End-to-End Energy-Bandwidth Tradeoff in Multi-hop Wireless Networks

Changhun Bae and Wayne E. Stark
Department of Electrical Engineering and Computer Science
University of Michigan

Abstract

In this paper, energy-constrained wireless multi-hop networks with a single source-destination pair are considered. A network model that incorporates both the energy radiated by the transmitter and the energy consumed by the circuits that process the signals is proposed. The rate of communication is the number of information bits transmitted (end-to-end) per coded symbol transmitted by any node in the network that is forwarding the data. The tradeoff between the total energy consumption and the end-to-end rate of communication is analyzed. The performance (either energy or rate) depends on the transmission strategy of each node, the location of the relay nodes, the data rate used by each node. Two communication schemes that capture the inherent constraints of networks, bandwidth and energy respectively are proposed. For a given distribution of relays, i.e., when the number of hops and the end-to-end distance are given, it is shown that the total energy consumption can be minimized with an optimal selection of end-to-end rate for both schemes. In the case of equi-spaced relays, analytical results for the tradeoff between the energy and the end-to-end data rate are provided.

Index Terms

Bandwidth efficiency, energy efficiency, link adaptation, relay networks

This research was supported in part by the Air Force Office of Scientific Research under grant FA9550-07-1-0456.

I. INTRODUCTION

Wireless ad-hoc networks may be widely deployed in the near future. Wireless nodes in such a network are typically equipped with a small battery. Thus, wireless nodes must operate to maintain the network performance as long as possible without replacing battery. Consequently node can only support a finite number of information bits, which makes minimizing the energy consumption a critical design issue for an energy-efficient communication. In addition, efficient use of the available spectrum to transmit information bits across the network is also an important design consideration. Two main challenges in the energy consumption and the bandwidth utilization motivate analyzing the energy-bandwidth tradeoff in wireless multi-hop networks.

Previous research in energy-constrained networks has mainly focused on transmission schemes to minimize the radiated energy. The capacity per cost with general capacity cost functions on a single link is considered in [1], and the bits-per-Joule capacity to measure the efficiency of finite data transmission is described in [2]. Some optimal signaling methods in the wideband regime that minimize the energy per bit while considering the fundamental energy-bandwidth tradeoff is described [3]. Optimal scheduling problems to minimize the transmission energy by varying packet transmission times are considered in [4]. Emphasis on the transmission energy is quite reasonable for typical long range communications where transmission energy dominates the total energy consumption. However, for short range communication, energy dissipation in the circuit is not negligible and results in a more complicated tradeoff in the power-limited regime [16][17]. In [5]-[7] the circuit energy consumption is incorporated to minimize the total energy consumption but the bandwidth efficiency is not considered. Besides the energy consumption, the end-to-end throughput of wireless networks or equivalently the bandwidth efficiency also needs to be considered at the same time. Important guidelines are provided for bandwidth efficient network design in [8] where the cooperative relaying is exploited, and in [9]-[13] where every relay is located equidistantly on a straight line. However, the energy-bandwidth tradeoff and the optimal routing to maximize the bandwidth efficiency are analyzed without considering the circuit energy consumption [11]-[13].

In this paper, we consider the tradeoff between the total energy consumption and the end-to-end rate in wireless multi-hop networks where each node can be placed arbitrarily between a single source and destination node. Our problem is restricted in the sense that multi-hop transmission is based on point-to-point communication and interference-free scheduling which requires perfect synchronization among all nodes. Specifically, we formulate the overall costs in terms of the end-to-end bandwidth utilization (channel uses) and energy consumption to deliver one information bit across the network in order to fairly evaluate different multi-hop routing strategies. The end-to-end rate measures how efficiently the network utilizes the bandwidth to forward one information bit. We also consider the total energy consumption that includes the transmission energy and the circuit processing energy to forward data. For this scenario, we propose two practical communication schemes namely (1) common rate scheme and (2) common power scheme. For both schemes, we characterize the tradeoff between energy efficiency E_b/N_0 and bandwidth efficiency R. This is similar in nature to the tradeoff in a single hop transmission for AWGN channel determined by Shannon. Typically, the energy-bandwidth tradeoff has been studied with the linear equi-distant placed nodes, and only the transmission energy is included in the model [1]-[4],[9]-[13]. Our work differs from previous work in two fundamental ways. First, to account for the general topologies, each relay node can be placed arbitrarily between source and destination with the constraint that the sum of relaying distance should be at least equal or larger than the end-to-end distance. Second, our energy consumption model incorporates both transmission energy and circuit processing energy.

In summary, the main contribution of this paper is twofold. First, we propose a framework to analyze the tradeoff between total energy consumption and end-to-end rate. In particular, we quantify the impact of the relay's configuration on the tradeoff between energy and rate. Secondly, we show that there is a minimum total energy consumption per information bit that is obtained by the optimal selection of the end-to-end rate. We find the minimum energy for the optimal transmission scheme and for more practical transmission schemes.

The rest of this paper is organized as follows. We describe the system model in Section II and the performance measures of wireless networks (energy and rate) in Section III. In Section IV, we investigate the performance in terms of the end-to-end energy-bandwidth tradeoff for two proposed strategies called common power transmission and common rate transmission. In Section V the special case of equi-spaced relays is considered. We provide numerical results in Section VI and summarize the results in Section VII.

II. System Model

Consider a wireless network where a source node communicates with a destination node, separated by a distance d_e, through a multi-hop route with the information traversing through $k-1$ relay nodes that are arbitrarily located between the source and the destination as shown in Fig. 1. We denote the relay distance of hop i as d_i and let $\alpha_i = d_i/d_e$ where $0 < \alpha_i < 1$ and $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$. We assume each relay is in the far-field region of the corresponding transmitter [21]. For RF carrier frequencies from 400 MHz to 10 GHz, the far-field assumption is satisfied by relays separated at least 1m [22]. The received power $P_{r,i}$ in link i in the far-field is modeled as

$$P_{r,i} = \frac{\beta}{d_i^{\eta}} P_{out}, \quad d_i > 1$$

where P_{out} is the transmitter output power and η is a path-loss exponent (typically between 2 and 4), and β is a constant from the antenna characteristics. For simplicity we assume $\beta = 1$.

A simple decode-and-forward protocol is considered in which relay i decodes the message sent from relay $i-1$, re-encodes it, and then forwards the message to relay $i+1$. We consider link connectivity only between intermediate neighboring relays, and thus any cooperative relaying scheme to exploit cooperative diversity is not allowed. Each transmission is assumed to employ capacity-achieving codes with the same time duration for each coded symbol for each link. We impose no peak power constraint for each transmitter output or delay constraints. The rate of transmission R in information bits per channel use must be less than the capacity C for reliable communication. That is

$$R < C(\gamma) \Longleftrightarrow \gamma > g(R)$$

where γ is the received energy per channel use-to-noise power spectral density ratio and $g(R) =$

$C^{-1}(R)$ is the inverse of channel capacity $C(\gamma)$. We now make the following assumptions on the channel capacity function $C(\gamma)$.

1) $C(\gamma)$ is continuous in $\gamma \geq 0$ and twice differentiable, non-decreasing and strictly concave in γ.
2) $C(\gamma)$ has an inverse function. That is, $C^{-1}(x) = g(x)$ which is also continuous, twice differentiable in γ.

For AWGN channel the capacity $C(\gamma)$ and its inverse function $g(R)$ are given by

$$C(\gamma) = \frac{1}{2}\log_2(1+2\gamma) \Longleftrightarrow g(R) = \frac{2^{2R}-1}{2}.$$

We assume that each node has the same circuitry which requires circuit energy E_p Joules to process a single received coded symbol. In practice, the circuit processing energy depends on a transceiver design and architecture, and coding and modulation techniques. In some cases the energy consumption of the receiver is dominated by the RF front-end. In this case the energy per symbol is just the product of the receiver power consumption and the time duration for each channel use (modulation symbol). We assume E_p is a constant throughout this paper.

We assume that each node operates in half-duplex mode, hence it cannot transmit and receive simultaneously. Thus each transmission needs to be scheduled to avoid conflicts. Inter-link interference is not considered here since in this paper we assume that the network operates without spatial reuse and perfect synchronization is achieved among nodes, hence each relay node transmits in its own slot (time or frequency) without any interference.

III. Problem Formulation and Performance Measures

In this section, we describe the performance measures considered, namely the total energy consumption per information bit by the network and the end-to-end rate. The total energy consumption includes the transmission energy and the circuit processing energy. The end-to-end rate is the number of information bits transmitted (end-to-end) per coded symbol transmitted by any node in the network that is forwarding the data.

A. End-to-end Rate

The end-to-end rate is defined as the number of information bits transmitted end-to-end per channel use across the network. Consider N total uses of the channel by the k links. Link i connecting node $i-1$ and node i is allocated N_i exclusive channel uses. Node $i-1$ transmits at rate R_i on link i. The maximum end-to-end rate that can be achieved is [11][12]

$$R_e = \max_{\sum_i N_i = N} \min \left\{ \frac{N_i}{N} R_i \right\} = \max_{\sum_i q_i = 1} \min\{q_i R_i\} \tag{1}$$

where $q_i = N_i/N$. The optimal solution of the above minimax problem can be obtained easily by letting $q_i R_i = q_j R_j$ which yields $q_i^* = R_i^{-1}/\sum_j R_j^{-1}$. Therefore the *end-to-end rate* is

$$R_e = \frac{1}{\sum_{i=1}^{k} R_i^{-1}} \text{ (bits/channel use).} \tag{2}$$

B. End-to-end Energy Consumption

Consider the i-th link which is communicating at rate R_i. The energy consumption per information bit of link i, E_i, consists of transmission energy consumption and receiver energy consumption to process received symbols. Thus, we have

$$E_i = \frac{E_{i,tx} + E_p}{R_i} \tag{3}$$

where $E_{i,tx}$ is the energy transmitted per coded symbol and E_p is the energy consumed by the circuitry at hop i for each coded symbol. To compare the end-to-end energy consumption with the single hop case, we normalize the energy $E_{i,tx}$ as

$$\frac{E_{i,tx}}{N_0} d_e^{-\eta} \quad = \quad \frac{E_{i,tx}}{N_0} \alpha_i^{\eta} d_i^{-\eta} = \alpha_i^{\eta} \gamma_i \tag{4}$$

where $\gamma_i = \frac{E_{i,tx}}{N_0} d_i^{-\eta}$ denotes the received energy per coded symbol-to-noise power spectral density ratio at hop i. Similarly a normalized circuit processing energy γ_c is defined as

$$\gamma_c \triangleq \frac{E_p}{N_0} d_e^{-\eta}. \tag{5}$$

Our major interest in this paper is the total energy consumption that includes the transmission energy and the circuit processing energy required to communicate a bit across the network. In order to fairly represent the total energy consumption for different hops, we define the *normalized total energy consumption per information bit-to-noise power spectral density ratio* as

$$\frac{E_{tot}}{N_0} \triangleq \sum_{i=1}^{k} \frac{E_i}{N_0} d_e^{-\eta} = \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i + \gamma_c}{R_i}. \tag{6}$$

Note that for low rates, the processing energy for the circuitry will dominate the total energy consumption whereas for high rates the transmitted energy will dominate.

For a single link the received energy is the usual performance measure as opposed to the transmitted energy. In order to be consistent with this, we define the *end-to-end energy received per information bit-to-noise power spectral density ratio* as

$$\frac{E_b}{N_0} \triangleq \sum_{i=1}^{k} \frac{E_{i,tx}}{R_i} \frac{d_e^{-\eta}}{N_0} = \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i}{R_i} \equiv \left(\frac{E_{tot}}{N_0}\right)_{\gamma_c = 0} \tag{7}$$

which denotes the sum of transmission energy per information bit in each link that is involved in the end-to-end transmission, and a special case of the total energy consumption when $\gamma_c = 0$.

Our fundamental goal of this paper is, for a given multi-hop routing path $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$, to find the optimal set of transmission energies $(E_{1,tx}, ..., E_{k,tx})$ or equivalently $(\gamma_1, ..., \gamma_k)$ that provide the smallest total energy consumption for a given end-to-end data rate R_e. This problem corresponds to scarce wireless multi-hop networks where only a few relays are available and thus location of relays are not design variables. This general problem of minimizing total energy consumption for a given end-to-end rate can be formulated as the following optimization problem.

