S_5, \bar{S}_6, S_7\}$	$N_{31}:\{\bar{S}_0, S_1, \bar{S}_2, S_3, \bar{S}_4, \bar{S}_5, \bar{S}_6, S_7\}$

The N_i and ID_i signals form the following multi-phase symmetry groups:

$OD_{00}:\{N_{00}, ID_{00}\}$	$OD_{08}:\{N_{08}, ID_{08}\}$	$OD_{16}:\{N_{16}, ID_{16}\}$	$OD_{24}:\{N_{24}, ID_{24}\}$
$OD_{01}:\{N_{01}, ID_{01}\}$	$OD_{09}:\{N_{09}, ID_{09}\}$	$OD_{17}:\{N_{17}, ID_{17}\}$	$OD_{25}:\{N_{25}, ID_{25}\}$
$OD_{02}:\{N_{02}, ID_{02}\}$	$OD_{10}:\{N_{10}, ID_{10}\}$	$OD_{18}:\{N_{18}, ID_{18}\}$	$OD_{26}:\{N_{26}, ID_{26}\}$
$OD_{03}:\{N_{03}, ID_{03}\}$	$OD_{11}:\{N_{11}, ID_{11}\}$	$OD_{19}:\{N_{19}, ID_{19}\}$	$OD_{27}:\{N_{27}, ID_{27}\}$
$OD_{04}:\{N_{04}, ID_{04}\}$	$OD_{12}:\{N_{12}, ID_{12}\}$	$OD_{20}:\{N_{20}, ID_{20}\}$	$OD_{28}:\{N_{28}, ID_{28}\}$
$OD_{05}:\{N_{05}, ID_{05}\}$	$OD_{13}:\{N_{13}, ID_{13}\}$	$OD_{21}:\{N_{21}, ID_{21}\}$	$OD_{29}:\{N_{29}, ID_{29}\}$
$OD_{06}:\{N_{06}, ID_{06}\}$	$OD_{14}:\{N_{14}, ID_{14}\}$	$OD_{22}:\{N_{22}, ID_{22}\}$	$OD_{30}:\{N_{30}, ID_{30}\}$
$OD_{07}:\{N_{07}, ID_{07}\}$	$OD_{15}:\{N_{07}, ID_{07}\}$	$OD_{23}:\{N_{23}, ID_{23}\}$	$OD_{31}:\{N_{31}, ID_{31}\}$

Appendix D Symmetry structure for C6288 expressed in terms of its partial products

Output	Symmetry Partition
545GAT(287)	1(2)
1581GAT(423)	1(2)
1901GAT(561)	1(2) 1(3)
2223GAT(700)	1(2) 1(3) 1(4)
2548GAT(840)	1(2) 1(3) 1(4) 1(5)
2877GAT(983)	1(2) 1(3) 1(4) 1(5) 1(6)
3211GAT(1128)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7)
3552GAT(1275)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8)
3895GAT(1423)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9)
4241GAT(1572)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10)
4591GAT(1722)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11)
4946GAT(1876)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12)
5308GAT(2031)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 1(13)
5672GAT(2187)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 1(13) 1(14)
5971GAT(2309)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 1(13) 1(14) 1(15)
6123GAT(2368)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 1(13) 1(14) 1(15) 1(16)
6150GAT(2378)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 1(13) 1(14) 2(15) 1(16)
6160GAT(2383)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 1(13) 2(14) 2(15) 1(16)
6170GAT(2388)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 1(12) 2(13) 2(14) 2(15) 1(16)
6180GAT(2393)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 1(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6190GAT(2398)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 1(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6200GAT(2403)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 1(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6210GAT(2408)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 1(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6220GAT(2413)	1(2) 1(3) 1(4) 1(5) 1(6) 1(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6230GAT(2418)	1(2) 1(3) 1(4) 1(5) 1(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6240GAT(2423)	1(2) 1(3) 1(4) 1(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6250GAT(2428)	1(2) 1(3) 1(4) 2(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6260GAT(2433)	1(2) 1(3) 2(4) 2(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6270GAT(2438)	1(2) 2(3) 2(4) 2(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6280GAT(2443)	2(2) 2(3) 2(4) 2(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6287GAT(2444)	1(1) 2(2) 2(3) 2(4) 2(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)
6288GAT(2447)	1(1) 2(2) 2(3) 2(4) 2(5) 2(6) 2(7) 2(8) 2(9) 2(10) 2(11) 2(12) 2(13) 2(14) 2(15) 1(16)

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