Problem Ia:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \min_{(\gamma_1, .., \gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i + \gamma_c}{R_i} \\ \text{s.t.} \quad & R_i = C(\gamma_i), \ i = 1, ..., k \\ & \textstyle\sum_{i=1}^{k} R_i^{-1} = R_e^{-1} \end{aligned} \tag{8}$$

where the optimization variables are γ_i's and $C(\gamma_i)$ denotes the channel capacity which is increasing concave. Problem Ia can be converted into an equivalent convex optimization problem as shown in Appendix A. The solution for the above problem can be obtained in a parametric form as follows.

Parametric Solution to Problem Ia: Let

$$x_i = \frac{1}{C(\gamma_i)} = \frac{1}{R_i} \tag{9}$$

and let $g(x) = C^{-1}(x)$ be the inverse function of the channel capacity function. By solving the following set of equations for x_i

$$\lambda = \alpha_i^\eta \left[g(x_i^{-1}) - \frac{1}{x_i} g'(x_i^{-1}) \right] + \gamma_c, \ i = 1, ..., k \tag{10}$$

where $x_i > 0$ and $\gamma_c \geq 0$, the solution to Problem Ia is as follows

$$\gamma_i = C^{-1}(x_i^{-1}) \tag{11}$$

$$R_e^{-1} = \sum_{i=1}^{k} x_i \tag{12}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} x_i \left[\alpha_i^\eta g(x_i^{-1}) + \gamma_c \right]. \tag{13}$$

Proof: See Appendix A. ■

We can obtain the solution to Problem Ia from the following procedure using numerical methods:

Step 1. Fix λ.

Step 2. Solve (10) for x_i for $i = 1, ..., k$.

Step 3. Determine γ_i, R_i and R_e from (9), (11) and (12).

Step 4. Determine E_{tot}/N_0 from (6) or (13).

By varying λ we can determine the tradeoff between E_{tot}/N_0 and R_e. For the case of AWGN channel, the channel capacity is given by

$$\frac{1}{C(\gamma_i)} = \frac{2}{\log_2(1 + 2\gamma_i)} = x_i \Rightarrow \gamma_i = C^{-1}(x^{-1}) = g(x^{-1}) \tag{14}$$

$$g(x^{-1}) = C^{-1}(x^{-1}) = \frac{1}{2}(2^{2/x} - 1). \tag{15}$$

Then the parametric solution to Problem Ia for AWGN channel is obtained by solving the following equation for x_i.

Solution to Problem Ia for AWGN channel: Let $u_i = 1 + \mathcal{W}\left(\frac{2\alpha_i^{-\eta}(\gamma_c-\lambda)-1}{e}\right)$. Then from (10) we obtain

$$2\lambda = \alpha_i^{\eta}\left[e^{2\ln 2/x_i} - 1 - \frac{2\ln 2 e^{2\ln 2/x_i}}{x_i}\right] + 2\gamma_c \quad (16)$$

$$\frac{1}{x_i} = \frac{1}{2\ln 2}u_i = C(\gamma_i) \Rightarrow \gamma_i = \frac{1}{2}(e^{u_i} - 1) \quad (17)$$

where $\mathcal{W}(\cdot)$ is the principal branch of the Lambert W-function, for which $\mathcal{W}(x) \geq -1$ [19]. Therefore the solution to Problem Ia for AWGN channel is as follows

$$R_e^{-1} = 2\ln 2\sum_{i=1}^{k}\frac{1}{u_i} \quad (18)$$

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \ln 2\sum_{i=1}^{k}\frac{\alpha_i^{\eta}(e^{u_i}-1)+2\gamma_c}{u_i} = \ln 2\sum_{i=1}^{k}\left[\frac{2\lambda}{u_i} + \alpha_i^{\eta}e^{u_i}\right] \quad (19)$$

where $x_i \geq 0$ for all i.

The next problem is to also optimize E_{tot}/N_0 over R_e.

Problem Ib:

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \sum_{i=1}^{k}\frac{\alpha_i^{\eta}\gamma_i + \gamma_c}{C(\gamma_i)} \quad (20)$$

where the optimization variables are γ_i's. Problem Ib can be converted into an equivalent convex problem as shown in Appendix B. The solution for Problem Ib can be obtained by solving the following equation

Solution to Problem Ib: From the optimal solution x_i^* to the following equation

$$\frac{g'(1/x_i)}{x_i} - g(1/x_i) = \alpha_i^{-\eta}\gamma_c \quad (21)$$

we can determine γ_i^*, R_e^*, and $E_{tot}(\overline{\alpha}, R_e^*)/N_0$ from (11)-(13).

Proof: See Appendix B. ■

Note that $\gamma_c = 0$ results in $x_i = \infty$ or equivalently $\gamma_i^* = 0$ since $h(x) \triangleq xg'(x) - g(x)$

is increasing in $x \geq 0$, which leads to $C(\gamma_i^*) = 0$ and $R_e^* = 0$ respectively. Hence using the following property, the minimum end-to-end received energy per information bit can be obtained when $\gamma_i \to 0$ or equivalently $R_e \to 0$.

Property III.1. $\left(\frac{E_{tot}}{N_0}\right)_{\gamma_c=0} = \frac{E_b}{N_0}$ *is strictly increasing in* $R_e > 0$.

Proof: See Appendix C. ■

Therefore the minimum end-to-end received energy per information bit for a given multi-hop routing path without any processing energy is as follows.

Solution to Problem Ib with $\gamma_c = 0$**:**

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{\gamma_i \to 0} \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i}{C(\gamma_i)} = \frac{1}{C'(0)} \sum_{i=1}^{k} \alpha_i^\eta \tag{22}$$

where $C'(0) = \lim_{\gamma \to 0} \frac{\gamma}{C(\gamma)}$.

For the case of AWGN channel, the solution to Problem Ib for the minimum E_{tot}/N_0 over R_e is as follows.

Solution to Problem Ib for AWGN channel: Let $u_i^* = 1+\mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) = 1+\mathcal{W}\left(\frac{2d_i^{-\eta}E_p - N_0}{eN_0}\right)$. Then the optimal solution to Problem Ib is given as follows:

$$\gamma_i^* = \frac{1}{2}(e^{u_i^*} - 1) \tag{23}$$

$$R_e^*(\overline{\alpha})^{-1} = 2\ln 2 \sum_{i=1}^{k} \frac{1}{u_i^*} \tag{24}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \ln 2 \sum_{i=1}^{k} \frac{\alpha_i^\eta (e^{u_i^*} - 1) + 2\gamma_c}{u_i^*} = \ln 2 \sum_{i=1}^{k} \alpha_i^\eta e^{u_i^*} = \ln 2 \sum_{i=1}^{k} \left(\frac{d_i}{d_e}\right)^\eta e^{u_i}. \tag{25}$$

Proof: See Appendix B. ■

From the above result for a fixed $\overline{\alpha}$, the normalized minimum total energy consumption increases as d_e. This is because u_i increases with d_i and d_i increases with d_e. We note from (25) that if the end-to-end distance is doubled by duplicating each link of a given distance then the total end-to-end energy consumption *without* the end-to-end distance normalization (at the optimal rate R_e^*) also doubles. Also note that $\gamma_c = 0$ results in $u_i = 0$ and $\gamma_i^* = 0$ since

$\mathcal{W}(-e^{-1}) = -1$, which leads to $R_i^* = 0$ and $R_e^* = 0$ respectively. Hence using the property III.1, the minimum end-to-end received energy per information bit can be obtained when $\gamma_i \to 0$ or equivalently $R_e \to 0$.

Solution to Problem Ib with $\gamma_c = 0$ for AWGN channel:

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{\gamma_i \to 0} \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i}{0.5 \log_2(1+2\gamma_i)} = \ln 2 \sum_{i=1}^{k} \alpha_i^\eta. \qquad (26)$$

Note that the above result can be verified directly from (24) by setting $u_i = 0$ for $i = 1, ..., k$.

Besides determining the best achievable performance, we also want to find the relation between $\frac{E_{tot}}{N_0}$ and R_e for practical transmission schemes. In the following section we consider two practical schemes for rate selection and transmission energy selection of physical layer at each hop and investigate the optimization problem in (20).

IV. Common Power, Common Rate Transmission Strategies

In this section we consider two practical schemes for selecting the rate and energy in each relay link. For the first scheme, we fix the transmission rate of each link and vary the transmission energy of each link. We call this the “Common Rate” scheme. For the second scheme, we fix the transmission energy of each symbol on each link and vary the rate of each link. We call this the “Common Power” scheme.

A. Performance of the Common Rate Model

We first consider multi-hop transmission where each node communicates at a common fixed rate, $R_i = R$ for all i. For reliable communication at each hop, the common rate R should be achievable in any link which is involved in forwarding data. Under multi-hop transmission where each link communicates using channel-capacity achieving codes over the channel, the relation between the rate and the signal-to-noise power spectral density ratio of each link is

$$\gamma_i = \gamma = g(R), \ \ i = 1, ..., k. \qquad (27)$$

Hence the transmission energy of link i is obtained as

$$E_{i,tx} = \gamma d_i^\eta N_0. \tag{28}$$

Notice that to communicate at a common rate, each link needs to achieve the same received energy per coded symbol-to-noise ratio. Therefore the transmission energy in each link is adjusted so as to achieve the same amount of received code symbol energy-to-noise ratio

$$E_{i,tx} = \left(\frac{d_i}{d_{max}}\right)^\eta E_{tx} \le E_{tx} \tag{29}$$

where $E_{i,tx}$ denotes the transmission energy per coded symbol in link i, and E_{tx} denotes the transmission energy per coded symbol in the link with the largest relay distance $d_{max} = \max_i d_i$. Therefore each link uses a rate R while it adjusts its transmission energy to $\left(\frac{d_i}{d_{max}}\right)^\eta E_{tx}$.

1) End-to-end rate: With a fixed rate at each hop, the end-to-end rate in (2) is simplified to

$$R_e = \frac{1}{\sum_{i=1}^k R^{-1}} = \frac{R}{k} = \frac{C(\gamma)}{k} \tag{30}$$

where the factor k^{-1} follows from bandwidth sharing among relays. From the above relation we observe the rate can be improved by decreasing number of hops or increasing link SNR.

2) End-to-end energy-bandwidth tradeoff: With the common rate scheme, the energy consumption for a given end-to-end rate is

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^k \frac{\alpha_i^\eta \gamma + \gamma_c}{R}. \tag{31}$$

From (27) and (30), solving for γ in terms of R_e and substituting into (31) yields the relation between total energy consumption and end-to-end rate as

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \frac{\gamma \sum_i \alpha_i^\eta + k\gamma_c}{C(\gamma)} = \frac{g(kR_e) \sum_{i=1}^k \alpha_i^\eta + \gamma_c}{R_e}. \tag{32}$$

Using (32) we can find the relation between the total energy and the end-to-end rate for the common rate transmission scheme.

3) Problem IIb: Consider the optimal end-to-end rate that minimizes the total energy consumption and the corresponding minimum total energy consumption. Problem IIb for the common rate scheme is the following optimization problem.

Problem IIb for the common rate case:

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{\gamma > 0} \quad \frac{\gamma \sum_{i=1}^{k} \alpha_i^{\eta} + k\gamma_c}{C(\gamma)} \tag{33}$$

where the optimization variable is γ. From (A.3) the above optimization problem can be converted into an equivalent convex problem with the transformation $x = 1/C(\gamma)$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{x > 0} \quad \left[\left(\sum_{i=1}^{k} \alpha_i^{\eta} \cdot x g(x^{-1}) \right) + k\gamma_c x \right]. \tag{34}$$

By taking the derivative with respect to x and setting it to 0, we have

$$\frac{g'(x^{-1})}{x} - g(x^{-1}) = \frac{k\gamma_c}{\sum_{i=1}^{k} \alpha_i^{\eta}}. \tag{35}$$

Let x^* be the solution to the above equation. Then the optimal end-to-end rate and the minimum total energy consumption are as follows

$$R_e^* \quad = \quad \frac{1}{kx^*} \tag{36}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} \quad = \quad g'(1/x^*) \cdot \sum_{i=1}^{k} \alpha_i^{\eta}. \tag{37}$$

For the case of AWGN channel, the optimal solution to (35) is given by

$$x^* \quad = \quad \frac{2 \ln 2}{\mathcal{W}\left(\frac{2k\gamma_c - \sum_{i=1}^{k} \alpha_i^{\eta}}{e \sum_{i=1}^{k} \alpha_i^{\eta}} \right) + 1}. \tag{38}$$

Therefore the optimal end-to-end rate and the minimum total energy consumption for AWGN channel are as follows

$$R_e^* = \frac{1}{kx^*} = \frac{1}{2k\ln 2}\left[\mathcal{W}\left(\frac{2k\gamma_c - \sum_{i=1}^k \alpha_i^\eta}{e\sum_{i=1}^k \alpha_i^\eta}\right) + 1\right] \tag{39}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \ln 2 \cdot 2^{2/x^*} = \left[\ln 2 \cdot \sum_{i=1}^k \alpha_i^\eta\right] \exp\left(\mathcal{W}\left(\frac{2k\gamma_c - \sum_i \alpha_i^\eta}{e\sum_i \alpha_i^\eta}\right) + 1\right). \tag{40}$$

4) Problem IIb with $\gamma_c = 0$: Now consider minimizing the total energy consumption when the circuit processing energy $\gamma_c = 0$. Problem IIb with $\gamma_c = 0$ is as follows.

Problem IIb for the common rate case with $\gamma_c = 0$:

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \min_{R_e > 0} \frac{g(kR_e)}{kR_e} \sum_{i=1}^k \alpha_i^\eta. \tag{41}$$

Note that the right hand side of (41) is increasing in $R_e > 0$ and thus the minimum occurs when $R_e \to 0$. Using L'Hôpital's rule, the minimum end-to-end received energy per information bit is

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{R_e \to 0} \frac{E_b}{N_0} = g'(0) \sum_{i=1}^k \alpha_i^\eta \tag{42}$$

where notice that for AWGN channel, $g'(0) = \ln 2$. Note that $\sum_i \alpha_i^\eta < 1$ represents the gain over multi-hop transmission. We see that increasing the number of hops, without violating the far-field constraint, is more energy-efficient in terms of the received energy per information bit in the power-limited regime. Comparing the result of (42) with the result of (37) or (40), we identify the penalty in the normalized signal-to-noise ratio, resulting from the circuit processing energy consumption.

B. Performance of the Common Power Model

Now consider multi-hop transmission where each node transmits with same energy per coded symbol, *i.e.* $E_{i,tx} = E_{tx}$ for all i while each rate of communication is determined by the received signal-to-noise ratio on each link. The received signal-to-noise ratio is determined by the location of relays. Therefore the common power scheme can be interpreted as an adaptive rate communication strategy. Normalizing by the end-to-end distance d_e and relay distance d_i, and noise power spectral density N_0, we have $\gamma_i \alpha_i^\eta = \gamma_j \alpha_j^\eta$. Using the minimum SNR from a link with the largest relaying distance, we have

$$\gamma_i = \pi_i \gamma_{min} \geq \gamma_{min} \tag{43}$$

$$\pi_i \triangleq \left(\frac{\alpha_{max}}{\alpha_i}\right)^\eta \tag{44}$$

where γ_{min} is the minimum link SNR and $\alpha_{max} = \max_i \alpha_i$, and π_i denotes the ratio of the maximum relay distance to the i-th relay distance.

1) End-to-end rate: With a common transmission energy at each hop, the rate on link i is

$$R_i = C(\gamma_i),\ i = 1, ..., k \tag{45}$$

where $\gamma_i = E_{tx} d_i^{-\eta} / N_0$. The end-to-end rate in (2) is given by

$$R_e = \frac{1}{\sum_i R_i^{-1}} = \left[\sum_{i=1}^{k} \frac{1}{C(\pi_i \gamma_{min})}\right]^{-1} \tag{46}$$

where $\gamma_{min} = \frac{E_{tx}}{N_0} d_{max}^{-\eta}$.

2) End-to-end energy-bandwidth tradeoff: The end-to-end energy consumption can be determined from the minimum signal-to-noise ratio.

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} \frac{E_{tx} + E_p}{R_i} \frac{d_e^{-\eta}}{N_0}. \tag{47}$$

Substituting the relation of (43) into (47) yields the relation between total energy consumption and end-to-end rate as

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} \frac{\alpha_{max}^\eta \gamma_{min} + \gamma_c}{C(\pi_i \gamma_{min})} = \frac{\alpha_{max}^\eta \gamma_{min} + \gamma_c}{R_e}. \tag{48}$$

Thus we can parametrically determine the end-to-end rate and the corresponding normalized total energy consumption. As we vary either E_{tx} or γ_{min} the end-to-end rate changes as does the total energy consumption.

3) Problem IIb: Consider the optimal end-to-end rate minimizing the total energy consumption and the corresponding minimum total energy consumption. Problem IIb of the common rate case can be simplified to the following optimization problem.

Problem IIb for the common power case:

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{\gamma_{min}>0} \sum_{i=1}^{k} \frac{\alpha_{max}^{\eta}\gamma_{min} + \gamma_c}{C(\pi_i \gamma_{min})}. \tag{49}$$

From (A.3) (49) can be converted into an equivalent convex problem with a transformation of variable $x_i = 1/C(\pi_i \gamma_{min})$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{x_1,\ldots,x_k} \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right]. \tag{50}$$

By taking the derivative with respect to x_i and setting it to 0, we have

$$\frac{g'(x_i^{-1})}{x_i} - g(x_i^{-1}) = \alpha_i^{-\eta}\gamma_c. \tag{51}$$

Let x_i^* be the solution to the above equation. Then the optimal end-to-end rate and the minimum total energy consumption as follows

$$R_e^* = \left(\sum_{i=1}^{k} x_i^*\right)^{-1} \tag{52}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \sum_{i=1}^{k} \alpha_i^{\eta} g'(1/x_i^*). \tag{53}$$

For the case of AWGN channel, the optimal solution to (51) is given by

$$x_i^* = \frac{2\ln 2}{\mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) + 1}. \tag{54}$$

Therefore the optimal end-to-end rate and the minimum total energy consumption for AWGN channel are as follows

$$R_e^* = \left(\sum_{i=1}^{k} x_i^*\right)^{-1} = \frac{1}{2\ln 2}\left[\sum_{i=1}^{k} \frac{1}{\mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) + 1}\right]^{-1} \tag{55}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \ln 2 \cdot \sum_{i=1}^{k} \alpha_i^{\eta} \cdot 2^{2/x_i^*} = \ln 2 \cdot \left[\sum_{i=1}^{k} \alpha_i^{\eta} \cdot \exp\left(\mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) + 1\right)\right]. \tag{56}$$

4) Problem IIb with $\gamma_c = 0$: Now consider minimizing the total energy consumption when the circuit processing energy $\gamma_c = 0$. From (50), we have

Problem IIb for the common power case with $\gamma_c = 0$:

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \min_{x_1,\ldots,x_k} \sum_{i=1}^{k} x_i \alpha_i^\eta g(x_i^{-1}). \tag{57}$$

It can be shown easily that the right hand side of (57) is increasing in R_e. Therefore the minimum end-to-end received energy per information bit is achieved when $R_e \to 0$ or equivalently $x_i \to \infty$. By taking the limit as $x_i \to \infty$ and using L'Hôpital's rule, the minimum end-to-end received energy per information bit is

$$\lim_{R_e \to 0} \frac{E_b}{N_0} = \lim_{x_i \to \infty} \sum_{i=1}^{k} x_i \alpha_i^\eta g(x_i^{-1}) = g'(0) \sum_{i=1}^{k} \alpha_i^\eta \tag{58}$$

which is same as the result of the common rate in (42). Within the far-field region, using multiple hops for low rates is more energy efficient compared to single hop when the circuit processing energy is not taken into account.

V. Performance of Equidistant Multi-hop Routing

In this section we derive the optimal end-to-end rate and number of hops to minimize the overall energy consumption of equi-spaced relays. The optimal solutions are obtained by decoupling the joint optimization problem in rate and number of hops into two sub-problems. We first obtain the optimal number of hops as a function of R_e. Then the optimal value of R_e is derived [6][23]. Finally we derive the solution to Problem IIb for the equi-spaced relays.

Consider the case of equi-spaced relays, $\alpha_i = 1/k$, which makes the common rate case and the common power case identical. The end-to-end rate R_e is then

$$R_e = \frac{R}{k} = \frac{C(\gamma)}{k} \tag{59}$$

where γ denotes the link SNR. From (6) and (59), the total energy consumption per information bit is

$$\frac{E_{tot}(k,R_e)}{N_0}=\frac{k^{-\eta}g(kR_e)+\gamma_c}{R_e}. \tag{60}$$

With a transformation of variable $e^{-x}=k$ and $R_e^{-1}=e^y$, the minimum total energy consumption can be converted into an equivalent optimization problem

$$\frac{E_{tot}(x^*,y^*)}{N_0}=\min_{y\in\mathbf{R},x\in\mathbf{R}} e^y\left[e^{\eta x}g(e^{-x-y})+\gamma_c\right]. \tag{61}$$

Proposition V.1. *The objective function* $E_{tot}(x,R_e)/N_0$ *is convex in* x,R_e *for* $R_e>0$ *and* $x\in\mathbf{R}$ *if* $g(u)$ *satisfies the following conditions*

1) $a'=u^2g''(u)-ug'(u)+g(u)>0$.
2) $a'c'-(b')^2>0$ *where* $b'=\eta ug'(u)-u^2g''(u)-ng(u)$ *and* $c'=u^2g''(u)+\eta^2g(u)+(1-2\eta)ug'(u)$.

Proof: See Appendix D. ∎

Therefore if the sufficient conditions are satisfied, the optimization problem is convex problem and thus the locally optimal solution is globally optimal. Let $\hat{x}^*$ and $\hat{y}^*$ be the locally optimal solution to (61). The locally optimal solutions can provide an upper bound to the global optimum

$$\frac{E_{tot}(x^*,y^*)}{N_0}\le\frac{E_{tot}(\hat{x}^*,\hat{y}^*)}{N_0} \tag{62}$$

where the equality holds if the convexity is guaranteed. For the case of AWGN channel, it can be easily verified that the objective function in (61) is convex (see appendix D). Other channels of interest are the binary input AWGN channel and the binary input hard decision AWGN channel whose channel capacity functions are given by respectively [24]

$$C_{BISO}(\gamma) = 1-\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x-\sqrt{2\gamma})^2}{2}\right)\log_2(1+e^{-2x\sqrt{2\gamma}})dx \tag{63}$$

and

$$C_{BIBO}(\gamma) = 1-H_2(Q(\sqrt{2\gamma})) \tag{64}$$

where $H_2(x) = -x\log_2(x) - (1-x)\log_2(1-x)$ is the binary entropy function. Numerical methods can verify the sufficient conditions for convexity of the problem for the above two channels.

A. Problem IIIa: Optimization of the number of hops

For a given end-to-end rate, treating the number of hops as a continuous variable, we optimize the total energy consumption over x

$$\frac{E_{tot}(\hat{x}^*, y)}{N_0} = \min_{x\in\mathbf{R}} e^y \left[e^{\eta x} g(e^{-x-y}) + \gamma_c \right]. \tag{65}$$

Setting the derivative with respect to x equal to zero yields $\eta g(e^{-x-y}) = e^{-x-y} g'(e^{-x-y})$. Let $\hat{u}^*$ be the solution to

$$\eta g(u) = u g'(u). \tag{66}$$

The optimal number of hops for a given end-to-end rate is

$$\hat{k}^* = e^{-x^*} = e^y \hat{u}^* = \frac{\hat{u}^*}{R_e}. \tag{67}$$

The minimum of the total energy consumption at the optimal number of hops is given by

$$\begin{aligned} \frac{E_{tot}(\hat{x}^*, y)}{N_0} &= g(\hat{u}^*)(\hat{u}^*)^{-\eta} e^{(1-\eta)y} + \gamma_c e^y && (68)\\ &= g(\hat{u}^*)(\hat{u}^*)^{-\eta} R_e^{\eta-1} + \gamma_c R_e^{-1}. && (69) \end{aligned}$$

B. Problem IIIb: Optimization of the end-to-end rate

The problem of determining the optimal end-to-end rate is formulated by substituting (67) into (61). This results in

$$\begin{aligned} \frac{E_{tot}(\hat{x}^*, \hat{y}^*)}{N_0} &= \min_y \min_x e^y \left[e^{\eta x} g(e^{-x-y}) + \gamma_c \right] && (70)\\ &= \min_y \left[g(\hat{u}^*)(\hat{u}^*)^{-\eta} e^{(1-\eta)y} + \gamma_c e^y \right]. \end{aligned}$$

Setting the derivative with respect to y equal to zero yields the optimal end-to-end rate $\hat{R}_e^* = e^{-\hat{y}^*}$ as follows

$$e^{-\hat{y}^*} = \hat{R}_e^* = \frac{{\gamma_c}^{1/\eta}\hat{u}^*}{[(\eta-1)g(\hat{u}^*)]^{1/\eta}}. \tag{71}$$

The optimal number of hops $\hat{k}^*(\hat{R}_e^*)$ is obtained by substituting (71) into (67),

$$\hat{k}^*(\hat{R}_e^*) = g(\hat{u}^*)^{1/\eta}\left[\frac{\gamma_c}{\eta-1}\right]^{-1/\eta} = g(\hat{u}^*)^{1/\eta}\left[\frac{E_p}{N_0}\frac{1}{(\eta-1)}\right]^{-1/\eta} d_e. \tag{72}$$

The resulting minimum total energy consumption is then

$$\frac{E_{tot}(\hat{x}^*, \hat{R}_e^*)}{N_0} = \frac{g(\hat{u}^*)^{1/\eta}}{\hat{u}^*}\eta\left[\frac{\gamma_c}{\eta-1}\right]^{1-1/\eta} \tag{73}$$

$$= \frac{g(\hat{u}^*)^{1/\eta}}{\hat{u}^*}\left[\frac{\eta}{(\eta-1)^{1-1/\eta}}\right]\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta} \tag{74}$$

$$= f_c(\eta)\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta} \tag{75}$$

where $f_c(\eta)$ is a function of the channel and the propagation loss exponent. From the above results we observe that the optimal rate decreases as d_e^{-1} and the optimal number of hops grows linearly with d_e. In addition, there are two key observations for the minimum normalized total energy consumption: it (i) grows as the $\frac{\eta-1}{\eta}$ power of the processing energy and (ii) decreases as $d_e^{1-\eta}$. Therefore we can conclude that the actual unnormalized minimum total energy consumption increases linearly with d_e from the relation between the normalized total energy consumption and the end-to-end distance in (6).

For the case of AWGN channel, it can be easily verified that the objective function in (61) is convex (see Appendix D). Therefore the locally optimal solution is indeed the globally optimal solution. The optimal solution to (66) is $u^* = a/2\ln 2$ where $a = \mathcal{W}(-\eta e^{-\eta}) + \eta$ which is determined from the path-loss exponent. Therefore the optimal number of hops $k^*(R_e^*)$ is

$$k^*(R_e^*) = \left[\frac{ae^a}{2\eta}\right]^{1/\eta}\left[\frac{\gamma_c}{\eta-1}\right]^{-1/\eta} \tag{76}$$

and the optimal end-to-end rate R_e^* is given by

$$R_e^* = \frac{a}{2\ln 2}\left[\frac{2\eta}{ae^a}\right]^{1/\eta}\left[\frac{\gamma_c}{\eta-1}\right]^{1/\eta}. \tag{77}$$

The minimum total energy consumption over the number of hops and the end-to-end rate is then

$$\begin{aligned}\frac{E_{tot}^*(k^*, R_e^*)}{N_0} &= \frac{\ln 2}{a}\left[\frac{ae^a}{2\eta}\right]^{1/\eta}\eta\left[\frac{\gamma_c}{\eta-1}\right]^{1-1/\eta} && (78)\\ &= \frac{\ln 2}{a}\left[\frac{ae^a}{2\eta}\right]^{1/\eta}\frac{\eta}{(\eta-1)^{1-1/\eta}}\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta}. && (79)\end{aligned}$$

Notice that the optimal energy consumption constant

$$f_c(\eta) = \frac{g(\hat{u}^*)^{1/\eta}}{\hat{u}^*}\frac{\eta}{(\eta-1)^{(1-1/\eta)}} \tag{80}$$

from (74) depends on the channel capacity function. For different AWGN channels, the resulting energy consumption constants are given in Table I along with the loss incurred relative to the AWGN channel.

C. Problem IVa: Optimization of the end-to-end rate

Now consider optimizing over the rate first and then the number of hops using the result of (37) and (53) because common rate case and common power case for equi-spaced relays are equivalence. The result of both cases in (34) and (50) gives the minimum total energy consumption per information bit

$$\frac{E_{tot}(k, R_e^*)}{N_0} = \min_{v>0}\quad [k^{1-\eta}vg(v^{-1}) + k\gamma_c v] = k^{1-\eta}g'(1/\hat{v}^*). \tag{81}$$

where $\hat{v}^*$ is the solution to $v^{-1}g'(v^{-1}) - g(v^{-1}) = k^\eta\gamma_c$ and $v = 1/C(\gamma)$. The corresponding optimal rate for k equi-spaced relays is given by

$$\hat{R_e}^* = \frac{1}{k\hat{v}^*}. \tag{82}$$

For the case of AWGN channel where the locally optimum is the global optimum, the optimal rate is given by

$$R_e^* = \frac{1}{2k\ln 2}\left[\mathcal{W}\left(\frac{2\gamma_c k^\eta - 1}{e}\right) + 1\right]. \tag{83}$$

and the resulting minimum energy consumption is

$$\frac{E_{tot}(k, R_e^*)}{N_0} = \left(k^{1-\eta}\ln 2\right)\exp\left(\mathcal{W}\left(\frac{2\gamma_c k^\eta - 1}{e}\right) + 1\right). \tag{84}$$

When the circuit processing energy is ignored, $\gamma_c = 0$, from (59) and (60), we have

$$\left(\frac{E_{tot}}{N_0}\right)_{\gamma_c=0} = \frac{E_b}{N_0} = \frac{g(kR_e)}{kR_e}k^{1-\eta} \tag{85}$$

which is an increasing function of R_e. Therefore the minimum end-to-end energy received per information bit for a given number of hops is given by

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{R_e \to 0}\frac{E_b}{N_0} = k^{1-\eta}g'(0) \tag{86}$$

which can be also verified in (81) for $x \to \infty$. Note that from (86), multi-hops can provide more energy-efficiency for low rates. However (85) is increasing in k for high rate that implies a single hop transmission is the most energy-efficient.

D. Problem IVb: Optimization of the number of hops

Now we further optimize the result of problem IVa over the number of hops similar to problem III. Since $v^{-1}g'(v^{-1}) - g(v^{-1}) = k^\eta\gamma_c$, by setting the derivative of (81) with respect to k equal to zero, we obtain

$$\hat{k}^* = \left[\frac{(\eta-1)\hat{v}^* g(1/\hat{v}^*)}{\gamma_c}\right]^{1/\eta}. \tag{87}$$

Substituting the above result into (81) yields

$$\frac{E_{tot}(k^*, R_e^*)}{N_0} = \left[\frac{\gamma_c}{\eta-1}\right]^{1-1/\eta}\left[\hat{v}^* g(1/\hat{v}^*)\right]^{1/\eta} g'(1/\hat{v}^*). \tag{88}$$

For AWGN channel since $\frac{\partial \mathcal{W}(x)}{\partial x} = \frac{\mathcal{W}(x)}{x(\mathcal{W}(x)+1)}$, by setting the derivative of (84) with respect to

k equal to zero, we obtain the optimal number of hops as

$$k^* = \left(\frac{b}{2\gamma_c}\right)^{1/\eta} \tag{89}$$

where b is the solution to

$$\frac{\eta-1}{\eta} = \frac{b\mathcal{W}(\frac{b-1}{e})}{(b-1)(\mathcal{W}(\frac{b-1}{e})+1)}. \tag{90}$$

Note that, as expected, the optimal number of hops varies inversely with the processing energy at each hop. Then substituting (89) into (84) yields

$$\frac{E^*_{tot}(k^*, R^*_e)}{N_0} = \ln 2 \left(\frac{2}{b}\right)^{\frac{\eta-1}{\eta}} e^{1+\mathcal{W}(\frac{b-1}{e})} (\gamma_c)^{\frac{\eta-1}{\eta}}. \tag{91}$$

By using numerical methods we can identify that the above results for the optimal number of hops and the minimum energy consumption are same as the result of (76) and (78).

VI. Numerical Results

In the results we assume the path-loss exponent $\eta = 4$, the end-to-end distance $d_e = 3000$m, the noise power spectral density $N_0 = -174$dBm/Hz and the circuit processing energy $E_p = 0.95\mu$J/symbol ($\gamma_c = 4.69$dB) and 0.095μJ/symbol ($\gamma_c = -5.31$dB) unless otherwise specified. Fig. 2 depicts the normalized energy-bandwidth tradeoff for various AWGN channel models for a two hop network where the ratio of relay distances is equal to 99. This is done by applying the procedures to find the solution of Problem Ia in (9) to binary input with hard decision AWGN channel (BIBO AWGN), binary input AWGN channel (BISO AWGN), and unquantized input/ouput AWGN channel (SISO AWGN). As expected it is observed that AWGN channels with input constraint show limited bandwidth utilization. The end-to-end rate of input constraint AWGN channels is bounded at 0.5 (bits/channel use) in a two hop network. In addition, the total energy consumption of input constraint AWGN channels shows a degraded performance compared to the unquantized AWGN channel because of their limited bandwidth utilization.

In Fig. 3 we plot the normalized energy-bandwidth tradeoff for a two hop network with the ratio of link distances equal to 99. This is done for the common rate transmission scheme from

(22) and common power transmission scheme from (34) and (36) as well as the optimal scheme obtained from the solution of Problem Ia. At low rates the energy consumption due to the transmitter alone decreases while the overall energy consumption increases due to the energy needed for processing. At high rates the energy consumption increases due to the increase in transmitted energy. Because of this there is an optimal nonzero rate that minimizes the total energy consumption. These curves also show that the common power transmission strategy is nearly as good as the optimal transmission scheme whereas the common rate scheme requires significantly higher energy. In Fig. 4 we plot the normalized energy-bandwidth tradeoff for $E_p = 0.095\mu$J/symbol. As the circuit processing energy becomes small, the optimal rate for minimum total energy consumption becomes small. The energy-bandwidth tradeoff shows similar performance without circuit processing energy because the transmission energy is more dominant at low processing energy. Fig. 5 plots the normalized energy-bandwidth characteristic without processing energy. The same minimum energy consumption is achieved when $R_e \to 0$ from the solution of Problem Ib in (26), the common rate scheme in (42) and the common power scheme in (58).

Fig. 6 plots the normalized energy-bandwidth tradeoff with various locations of relays. The common power case outperforms the common rate case by allowing adaptive rate in each link. In particular the common power case with an irregular distribution of relays may outperform the equi-spaced relays case at high rates. This is because the gain from the irregular distribution of relays at high SNR dominates the performance. It implies that the equi-spaced relay case is not always an optimal route for a given number of hops, which further suggests to reduce the bandwidth sharing at high SNR. Therefore at high rates a single hop transmission shows the best performance while the minimum energy consumption is achieved at the equi-spaced relays. In Fig. 7 we plot the result of (69) and the minimum energy consumption for single and two hop networks for comparison. It is observed that the use of time sharing between single hop and multi-hop transmissions can improve the performance. By using time sharing between single hop and two hops, there exists a region in which time sharing between different multi-hop transmissions can improve the energy efficiency. For instance the total energy consumption can be reduced around $R_e = 2$ by using time sharing between single hop and equi-spaced two hops.

Fig. 8 plots the normalized energy-bandwidth characteristic for various locations of relays without processing energy. As expected multi-hop transmissions within the far-field region is more energy-efficient at low rates without considering processing energy. Fig. 9 and Fig. 10 plot the normalized energy-bandwidth characteristic for the equi-spaced relays from the result of (69). Fig. 11 and Fig. 12 compare the normalized energy-bandwidth tradeoff for various AWGN channel models from the result of (69). Since we approximated k by a continuous variable, the optimal number of hops could be less than 1 at high rates which is impossible practically. To represent the practical optimal number of hops, the curves for single hop case, $k = 1$ are plotted respectively for BIBO and BISO AWGN channels. The threshold end-to-end rate where a single hop is the most energy-efficient is 0.907, 0.894 and 2.856 for BIBO, BISO and SISO AWGN channel respectively. This threshold does not depend on γ_c. As expected, SISO AWGN channel outperforms the input constrained AWGN channels, BIBO and BISO AWGN channels because both channels suffer from their limited channel utilization.

Fig. 13 depicts the energy-bandwidth characteristic for the equi-spaced relays with selected end-to-end distances and number of hops. At low end-to-end rates the transmission energy consumption is $k^{1-\eta} \ln 2$ which is negligible compared to the receiver processing energy consumption. Hence from (60) the energy consumption is dominated by the receiver energy consumption. For low end-to-end rates the receiver energy depends on the end-to-end distance but not the number of hops. However, at high rates the receiver energy consumption is negligible compared to the transmission energy consumption. Therefore the energy consumption at high rates depends on the number of hops but not the end-to-end distance.

Finally consider a source, destination and a single relay. We have shown that when the three nodes are located on a straight line at high rates the single hop routing strategy is better than a two hop strategy while at low rates the two hop strategy is more energy efficient. However, when the nodes are not collinear then the additional distance for a two hop strategy may require more energy than a single hop strategy. In Fig. 14 we plot the feasible region of relay locations for selected end-to-end rates where the minimum energy consumption of a two hop network for a given end-to-end rate is less than that of a single hop network using the result of (10) and (11). As the end-to-end rate increases, the feasible region of relays shrinks because at high end-to-end rates a single hop network achieves the minimum energy consumption. Observe that in the region where a two hop network is more energy efficient than a single hop network, the optimal distribution of relays is that of equi-distant relays. Note that the processing energy and the end-to-end distance, *i.e.* γ_c do not affect the region for a given end-to-end rate. However in terms of the minimum total energy consumption over the end-to-end rate, the feasible region depends on the processing energy and distance through γ_c. This is because the gain from multi-hop routing decreases as the circuit energy consumption increases. In Fig. 15 we plot the feasible region of relay locations for selected receiver processing energies where two hop routing is better than single hop in terms of the minimum total energy consumption at the optimal end-to-end rate. As explained the feasible region of two hop network becomes smaller as γ_c increases.

VII. Conclusion

In this paper, we considered interference-free communication through multi-hop routing where each relay is not necessarily placed on a line between a single source-destination pair. We formulated the normalized total energy consumption and the end-to-end bandwidth utilization to transmit information bits for the optimal transmission energy and rate on each hop and two practical communication schemes: (1) common rate scheme and (2) common power scheme. The results showed that the common power case yields better energy-bandwidth tradeoff than the common rate case due to the rate adaptation in each link, while both cases have the same asymptotic performance in the power-limited regime when the circuit processing energy is not taken into account. In addition, we showed that an irregular distribution of relays with common power scheme can outperform equi-spaced relays at high end-to-end rates, which implies reducing the number of hops is more energy-efficient. We also showed that the total energy consumption can be minimized by optimally choosing the rate, determined from the location of relays and the end-to-end distance. Therefore, by comparing the total energy consumption for different location of relays, a routing path which achieves the best energy-bandwidth tradeoff can be obtained. In addition, we showed that by optimizing jointly over the end-to-end rate and number of hops for equi-spaced relays, the optimal number of hops and the optimal end-to-end rate can be found.

APPENDIX A
CONVEXITY AND SOLUTION OF PROBLEM IA

Proposition A.1. *Suppose* $f : \mathbf{R} \rightarrow \mathbf{R}$ *is increasing and concave. Let* g *denote its inverse, i.e.,* $g(f(x)) = x$*. Then* $y = g(x)$ *is an increasing convex function.*

Proof: Since $g(f(x)) = x$, we have $f'(g(x))g'(x) = 1$. Hence the first derivative with respect to x is given by

$$g'(x) = \frac{1}{f'(g(x))} > 0 \tag{92}$$

since $f'(x) > 0$. Further the second derivative with respect to x is given by

$$g''(x) = -\frac{f''(g(x))g'(x)}{[f'(g(x))]^2} > 0 \tag{93}$$

since $g'(x) > 0$ and $f''(x) < 0$. Therefore $g(x)$ is an increasing convex function. ■

Proposition A.2. *The function* $y = x(c_1 g(x^{-1}) + c_2)$ *is convex in* x *for* $x \geq 0$ *and* $c_1 > 0, c_2 \geq 0$ *if* $g(x)$ *is increasing convex.*

Proof: The first order partial derivative with respect to x is given by

$$\frac{\partial y}{\partial x} = c_1 g(x^{-1}) - c_1 \frac{g'(x^{-1})}{x} + c_2 \tag{94}$$

and further the second order derivative with respect to x is given by

$$\frac{\partial^2 y}{\partial x^2} = \frac{c_1 g''(x^{-1})}{x^3} > 0 \tag{95}$$

since $g''(x) > 0$ for all x. ■

For a given location of relays, the energy optimization problem for a given end-to-end rate

R_e from (8) is

$$\begin{array}{rl} \min\limits_{(\gamma_1,..,\gamma_k)} & \sum\limits_{i=1}^{k} \dfrac{\alpha_i^{\eta}\gamma_i + \gamma_c}{C(\gamma_i)} \\ \text{s.t.} & \sum_i C(\gamma_i)^{-1} = R_e^{-1} \end{array} \tag{96}$$

where the optimization variables are γ_i's. Let $x_i = \frac{1}{C(\gamma_i)}$ and $C^{-1}(x) = g(x)$. Then the optimization problem can be converted into an equivalent convex optimization problem

$$\begin{array}{rl} \min\limits_{(x_1,..,x_k)} & \sum\limits_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right] \\ \text{s.t.} & \sum_i x_i = R_e^{-1} \end{array} \tag{97}$$

where the optimization variables are x_i's and the equality constraint is affine.

Proposition A.3. *The function* $f(x_1, ..., x_k) = \sum\limits_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right]$ *is convex in* x_i *for* $x_i \geq 0$ *for* $i = 1, ..., k$ *and* $\gamma_c \geq 0$.

Proof: To prove convexity of $f(x_1, ..., x_k)$, it is sufficient to prove convexity of $h(x_i) = x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right]$ for any i since a non-negative sum of convex functions is convex. From (A.1), $g(x) = C^{-1}(x)$ is increasing convex since the channel capacity function $C(x)$ is increasing concave. Then using the result from (A.2), $h(x_i)$ is convex for $x_i > 0$. ■

The optimal solution can be obtained using the Lagrangian function of (97)

$$L(x_1, ..., x_k, \lambda) = \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right] - \lambda(\sum_i x_i - R_e^{-1}) \tag{98}$$

where $x_i \geq 0$ for all i. Therefore the Karush-Kuhn-Tucker (KKT) conditions give the following parametric solution which can be evaluated by numerical techniques,

$$\lambda = \alpha_i^{\eta} \left(g(x_i^{-1}) + x_i \frac{\partial g(x_i^{-1})}{\partial x_i} \right) + \gamma_c \tag{99}$$

$$= \alpha_i^{\eta} \left(g(x_i^{-1}) - \frac{1}{x_i} g'(x_i^{-1}) \right) + \gamma_c \tag{100}$$

$$\sum_{i=1}^{k} x_i = \frac{1}{R_e} \tag{101}$$

where $x_i \geq 0$ for all i.

Appendix B

Convexity and solution of Problem Ib

For a given location of relays, the energy optimization problem is

$$\min_{(\gamma_1,..,\gamma_k)} \quad \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i + \gamma_c}{C(\gamma_i)} \tag{102}$$

where the optimization variables are γ_i's. Let $x_i = \frac{1}{C(\gamma_i)}$ and $C^{-1}(x) = g(x)$. Then the optimization problem can be converted into an equivalent convex optimization problem from (A.3)

$$\min_{(x_1,..,x_k)} \quad \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right] \tag{103}$$

where the optimization variables are x_i's.

Setting the first derivative to zero yields

$$\frac{g'(1/x_i)}{x_i} - g(1/x_i) = \alpha_i^{-\eta}\gamma_c \tag{104}$$

where $x_i \geq 0$ for all i. Therefore from the optimal solution of the above equation, we obtain the optimal SNR for i-th hop

$$\gamma_i^* = g(1/x_i^*) = C^{-1}(1/x_i^*) \tag{105}$$

where $x_i^* \geq 0$ for all i.

For the case of AWGN channel, from (104) we obtain

$$\frac{\ln 2}{x_i^*} e^{2\ln 2/x_i^*} - \frac{1}{2}(e^{2\ln 2/x_i^*} - 1) = \alpha_i^{-\eta}\gamma_c \tag{106}$$

where $x_i^* \geq 0$ for all i. Substituting $2\ln 2/x_i^* = u_i^* + 1$ yields

$$u_i^* = \frac{2\ln 2}{x_i^*} - 1 = \mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) \tag{107}$$

$$\frac{1}{x_i^*} = C(\gamma_i^*) = \frac{1}{2}\log_2(1 + 2\gamma_i^*) \tag{108}$$

$$\gamma_i^* = \frac{1}{2}\left[\exp\left(\mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) + 1\right) - 1\right]. \tag{109}$$

Using (109), it is straightforward to obtain the optimal end-to-end rate and the minimum total energy consumption in (24). Further using $u_i = 1 + \mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right)$, we obtain $e^{u_i} - 1 = u_i e^{u_i} - 2\alpha_i^{-\eta}\gamma_c$ which yields the second part of the minimum total energy consumption in (24).

APPENDIX C

PROOF OF PROPERTY III.1

Property III.1 $\left(\frac{E_{tot}}{N_0}\right)_{\gamma_c=0} = \frac{E_b}{N_0}$ is strictly increasing in $R_e > 0$.

Proof: Suppose that for a given R'_e, $\{\gamma'_1, ..., \gamma'_k\}$ is the set of received SNR that achieves the rate at each hop R'_i and the end-to-end received energy per information bit E'_b/N_0. Now suppose we are allowed to have $\lambda R'_e$ where $\lambda > 1$. Then it is easy to see that the rate at each hop has increased by λ from the fact that $\frac{1}{\lambda R'_e} = \sum_{i=1}^{k} \frac{1}{\lambda R'_i}$. Since this new received SNR γ_i resulting from the increased rate of each hop should only be greater than γ'_i and $\frac{\gamma_i}{C(\gamma_i)}$ in (7) for $i = 1, ..., k$ is an increasing function on γ_i, E_b/N_0 is strictly increasing in R_e. ■

APPENDIX D

CONDITIONS FOR CONVEXITY OF PROBLEM III

Proposition V.1 The function $f(x, y) = e^y\left[e^{\eta x} g(e^{-x-y}) + \gamma_c\right]$ is convex in x, y for $x, y \in \mathbf{R}$ if $g(u)$ satisfies the following conditions for $u \geq 0$

1) $a' = u^2 g''(u) - u g'(u) + g(u) > 0$.
2) $a'c' - (b')^2 > 0$ where $b' = \eta u g'(u) - u^2 g''(u) - n g(u)$ and $c' = u^2 g''(u) + \eta^2 g(u) + (1 - 2\eta) u g'(u)$.

Proof: To prove convexity of $f(x,y)$ in x and y, it is sufficient to prove convexity of $f_1(x,y) = e^{\eta x+y} g(e^{-x-y})$ since $f_2(y) = \gamma_c e^y$ is convex in y for $\gamma_c \geq 0$ and a non-negative sum of convex functions is convex. The second order partial derivative of $f_1(x,y)$ with respect to x is given by

$$\frac{\partial^2 f(x,y)}{\partial x^2} = e^{y+\eta x}\left[g(e^{-x-y}) - g'(e^{-x-y})e^{-x-y} + g''(e^{-x-y})e^{-2x-2y}\right] \triangleq a \tag{110}$$

and with respect to y

$$\frac{\partial^2 f(x,y)}{\partial y^2} = e^{y+\eta x}\left[\eta^2 g(e^{-x-y}) + (1-2\eta)g'(e^{-x-y})e^{-x-y} + g''(e^{-x-y})e^{-2x-2y}\right] \triangleq c. \tag{111}$$

Finally with respect to x and y, we have

$$\frac{\partial^2 f(x,y)}{\partial x \partial y} = e^{y+\eta x}\left[\eta g(e^{-x-y}) - \eta g'(e^{-x-y})e^{-x-y} + g''(e^{-x-y})e^{-2x-2y}\right] \triangleq b. \tag{112}$$

Therefore, the Hessian of $f(x,y)$ is given by

$$\mathbf{H} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

A 2×2 matrix of the form $\mathbf{H}$ with $a > 0$ is positive definite if and only if $\Delta = ac - b^2 > 0$ (Schur's complement condition [23]). Let $u = e^{-x-y} \geq 0$. Then for $u \geq 0$ and $\gamma_c \geq 0$, it is sufficient for its Hessian to be positive definite that the following conditions be satisfied for any $\gamma_c \geq 0$.

1) $a' = u^2 g''(u) - u g'(u) + g(u) > 0$.
2) $a'c' - (b')^2 > 0$, where $b' = \eta u g'(u) - u^2 g''(u) - n g(u)$ and $c' = u^2 g''(u) + \eta^2 g(u) + (1 - 2\eta) u g'(u)$.

■

For the case of AWGN channel it is sufficient to show that $g(u) = e^u - 1$ satisfies the conditions. For the first condition, we have

$$a' = u^2 e^u - u e^u + e^u - 1 > 0, \text{ for } u \geq 0 \tag{113}$$

since a' is increasing with $u \geq 0$ and $f_1(0) = 0$. For the second condition, we obtain $b' = (\eta - \eta u + u^2)e^u - \eta$ hence

$$a'c' - (b')^2 = (\eta - 1)^2 u e^u (e^u - u - 1) > 0 \text{ for } u \geq 0. \quad (114)$$

Therefore the function $E_{tot}(x,y)/N_0$ for AWGN channel is convex in $x, y \in \mathbf{R}$.

REFERENCES

[1] S. Verdú, “On channel capacity per unit cost,” *IEEE Trans. Inform. Theory*, vol. 36(5), pp. 1019-1030, Sept. 1990.

[2] V. Rodoplu and T. H. Meng, “Bits-per-Joule capacity of energy-limited wireless networks,” *IEEE Trans. Wireless Communications*, vol. 6(3), pp. 857-865, March 2007.

[3] S. Verdú, “Spectral efficiency in the wideband regime,” *IEEE Trans. Inform. Theory*, vol. 48(6), pp. 1319-1343, June 2002.

[4] E. Uysal-Biyikoglu and A. E. Gamal, “On adaptive transmission for energy efficiency in wireless data networks,” *IEEE Trans. Inform. Theory*, vol. 50(12), pp. 3081-3094, Dec. 2004.

[5] S. Cui, A. J. Goldsmith, and A. Bahai, “Energy-constrained modulation optimization,” *IEEE Trans. Wireless Communications*, vol. 4(5), pp. 2349-2360, Sep. 2005.

[6] T. Shu, M. Krunz, and S. Vrudhula, “Joint optimization of transmission power-time and bit energy efficiency in CDMA wireless sensor networks,” in *IEEE Trans. Wireless Communications*, vol. 5(3), pp. 3109-3118, Nov. 2006.

[7] S. Cui, R. Madan, and A. J. Goldsmith, and S. Lall, “Cross-layer energy and delay optimization in small-scale sensor networks,” *IEEE Trans. Wireless Communications*, vol. 6(10), pp. 3688-3699, Oct. 2007

[8] G. Kramer, M. Gastpar and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” in *IEEE Trans. Inform. Theory*, vol. 51(9), pp. 3037-3063, Sep. 2005.

[9] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, “Bandwidth and power efficient routing in linear wireless networks,” *Joint Special Issue of IEEE Trans. Inform. Theory and IEEE Trans. Networking*, vol. 52, pp. 2624-2633, June 2006.

[10] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, “On the optimum number of hops in linear wireless networks,” *Proc. IEEE Information Theory Workshop (ITW’04)*, Oct. 2004

[11] Ö. Oyman and S. Sandhu, “A Shannon-theoretic perspective on fading multihop networks,” in *Proc. Conf. Inform. Sci. and Syst.(CISS)*, 2006.

[12] Ö. Oyman and S. Sandhu, “Non-ergodic power-bandwidth tradeoff in linear multihop wireless networks,” in *Proc. IEEE International Symposium on Information Theory(ISIT’06)*, 2006.

[13] D. Chen, M. Haenggi, and J. N. Laneman, “Distributed spectrum-efficient routing algorithms in wireless networks,” in *Proc. Conf. Inform. Sci. and Syst.(CISS)*, 2007.

[14] W. E. Stark, H. Wang, A. Worthen, S. Lafortune, and D. Teneketzis “Low-energy wireless communication network design,” *IEEE Wireless Communications*, pp. 60-72, Aug. 2002.

[15] H. Wang, “Analysis and design of efficient wireless networks,” University of Michigan Ph.D. Thesis, 2004.

[16] C. Bae and W. E. Stark, “Energy and bandwidth efficiency in wireless networks,” in *Proc. International Conference on Communications, Circuits and Systems(ICCCAS’06)*, vol. 2, pp.1297-1302, June 2006.

[17] C. Bae and W. E. Stark, “A tradeoff between energy and bandwidth efficiency in wireless networks,” in *Proc. IEEE Military Communications Conference (MILCOM’07)*, vol. 2, pp.1297-1302, Nov. 2007.

[18] C. Bae and W. E. Stark, “Joint Optimization of Transmission Energy and Relay Location in Wireless Networks,” *in preparation*, 2008.

[19] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, “On the Lambert-W function,” in *Advances in Computational Mathematics*, vol. 5(1), pp. 329-359, 1996.

[20] N. Blaunstein and C. Christodoulou, *Radio propagation and adaptive antennas for wireless communication links*, John Wiley & Sons, Hoboken, NJ, USA, 2007

[21] T.S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, Englewood Cliffs, NJ, USA, 1996

[22] S. L. Howard, C. Schlegel, and K. Iniewski, “Error control coding in low-power wireless sensor networks:when is ECC energy-efficient?,” in *EURASIP Journals on Wireless Communication and Networking*, vol.2006, pp.1-14, 2006.

[23] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, Cambridge, U.K., 2004

[24] J. G. Proakis, *Digital Communications*, McGraw-Hill Higher Education, fourth edition, 2000

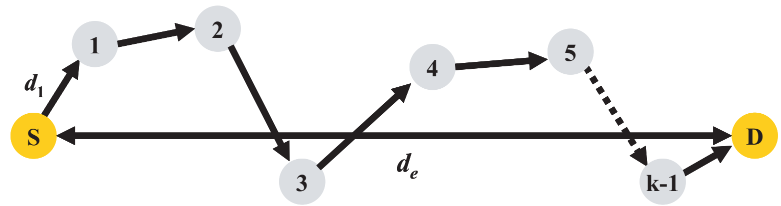


Fig. 1. Illustration of network model.

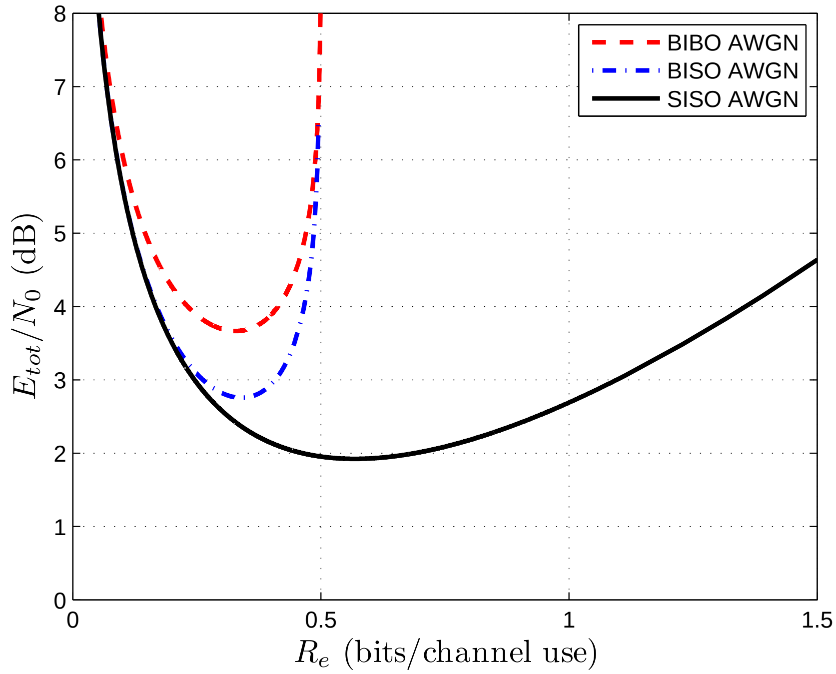


Fig. 2. Normalized energy-bandwidth characteristic for various AWGN channel models when $\gamma_c = -5.31$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

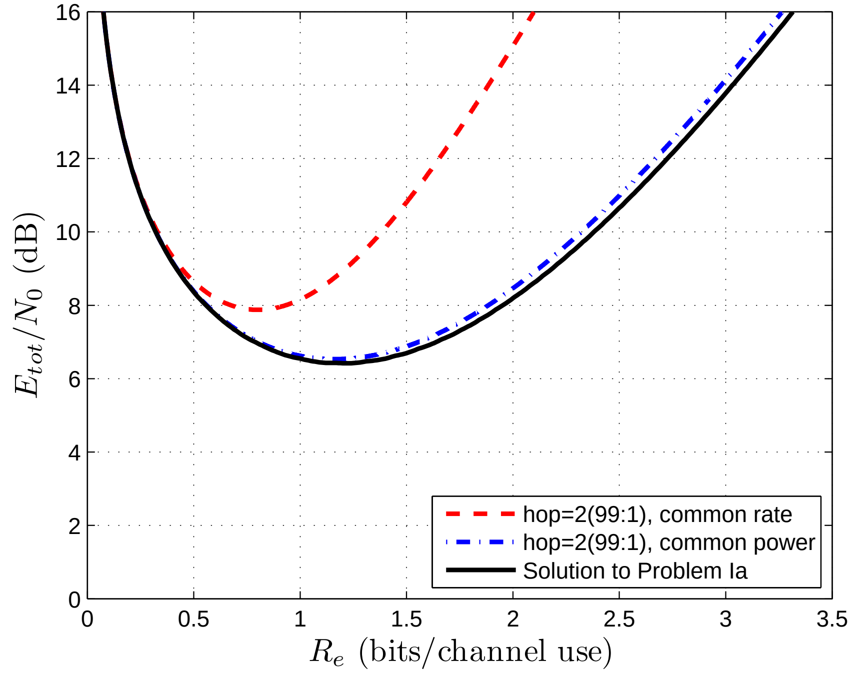


Fig. 3. Normalized energy-bandwidth characteristic when $\gamma_c = 4.69$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

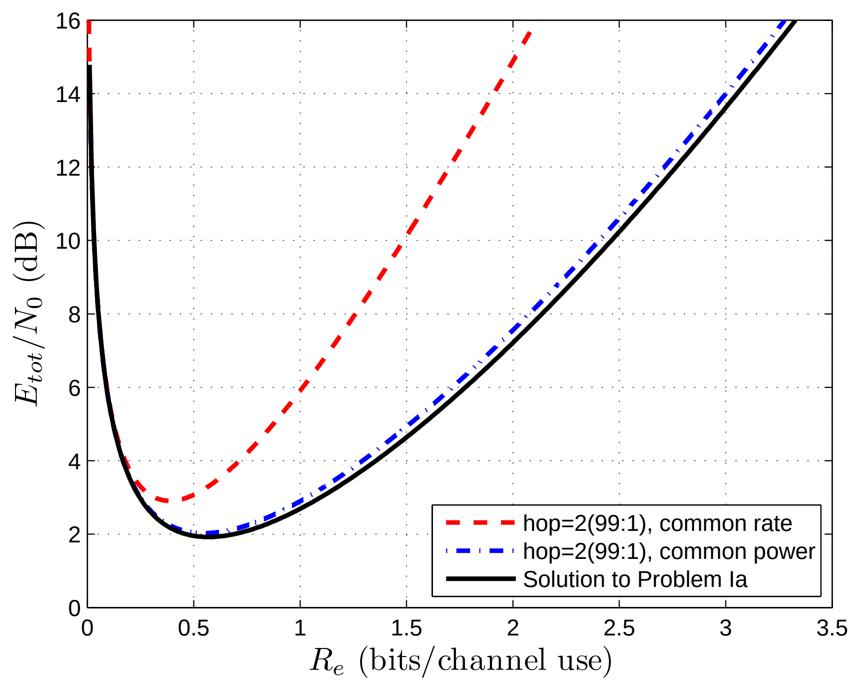


Fig. 4. Normalized energy-bandwidth characteristic when $\gamma_c = -5.31$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

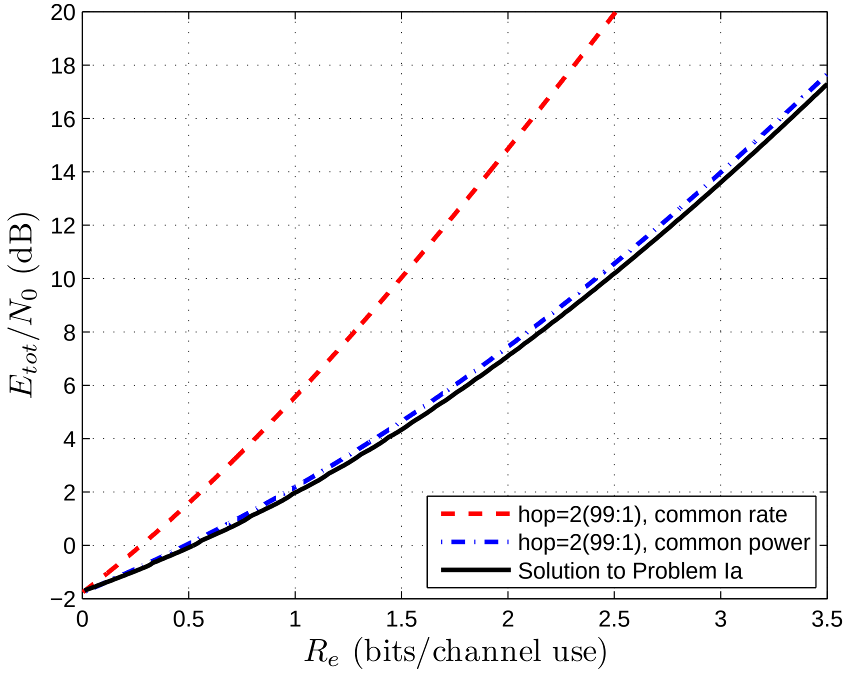
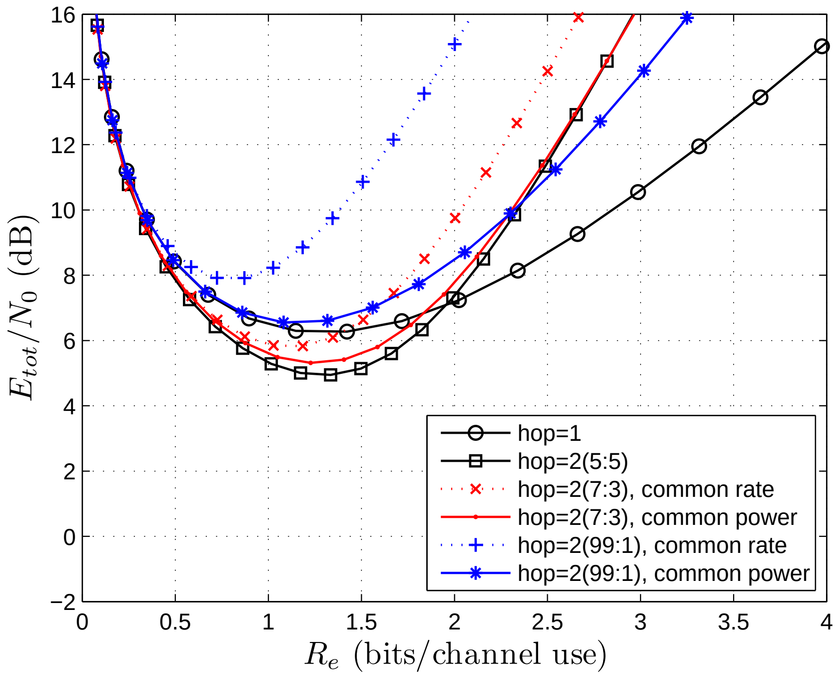


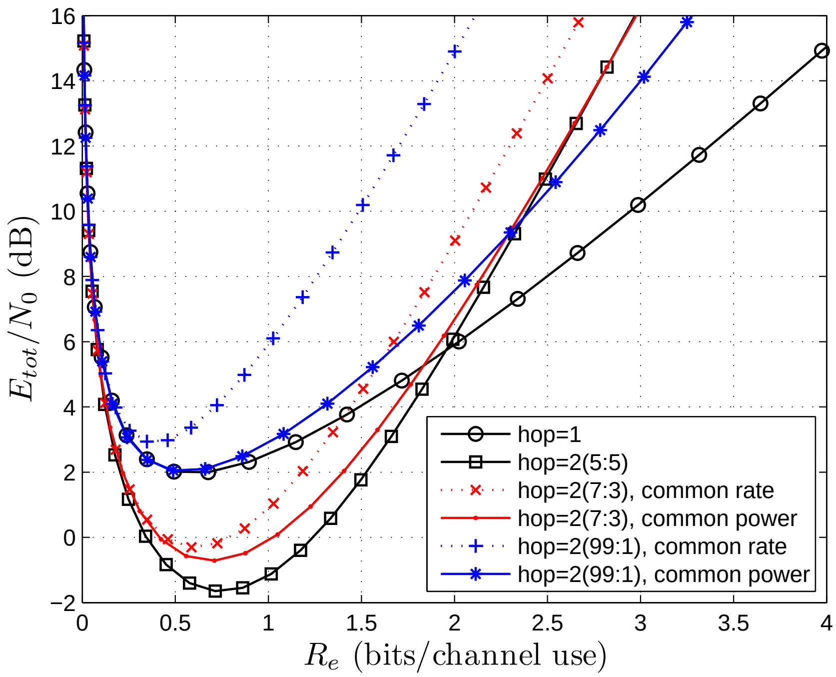
Fig. 5. Normalized energy-bandwidth characteristic when $\gamma_c = 0$ for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

TABLE I
OPTIMAL ENERGY CONSUMPTION CONSTANTS FOR VARIOUS AWGN CHANNEL

AWGN Channel type	Energy consumption constant			Loss (dB)		
	$\eta = 2$	$\eta = 3$	$\eta = 4$	$\eta = 2$	$\eta = 3$	$\eta = 4$
Soft input/Soft output	2.43	1.85	1.38	-	-	-
Binary input/Soft output	2.76	2.55	2.28	-0.55	-1.39	-2.18
Binary input/Binary output	3.26	2.82	2.44	-1.27	-1.83	-2.47



(a) Normalized total energy consumption per information bit vs end-to-end rate with $\gamma_c = 4.69$dB



(b) Normalized total energy consumption per information bit vs end-to-end rate with $\gamma_c = -5.31$dB

Fig. 6. Energy-bandwidth characteristic when $\gamma_c \neq 0$ with single hop ($k = 1$) and multi-hop ($k = 2$) for selected location of relays, $\alpha_1 : \alpha_2 = 5 : 5, 7 : 3$, and $99 : 1$ at d_e=3000m.

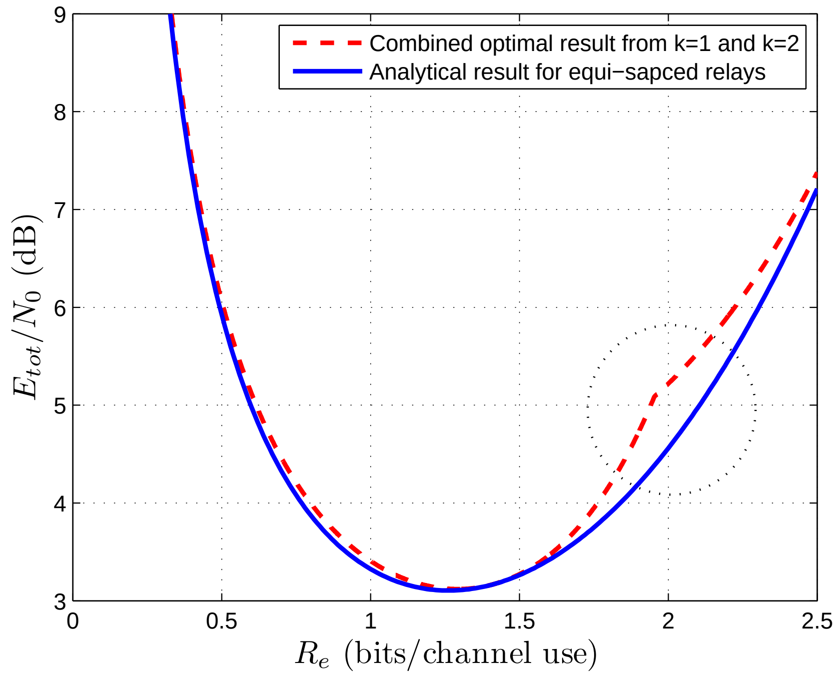


Fig. 7. Energy-bandwidth characteristic comparison between the numerical optimization and equi-spaced relays case.

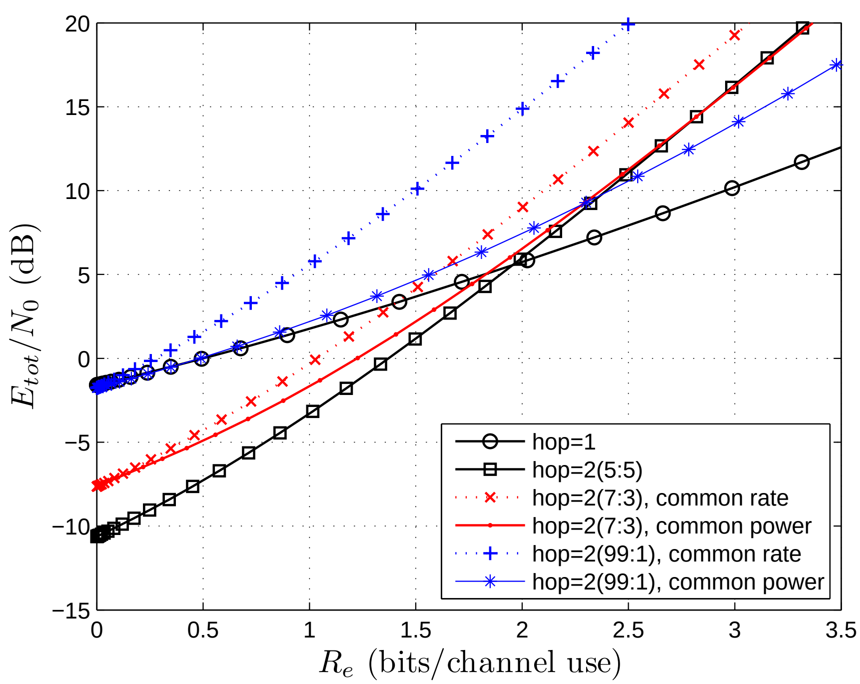


Fig. 8. Normalized energy-bandwidth characteristic when $\gamma_c = 0$ with $k = 1$ single hop and $k = 2$ multi-hops for selected location of relays, $\alpha_1 : \alpha_2 = 5 : 5, 7 : 3$, and $99 : 1$ at d_e=3000m.

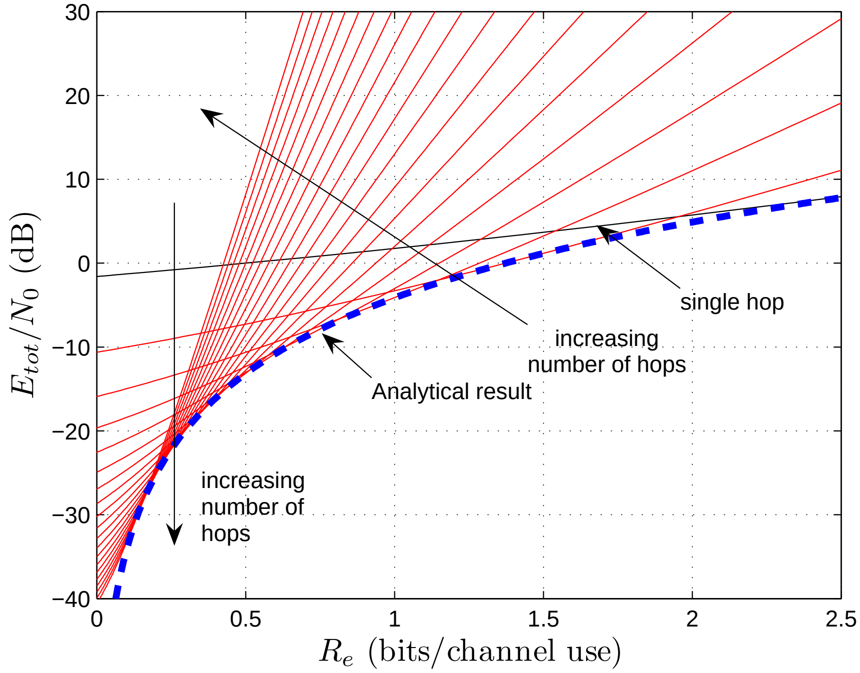


Fig. 9. Normalized energy-bandwidth characteristic when $\gamma_c = 0$ for equi-spaced relays with $k = 1, ..., 20$, and the analytical result in (69) at d_e=3000m.

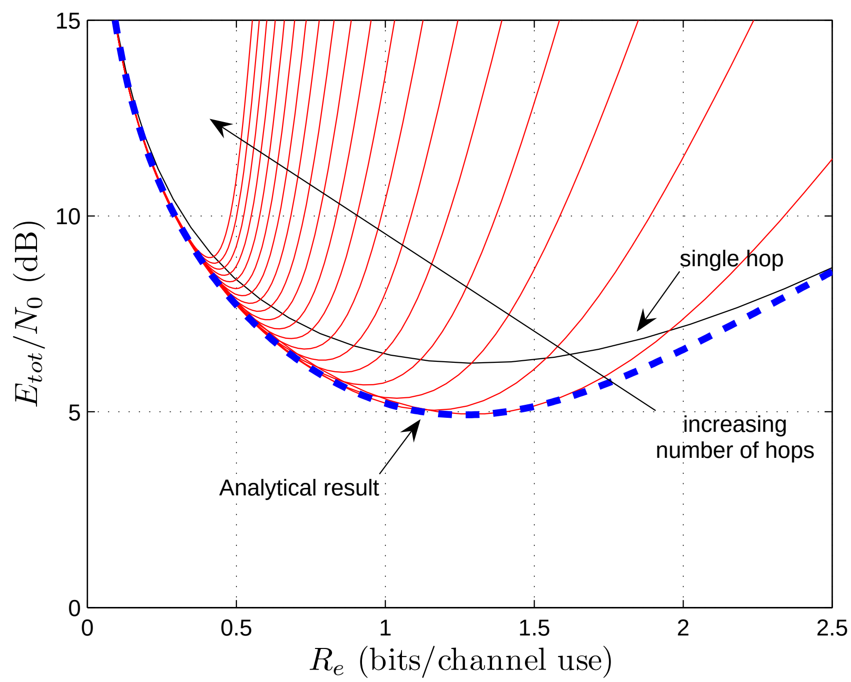


Fig. 10. Normalized energy-bandwidth characteristic when $\gamma_c = 4.69$dB for equi-spaced relays with $k = 1, ..., 20$, and the analytical result in (69) at d_e=3000m.

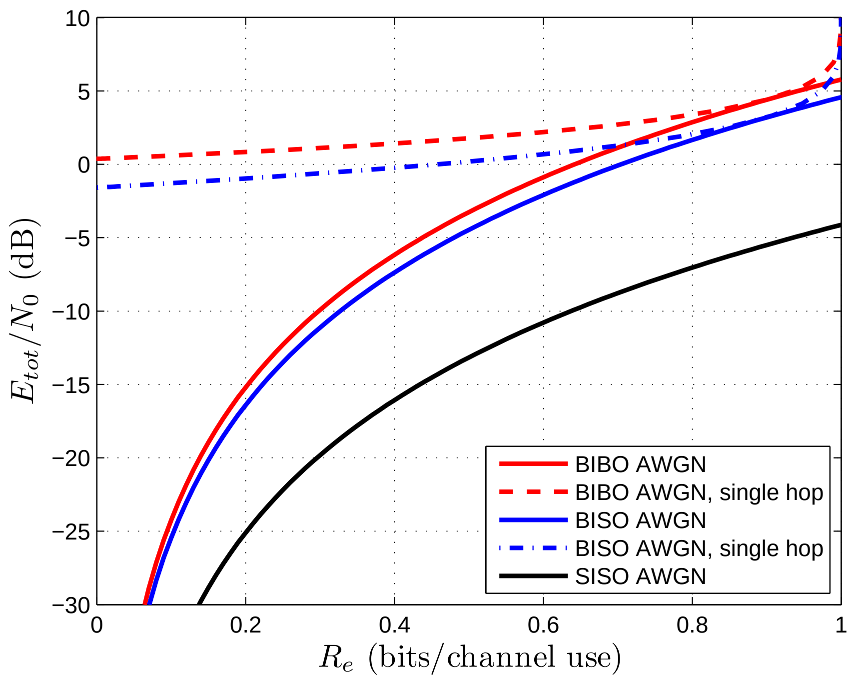


Fig. 11. Normalized energy-bandwidth characteristic of the analytical result in (69) with the single hop case when $\gamma_c = 0$ for various AWGN channel at d_e=3000m.

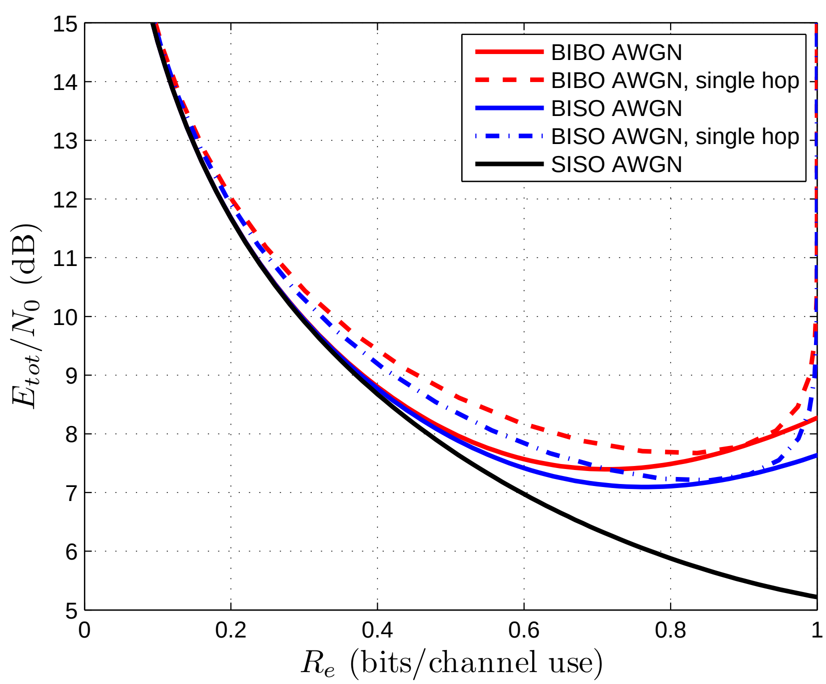


Fig. 12. Normalized energy-bandwidth characteristic of the analytical result in (69) with the single hop case when $\gamma_c = 4.69$dB for various AWGN channel at d_e=3000m.

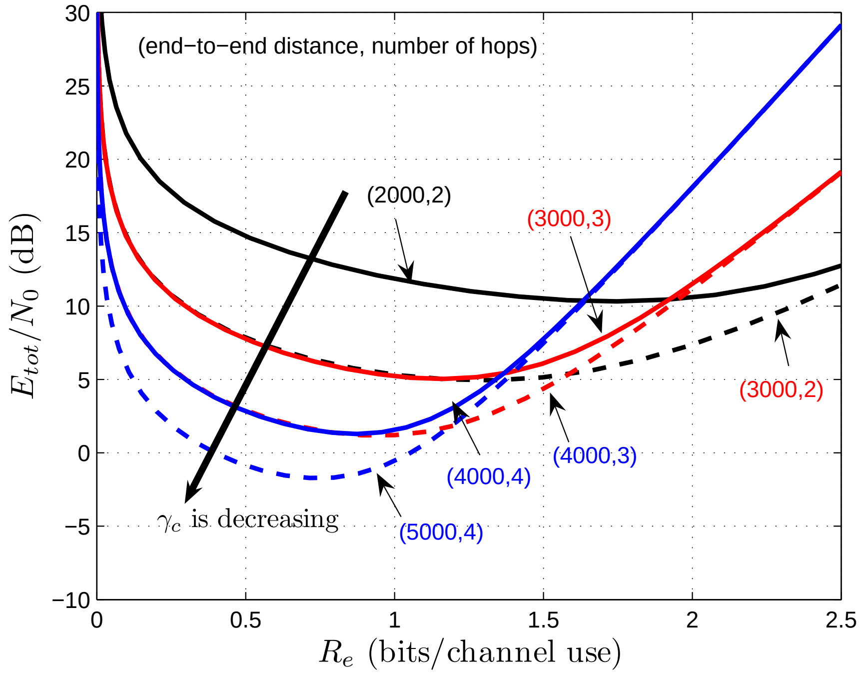


Fig. 13. Normalized energy-bandwidth characteristic of different end-to-end distance and number of hops for equi-spaced relays when $E_p = 0.95\mu$J/symbol.

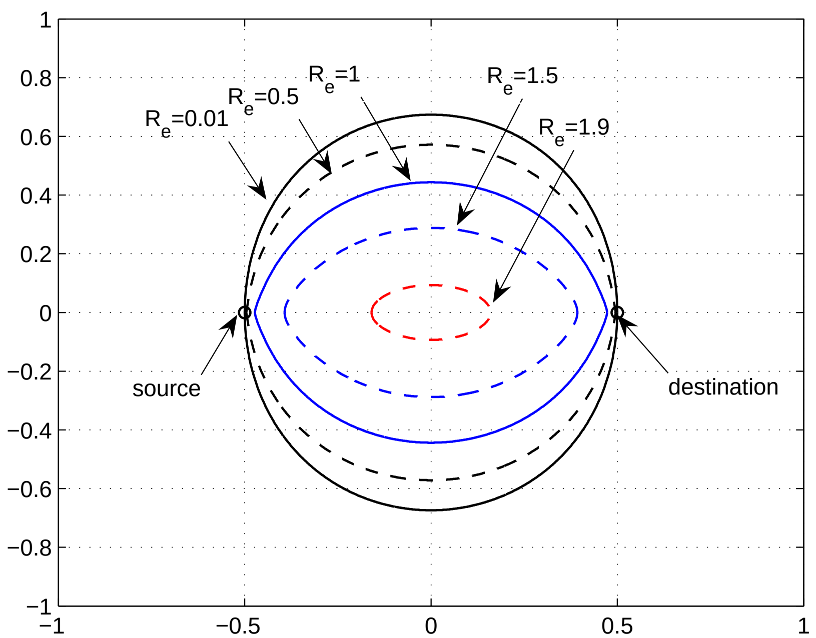


Fig. 14. Feasible region of the relay location where a two hop network is more energy efficient than a single hop network for selected end-to-end rates.

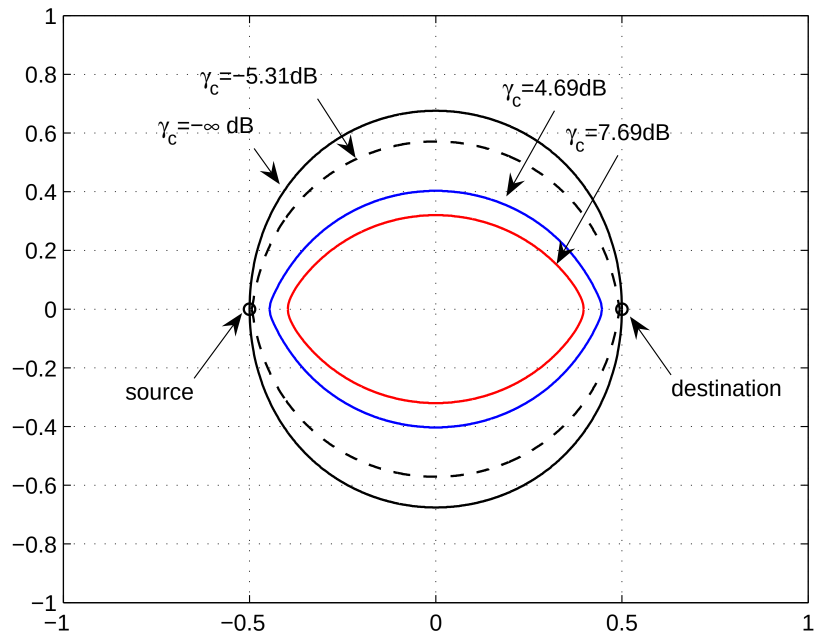


Fig. 15. Feasible region of the relay location where a two hop network is better than a single hop network in terms of the minimum total energy consumption at the optimal end-to-end rate for selected receiver circuit processing energy.